Problem Set 1

discussion on October 31, 2019

Problem 1.1  Velocity dispersions in the Coma cluster

The first indication for dark matter was found by Fritz Zwicky in 1933.\(^1\) He applied the virial theorem to the Coma cluster to determine its mass from the dispersion of radial velocities of galaxies in the cluster. The virial theorem relates the total kinetic energy \(T\) and the total potential energy \(U\) of a cluster via

\[
2\langle T \rangle = -\langle U \rangle,
\]

where the brackets denote the average over time.

a) Derive a general expression for the total mass \(M\) of a cluster of \(N\) galaxies as a function of the velocity dispersion \(\langle v^2 \rangle\). You can assume that all galaxies have the same mass \(m\).

For the Coma cluster, Zwicky observed about 1000 galaxies with an average distance of \(\langle r \rangle = 1.9 \times 10^{22}\) m and a velocity dispersion of \(\langle v^2 \rangle = 5 \times 10^{11} \left(\frac{m}{\text{s}}\right)^2\). The average luminosity per galaxy is \(L = 8.5 \times 10^7 L_{\text{sun}}\), where \(L_{\text{sun}}\) is the solar luminosity.

b) Derive the mass-to-light ratio for the Coma cluster. Compare your result with the expected ratio of \(3M_{\text{sun}}/L_{\text{sun}}\), where \(M_{\text{sun}}\) is the mass of the sun. What does it tell you?

Problem 1.2  Alternatives to particle dark matter

In this course we focus on the hypothesis that dark matter is some kind of new elementary particles. But this is not the only hypothesis we pursue today.

a) Consult the internet (or your favorite source of information) for alternative explanations of the cosmological and astrophysical observations we discussed in the lecture. Make a brief summary.

b) What are the strong and weak points of each alternative you could find? How well can they explain the various observations (galactic rotation curves, gravitational lensing in galaxy clusters, cosmic microwave background, structure formation)?