Problem 3.1  Neutralino dark matter

Supersymmetry (SUSY) is a symmetry between bosons and fermions. In SUSY theories each boson of the standard model has a fermionic supersymmetric partner, and each standard-model fermion has a bosonic partner with the same gauge quantum numbers.

The superpartners of the charged weak gauge bosons are called charginos; the superpartners of the neutral weak gauge bosons and the Higgs boson are called neutralinos. There are 2 charginos, $\chi^+ \text{ and } \chi^-$, and 4 neutralinos $\chi_i^0$, with $i = 1, \ldots, 4$ and masses $m_1 < m_2 < m_3 < m_4$. In the minimal supersymmetric standard model, all superpartners couple in pairs to standard-model particles.

a) Write down the Feynman diagrams for all possible couplings of charginos and neutralinos to the $W$, $Z$ and Higgs bosons. They should be invariant under Lorentz transformations and neutral with respect to electromagnetism.

b) What conditions does a superpartner need to fulfill to be a viable dark matter candidate? Which of the superpartners a-priori fulfill these conditions?

c) Assume that the relic abundance of your dark matter candidate(s) from b) is generated through thermal freeze-out. Write down the Feynman diagrams for dark matter annihilation, based on the couplings you found in a). Can you say something about the masses and couplings of the involved particles?

Problem 3.2  Sterile neutrinos

In 1993, Scott Dodelson and Lawrence M. Widrow proposed that dark matter could be explained by sterile neutrinos:  

1By far the simplest dark matter candidate, at least from the point of view of particle physics, is the neutrino. Massive neutrinos require only the addition of right-handed or sterile neutrino fields to the standard model.

The sterile neutrino couples to the standard-model neutrino through the Lagrangian

$$L = -y \bar{\nu}_L N_R (h^0 + v) + M N_R C N_R + h.c.$$  \hspace{1cm} (1)

The interaction eigenstates $\nu = |\nu_a\rangle$ and $N = |\nu_b\rangle$ mix through the Yukawa term $y \bar{\nu}_L N_R$ into mass eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$. The mixing can be parametrized by an angle $\theta \sim yv/M$, so that

$$\begin{pmatrix} |\nu_a\rangle \\ |\nu_b\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} \equiv U \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}. \hspace{1cm} (2)$$

1If you are interested in the details, have a look at Section 8.2 in Stephen Martin’s SUSY primer on https://arxiv.org/abs/hep-ph/9709356.
3Here $C = i\gamma_0\gamma_2$ is the charge conjugation matrix in Lorentz space, but this is not so important for us here.
An efficient way to produce sterile neutrinos is via oscillations $\nu_L \rightarrow N_R$.

a) Assume that the neutrino is in its interaction eigenstate $|\nu_a\rangle$ at time $t_0 = 0$. The time evolution into a state $|\nu_a(t)\rangle$ at time $t$ can be written as

$$
|\nu_a(t)\rangle = \sum_{k=1,2} U_{ak} \exp(-iE_k t)|\nu_k\rangle, \tag{3}
$$

where $E_k$ is the energy of the mass eigenstate $|\nu_k\rangle$.

Calculate a general expression for the probability to find a neutrino in interaction state $|\nu_b\rangle$ at time $t$,

$$
P(\nu_a \rightarrow \nu_b, t) = |\langle \nu_b | \nu_a(t) \rangle|^2. \tag{4}
$$

b) In the ultra-relativistic limit, we can approximate the energies of the neutrino mass eigenstates by $E_k \approx p + m_k^2 / 2p$, where $p$ is the momentum of each neutrino.

Show that the oscillation probability for a standard-model neutrino $\nu_L$ into a sterile neutrino $N_R$ is given by

$$
P(\nu_L \rightarrow N_R, t) = \sin^2(2\theta) \sin^2 \left( \frac{\Delta m^2}{4p} t \right), \quad \Delta m^2 = m_2^2 - m_1^2. \tag{5}
$$

c) Can you think of alternative ways to produce sterile-neutrino dark matter? \[4\]

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