Phenomenology of charged leptons

1. Charged lepton flavor violation

With massive neutrinos and neutrino oscillation LFV is in principle possible at loop level:
\[
\begin{align*}
\nu^+ &\rightarrow e^+ \gamma \\
\nu^+ &\rightarrow e^+ e^+ e^- \quad \text{(additional suppression)}
\end{align*}
\]

The BR is suppressed by \(\frac{(\Delta m^2)^2}{M_W^4} \times 10^{-50}\) and is found to be \(\lesssim 10^{-54}\) in SM, i.e. it is effectively forbidden.

However SUSY model like many other "beyond SM" (BSM) physics models introduce naturally LFV and can enhance the BR \(\nu \rightarrow e \gamma\) to \(\mathcal{O}(10^{-12})\):

\[
\begin{align*}
\nu^+ &\rightarrow e^+
\end{align*}
\]

\(\rightarrow\) search for LFV in charged lepton decay is an excellent probe for NP.

\[
\begin{align*}
\nu &\rightarrow e^+ \gamma \\
N^+ &\rightarrow e^-N \\
\nu^+ &\rightarrow e^+ e^- e^-
\end{align*}
\]

Ways to search for LFV in neutrino decays: 

\[
\begin{align*}
\nu &\rightarrow e^+ \\
\nu &\rightarrow e^+ e^-
\end{align*}
\]
Kehematically:

2 body - decays:
- "2-body decay": 3-body decay, mono-energetic $\gamma, e$.
- Back-to-back.

Background:
- accidental backgrounds
- $\nu$ decay in orbit
- antiprotons
- proton decay
- continuous $\nu$ beam
- pulsed $\nu$ beam
- continuous $\mu$ beam

Expts: MEG/MEGII
- Mue2e/Termilab
- Mu3e (Sindriu)

1. $\nu \rightarrow e\gamma$: MEG/MEGII experiments

Requires a high intensity $\mu$ beam stopped inside detector.

PSI (Paul Scherrer Institute, Villigen, Switzerland) $\mu$ beam.

28 MeV/c, $\mu^+$ beam with intensity of up to $10^{9}/\text{sec}$ (1.2 MW proton).

Discussions ongoing to build new beamline of $\sim 10^{9}/\text{sec}$.

Principal:
- Muon beam is stopped on target inside detector.
- Back-to-back decay; detector reflects $\nu$ topology.

See Fig: MEG detector.

Signature:

$e^{-} \rightarrow e^{-}$

Correlated background:

$e^{-} \rightarrow e^{-}$ from different source (e.g., from annihilation).

Accidental Bckgr:

$\nu \rightarrow \bar{\nu}$
4.3 \( \mu \rightarrow 3e \) : Mu3e at PSI (SINDRUM is the past)

Signal topology

\[ \sum E_i = m_\mu \]
\[ \sum P_i = 0 \]

Backgrounds: radioactive decay with conversion

\[ E_{tot} = \sum E_i + m_\mu \]
\[ Z \rightarrow p_i/\gamma \]

If one wants to set

BR limit of \( O(10^{-13}) \)

one needs energy resolution better than \( 1 MeV \)!!

Additional Background: accidents

\( \approx \) overlay of two ordinary \( \mu \rightarrow e \) decay with another (fake) electron (\( e^- \))

- excellent vertex resolution
- poor resolution

Fig of Mu3e detector principle

Ultimate goals: 

\[ \text{BR} (\mu \rightarrow 3e) < 10^{-14} \]  (Phase 1)
\[ \text{Mu3e} < 10^{-16} \]  (Phase 2)

Sindram (1988) \( < 1 \times 10^{-12} \)

ME6 - Results:

\[ \mu^+ \text{ stopped (2009-13): } 7.5 \times 10^{14} \rightarrow \text{expected sensitivity: } 5 \times 10^{-13} \]

\[ \operatorname{BR}(\mu^+ \rightarrow e^+ \gamma) < 4.2 \times 10^{-13} \quad \text{cred: } 1605.05081 \]

\( \text{Eqn 1, p14, Sec. 1.3} \)

ME6 II Upgrade aims to improve upper limit by one order of magnitude

- Larger detector acceptance (large Li:Ce calor)
- Unique volume drift chamber
- Higher beam intensities

1.2 \( \mu^- N^+ \rightarrow e^- N \) : Mu2e (Fermilab), Comet at J-PARC

Stopps \( \mu^+ \) on a Al target:

![Diagram of decay path](image)


\text{Nuon is trapped by Al nuclei}\]

- \( \mu^- \) orbit is much smaller than Al-\( e^- \) orbit

- Monoenergetic electron:
  \( E_e = m_p - E_\mu - E_{\text{esc}} \approx 109.97 \text{ MeV} \)

Best limits so far from SINDRUM II (\( \mu^- \)):

\[ R_\mu e = \frac{N^- N^+ \rightarrow e^- N^+}{N^- N^+ \rightarrow \text{capture}} < 7 \times 10^{-13} \]

Expected future limits: Al: \( R_\mu e < 10^{-14} \) (\( \mu^- \) upgrades < \( 10^{-15} \))
Tau pairs are abundantly produced at the $e^+e^-$ B factories.

$$ e^+e^- \rightarrow \tau\tau \quad \text{with} \quad \sigma' (\sqrt{s} = 104.6 GeV) = 4 \text{nb} = 0.6\text{ nb} $$

$\tau \rightarrow 3\mu$

Signature: $\tau \rightarrow 3\mu$ is an easy to trigger and to reconstruct.

Signatures:

$$ BR < 2 (4) \times 10^{-9} \quad \text{(Belle/BaBar)} $$

- Prospects for SuperB: $BR < 10^{-9}$ ($\sim 9 \times 10^8 \text{mc at Belle}$)

- $\tau \rightarrow \mu \gamma$: \quad Belle $< 4.5 \times 10^{-8}$

- Prospects for SuperB (Belle II): $BR < 5 \times 10^{-8}$

Remark: LHCB has also projected a search for $\tau \rightarrow 3\nu$ in Run-I data.

$$ BR < 4.6 \times 10^{-8} $$

(result is limited by backgrounds -> difficult to improve)
High-order corrections to the photon-electron vertex lead to deviations of the magnetic moment $\tilde{\mu} = g \mu_B \frac{e}{m_e}$ from the Dirac prediction $(g = 2)$:

\[ \frac{g}{2} = 1 + a_W + a_H + a_E + a_{\mu} \]

$\sim \frac{m_e^2}{M^2}$: not important for the electron, irrelevant for $\mu$.

**Hadronic vacuum polarization:** (important for precise calculation of $(g-2)_\mu$)

Cannot be calculated by first principles; instead one relies on the dispersion relation:

\[ \text{Im} [\Pi] \sim \int \frac{K(s) \text{Re} \Pi(s)}{s} ds \]

or

\[ a_H = \frac{2}{3} \left( \frac{\alpha}{\pi} \right)^2 \int \frac{K(s) \text{Re} \Pi (s)}{s} ds \]

$\Pi(s)$ from Höcker (e$^+e^-$ → $\mu^+\mu^-$).

Thus, the experimental determination of $\tilde{\mu}$ and integrate numerically. Fig. 6 from Höcker (e$^+e^-$ → $\mu^+\mu^-$).

Different groups have performed the numerical determination of $\tilde{\mu}$.

Experimental program to include Hadron contribution. Alternatively: $e^+e^- \rightarrow$ Hadrons.
2.1 9-2 of the electron (Gabrielse et al., 2006, 2008)

Experimental method: Quantum cyclotron

- bind a single electron in a band of 
  "artificial atom" made up by a Penning trap,
  put into a strong external magnetic field.

Cooling down "artificial atom" at ~70 mK -> only certain energy levels
and circular cyclotron radius are allowed -> quantum cyclotron
with discrete ladder of energy levels spaced by $h\omega_c$ ($\omega_c$. cyclotron)
Energy levels also depend on spin - different for spin 0 and 1.$^*$

A flip of spin $\frac{1}{2}$ to $\frac{3}{2}$ -> cyclotron levels measured by hfs.

$$g = \frac{3}{2} \cdot \frac{h}{m_c} \quad \text{(Spin rotation frequency)}$$

Measurement:

In addition to the cyclotron and the betatron motion (see Fig.)
the electron also performs an axial oscillation inside the cavity of the trap - $f_a$: axial motion depends on $\mu$ and spin angular (because of the coupling)

One measures $f_c$ and $f_a = f_c - f_c$ to determine $g_f$: $g_f = 1 + \frac{f_a}{f_c}$

- $f_a$ is measured using the so-called quantum-jump spectroscopy:
electron in lowest energy level + microwave photon twist -> Asterisk.

- $f_a$ in upper $\quad$ ground state ($n=0$) $\quad$ transition $\quad$ spin $\quad$ flip

after the spontaneous transition to $n=1$ check whether spin has flipped
(Shuttled transition)
2.2 $g-2$ of the muon

Measurement principle:
- Store polarized muon in a storage ring with magnetic dipole fields
- Muon revolution with cyclotron frequency $\omega_c = \frac{eB}{2mc}$
- Measure spin precession frequency $\omega_s = \frac{g\mu_B B}{2mc}$

$\Rightarrow \omega_s = \omega_s - \omega_c$

with

$$\omega_s = -\frac{e}{mc} \left[ \frac{1}{\mu_B} B - \frac{1}{\gamma^2 - 1} \beta \times \vec{E} \right]$$

$\mu_B$: magnetic moment

Effect of electrical

Storage ring, focusing fields

Magnetic monopole: $\mu_B = \frac{1}{\gamma^2 - 1} \Rightarrow \mu_B = 0$

$\Rightarrow \mu_B = 0.00116 5957 (12)$

First measurement CERN 1970s:

- $\mu_- = 0.00116 5959 (14)$

Muonic production using pion decay:

- Forward going muon a
  - Oppositely polarized

Measurement of polarization at decay:

$\nu \rightarrow \mu^+ \rightarrow e^+ e^- \rightarrow \nu_e$

$\nu_e \rightarrow e^+ e^- \rightarrow \nu_e$

$\Rightarrow \nu$ helicity forced the electron to be in the direction of the polarization.
Measurement of $\omega_a$; see Fig.

\[ \alpha_p = \frac{e}{\hbar} \langle B \rangle \]

depends on the effective $\langle B \rangle$ field.

\[ \Rightarrow B \text{-field measured using NMR with magnetic resonance} \]

frequency of the proton $\omega_p \Rightarrow \langle B \rangle = \frac{h \omega_p}{2 \mu_p} \]

\[ \Rightarrow \text{reduces to the measurement of a ratio of frequencies,} \]

\[ \alpha_p = \frac{\omega_a}{\omega_p} \]

\[ \frac{N_p / \mu_p - \omega_a / \omega_p}{\text{ratio of magnetic moments}} \]

\[ \Rightarrow \text{measured via the ground state hyperfine structure of muonium:} \]

\(\Delta V \text{ known to } \pm 12 \text{ ppb} \Rightarrow \frac{N_p / \mu_p}{\text{to } \pm 120 \text{ ppb}}\)

\[ \alpha_p = 1.658208 \times 10^{-10} \quad (\pm 500 \text{ ppb}) \]

\[ \Rightarrow \approx 3.5\sigma \text{ deviation to the EM prediction.} \]

- experimental/theoretical problem?
- contributions from "new physics"?

New Fermilab $g-2$ experiment; $\pm 540 \text{ ppm} \Rightarrow 140 \text{ ppm}$

(If deviation will stay, effect becomes of "54\sigma" effect)

Should start in the course of this year.