Problem Set 2

discussion on November 25, 2019

Problem 2.1 Re却没有 abundance of neutrinos

From observations like neutrino oscillations and tritium beta decay, we know that neutrinos are very light particles with masses of $\mathcal{O}(\text{eV})$. Since their mass is much smaller than the decoupling temperature $T_\nu \approx 0.8 \text{ MeV}$, they are relativistic at freeze-out.

a) Calculate the yield for one species of relativistic neutrinos in thermal equilibrium at $T \gtrsim T_\nu$,

$$Y_{\text{eq}}(T) = \frac{n_{\nu,\text{eq}}(T)}{s(T)}.$$  

Here $n_{\nu,\text{eq}}$ is the number density of neutrinos in thermal equilibrium and $s(T)$ is the entropy density of the universe at temperature $T$.

b) After decoupling, neutrinos become non-relativistic as the temperature decreases. Calculate the energy density of neutrinos today, $\rho_{\nu,0}$, as a function of the photon number density, $n_{\gamma,0}$.

c) Use your result from b) to express the relic neutrino density, $\Omega_\nu h^2$, as a function of the photon density, $\Omega_\gamma h^2 = 2.45 \times 10^{-5}$, and the temperature of the cosmic microwave background, $T_{\gamma,0}$. Show that

$$\Omega_\nu h^2 = \frac{m_\nu}{94 \text{eV}}.$$  

d) Assuming that the neutrino density should not overclose the universe, $\Omega_\nu h^2 < 1$, derive an upper bound on the neutrino mass.

Problem 2.2 Freeze-in

In case dark matter never reaches thermal equilibrium with the photon bath, the relic abundance cannot be determined from freeze-out. Instead, a dark matter density can be created from the annihilation of particles in thermal equilibrium. This mechanism is called freeze-in.\footnote{L. Hall et al., https://arxiv.org/abs/0911.1120.}

For simplicity, let us assume that dark matter pairs are produced from electron-positron annihilation

$$e^+ e^- \rightarrow \chi \chi$$

by exchanging a light mediator particle $\eta$ with mass $m_\eta \ll m_e, m_\chi$ and couplings $g_e$ and $g_\chi$ to electrons and dark matter, respectively. The velocity-averaged annihilation cross section at a temperature $T$ is roughly given by

$$\langle \sigma v \rangle \simeq \frac{g_e^2 g_\chi^2}{T^2}.$$
The dark matter number density $n_\chi$ is then obtained as

$$n_\chi \simeq \frac{n_e^2 \langle \sigma v \rangle}{H},$$

where $H$ is the Hubble parameter and $n_e$ is the electron number density.

a) Calculate the yield of dark matter, $Y(T)$, as a function of temperature.

Dark matter production stops, i.e., dark matter freezes in,

i) if the electrons decouple from the thermal bath around $T_{fi} \approx m_e$,

ii) or if dark matter is too heavy to be produced, around $T_{fi} \approx m_\chi$.

b) Calculate the relic dark matter abundance $\Omega_\chi h^2$ for $m_\chi < m_e$ and $m_\chi > m_e$.

You can assume that the yield today is well approximated by the yield at the freeze-in temperature, $Y_0 \approx Y(T_{fi})$.

c) From the condition that dark matter should not overclose the universe, derive an upper bound on the product of couplings, $g_e g_\chi$, as a function of the dark matter mass $m_\chi$. Distinguish between the two cases $m_\chi < m_e$ and $m_\chi > m_e$. 
