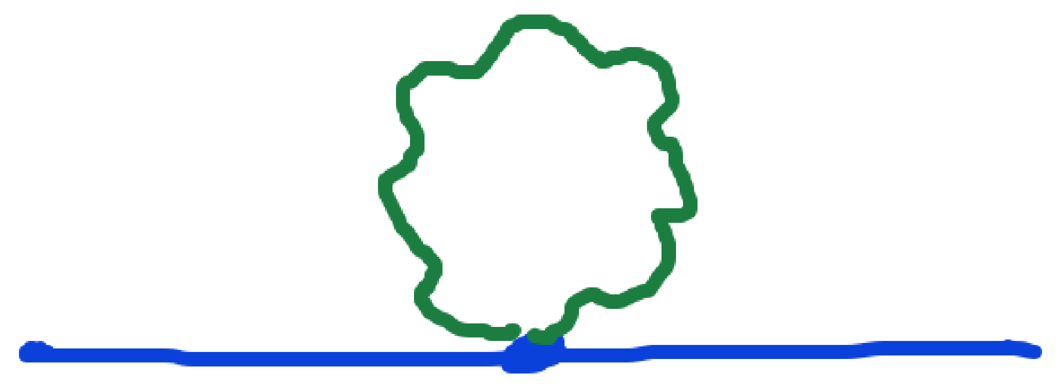
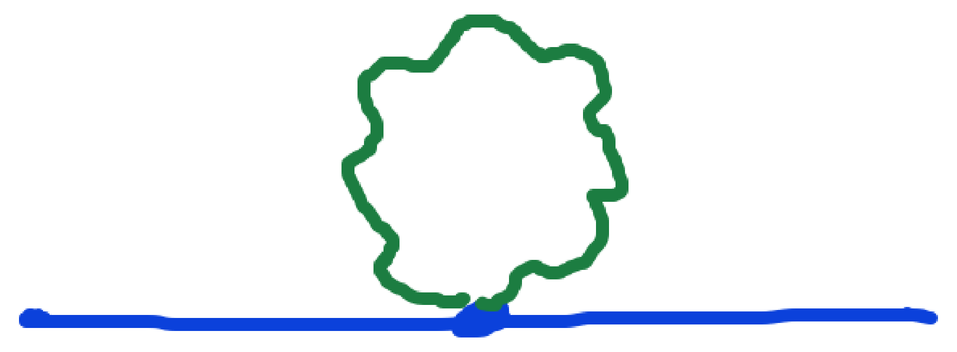
What is Quantum Gravity ?



???

- Quantisation of space ?
- No time ?
- Wave function of the Universe ?
- Change of quantum mechanics ?

Fluctuations of metric field matter



Quantum gravity needs method to take them into account

Quantum gravity

- Gravity is field theory. Similar to electrodynamics. Metric field.
- Gravity is gauge theory. Similar to QED or QCD. Gauge symmetry: general coordinate transformations (diffeomorphisms)
- Quantum gravity: include metric fluctuations in functional integral

Quantum gravity

- Quantum gravity is similar to other quantum field theories
- Difference: metric is tensor, gauge bosons are vectors (vierbein, spin connection : vectors)
- Difference: Quantum (Einstein-) gravity is not perturbatively renormalizable
- no small dimensionless coupling constant, effective coupling q^2/M^2

Strength of gravity

Characteristic mass scale:

Planck mass $M=2.4\ 10^{18}$ GeV , $M^{-1}=8\ 10^{-33}$ cm

Proton mass 1 GeV , LHC energy 10^3 GeV

- Strength of gravity q^2/M^2
- Very weak interaction between particles on energy scales of everyday life – much weaker than electromagnetic forces
- Important due to collective effect : many atoms
- Interaction becomes strong for $q^2 >> M^2$

Quantum gravity

Quantum gravity is

not perturbatively renormalizable

Asymptotic safety : non-perturbative renormalizabilty

Weinberg, Reuter, …

Use functional renormalization !

Fields

Fields

Fields have a value for

every position in space (x,y,z)

and for every time t

example:

electric field

$$\vec{E}(t, x, y, z)$$

$$\vec{E} = (E_x, E_y, E_z)$$

magnetic field

$$\vec{B}(t, x, y, z)$$

$$\vec{B} = (B_x, B_y, B_z)$$

Gravitational potential : field

$$\Phi(t,x,y,z)$$

- For every moment in time t , and for every position(x,y,z) the gravitational potential has a value Φ

metric field

$$g_{\mu\nu} : \begin{pmatrix} g_{00} & , & g_{01} & , & g_{02} & , & g_{03} \\ g_{10} & , & g_{11} & , & g_{12} & , & g_{13} \\ g_{20} & , & g_{21} & , & g_{22} & , & g_{23} \\ g_{30} & , & g_{31} & , & g_{32} & , & g_{33} \end{pmatrix}$$

$$g_{\mu\nu} = g_{\nu\mu}$$

Newton – approximation : gravitational potential

$$g_{\mu\nu} : \begin{pmatrix} -(1+2\phi) & , & 0 & , & 0 & , & 0 \\ 0 & , & 1 & , & 0 & , & 0 \\ 0 & , & 0 & , & 1 & , & 0 \\ 0 & , & 0 & , & 0 & , & 1 \end{pmatrix}$$

action

- the action is a functional of fields
- it associates to every field (- configuration) a value $S[\ \Phi\ ] = S[\ \Phi(t,x,y,z)\ ]$
- variation with respect to the fields yields field equations (functional derivative)
- symmetry: S is invariant under a particular field transformation

Maxwell action

$$S = \int d^4x \sqrt{g} \, \frac{Z_F}{4} F^{\mu\nu} F_{\mu\nu}$$

A_μ: gauge field

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$g = \det(g_{\mu\nu})$$

$$F^{\mu\nu} = g^{\mu\rho} g^{\nu\sigma} F_{\rho\sigma}$$

inverse metric $g^{\mu\nu}$

$$\sum_{\rho=0}^{3} g_{\mu\rho} g^{\rho\nu} = \delta_\mu^\nu = \begin{cases} 1 & \text{wenn } \mu = \nu \\ 0 & \text{sonst} \end{cases}$$

For $g_{\mu\nu} = \eta_{\mu\nu}$ field equations derived from S are Maxwell equations

Einstein Hilbert action

$$S = \int d^4x \sqrt{g} \left\{ -\frac{M^2}{2} R + \frac{Z_F}{4} F^{\mu\nu} F_{\mu\nu} + \dots \right\}$$

- Field equations derived from Einstein – Hilbert action are the Einstein equations

Riemann tensor and curvature scalar

$$R_{\mu\nu} = \sum_{\rho=0}^{3} \left(\frac{\partial \Gamma_{\mu\nu}{}^{\rho}}{\partial x^{\rho}} - \frac{\partial \Gamma_{\nu\rho}{}^{\rho}}{\partial x^{\mu}} \right) + \sum_{\rho=0}^{3} \sum_{\tau=0}^{3} \left(\Gamma_{\mu\nu}{}^{\tau} \Gamma_{\tau\rho}{}^{\rho} - \Gamma_{\mu\rho}{}^{\tau} \Gamma_{\nu\tau}{}^{\rho} \right)$$

Riemann tensor

$$\Gamma_{\mu\nu}{}^{\rho} = \frac{1}{2} \sum_{\tau=0}^{3} g^{\rho\tau} \left(\frac{\partial g_{\nu\tau}}{\partial x^{\mu}} + \frac{\partial g_{\mu\tau}}{\partial x^{\nu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\tau}} \right)$$

connection

inverse metric $g^{\mu\nu}$

$$\sum_{\rho=0}^{3} g_{\mu\rho} g^{\rho\nu} = \delta_{\mu}^{\nu} = \begin{cases} 1 & \text{wenn } \mu = \nu \\ 0 & \text{sonst} \end{cases}$$

$$R = \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} g^{\mu\nu} R_{\mu\nu}$$

curvature scalar

Einstein equation

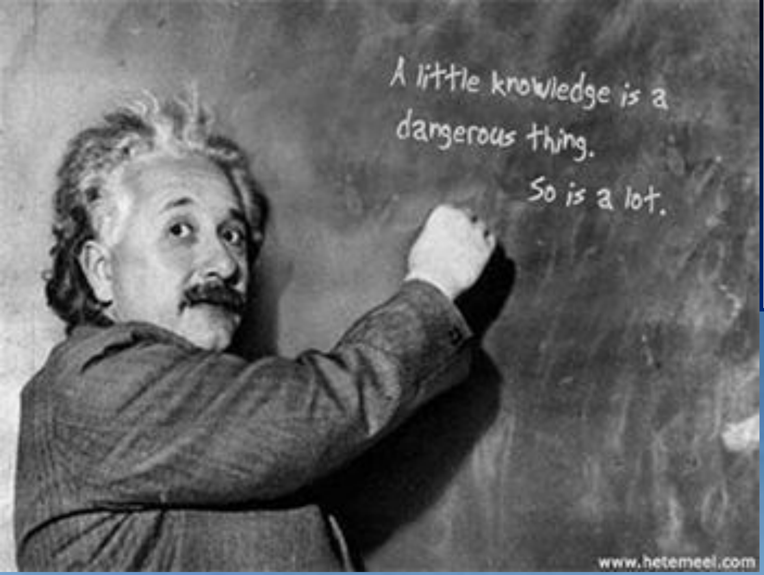
$$M^2(R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu}) = T_{\mu\nu}$$

- Differential equation for gravitational field (metric)
- Links geometry and matter

(energy-momentum –tensor $T_{\mu\nu}$)

- Needs additional equation for energy momentum tensor (equation of state)

$$S = \int d^4x \sqrt{g} \left\{ -\frac{M^2}{2} R + \frac{Z_F}{4} F^{\mu\nu} F_{\mu\nu} + \dots \right\}$$



$$M^2(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}) = T_{\mu\nu}$$

Einstein equation

$$M^2\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) = T_{\mu\nu}$$

M : Planck-Masse
verknüpft mit
Gravitationskonstante G

$$G = \frac{1}{8\pi M^2}$$


E=MC²
3.9512721
66⁴
ZZZZ
.0927864
6(3.78324) 2π
Y
0
X
Y. NC⁴ / 3.6 ÷ √(X/3)
gik; l = 0;
= 0 → O₃ → K₃
Rik = 0; g.5
=0

Newton Gravitation

$$g_{\mu\nu} : \begin{pmatrix} -(1+2\phi) & , 0 & , 0 & , 0 \\ 0 & , 1 & , 0 & , 0 \\ 0 & , 0 & , 1 & , 0 \\ 0 & , 0 & , 0 & , 1 \end{pmatrix}$$

Einstein Gleichung
für zeitunabhängiges Φ :

$$-M^2\vec{\nabla}^2 g_{00} = T_{00}$$

$$\vec{\nabla}^2\phi = \frac{\rho}{2M^2}$$

Newton - Gravitation

$$\vec{\nabla}^2 \phi = \frac{\rho}{2M^2}$$

$$\phi = -\frac{m}{8\pi M^2 r} = -\frac{Gm}{r}$$

$$G = \frac{1}{8\pi M^2}$$

$$\frac{d^2\vec{x}}{dt^2} = -\vec{\nabla}\phi$$

metric of the Universe

homogenous and isotropic

$$g_{\mu\nu} : \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a^2(t) & 0 & 0 \\ 0 & 0 & a^2(t) & 0 \\ 0 & 0 & 0 & a^2(t) \end{pmatrix}$$

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathrm{d}t^2 + a^2(t)\mathrm{d}x^i \mathrm{d}x^j \delta_{ij}$$

Space distances are proportional a(t)

Expansion of space if a(t) increases

Expansion of space

- Field changes its value
- That’s all
- not: “space is created” etc.

Friedmann Gleichung for cosmic evolution

$$\frac{\partial a}{\partial t} = \sqrt{\frac{\rho}{3M^2}}\ a$$

a(t) : scale factor
ρ(t) : energy-density
t : cosmic time

M : Planck-mass

$$G = \frac{1}{8\pi M^2}$$

Friedmann equation

- describes evolution
- of homogeneous and
- isotropic universe

- special case of

  Einstein Gleichung

  (no spatial curvature)

$$\frac{\partial a}{\partial t} = \sqrt{\frac{\rho}{3M^2}} \quad a$$

Hubble parameter

$$\frac{\partial a}{\partial t} = v$$

$$\dot{a} = v$$

$$v = aH$$

$$\dot{a} = Ha$$

$$H = \frac{\dot{a}}{a}$$

$$\frac{\partial a}{\partial t} = \sqrt{\frac{\rho}{3M^2}}\ a$$

$$3M^2H^2 = \rho$$

What is behind the Einstein equation ?

Symmetry under

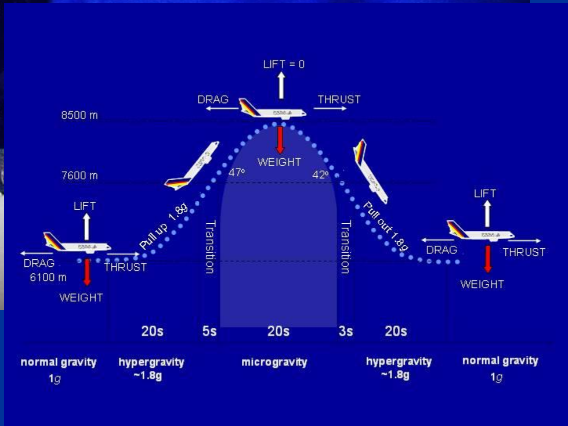
- general coordinate transformations (diffeomorphisms)
- action S invariant
- general relativity

Symmetry principle

Physics is the same in

all free falling systems


4
DLR
NOVESPACE
A300 ZERO-G
DLR


LIFT = 0
DRAG
THRUST
8500 m
WEIGHT
7600 m
47°
42°
LIFT
LIFT
Pull up 1.8g
Transition
Transition
Pull out 1.8g
DRAG
THRUST
DRAG
THRUST
6100 m
WEIGHT
WEIGHT
20s
5s
20s
3s
20s
normal gravity
1g
hypergravity
~1.8g
microgravity
hypergravity
~1.8g
normal gravity
1g

guiding principle : symmetry
general relativity

Functional integral
for
effective action

quantum field theory

- probabilistic theory
- fluctuations matter
- expectation values obtain by summing over fluctuations with a weight factor
- quantum field theory is defined by a functional integral
- sum over fields (functions)

The need for quantum gravity

- Energy momentum tensor is expectation value for a quantum field theory
- Metric must be expectation value of a microscopic metric field (fluctuating field)

$$M^2(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}) = T_{\mu\nu}$$

Effective action

- Effective action is functional of field expectation values
- Field equation obtained from variation of effective action is exact
- Effective action replaces classical action
- It can be computed from the action

$$\Gamma[\chi] = -\ln \int \mathcal{D}\hat{\chi} \, \exp\left\{ -S[\chi + \hat{\chi}] + \int_x \frac{\partial \Gamma}{\partial \chi(x)} \hat{\chi}(x) \right\}$$

Functional integral

Integral over fluctuations

$$\Gamma[\chi] = -\ln \int \mathcal{D}\hat{\chi}\, \exp\left\{ -S[\chi + \hat{\chi}] + \int_x \frac{\partial \Gamma}{\partial \chi(x)} \hat{\chi}(x) \right\}$$

$\chi'(x)$	microscopic field
$\chi(x)$	expectation value of microscopic field = macroscopic field
$\hat{\chi}(x) = \chi'(x) - \chi(x)$	fluctuation field

Functional integral

Solving a quantum field theory =

Computation of effective action

- Too complex to be done exactly if interactions are present
- Approximations are needed
- Perturbative expansion in strength of interaction diverges

$$\Gamma[\chi] = -\ln \int \mathcal{D}\hat{\chi}\, \exp\left\{ -S[\chi + \hat{\chi}] + \int_x \frac{\partial \Gamma}{\partial \chi(x)} \hat{\chi}(x) \right\}$$

Functional renormalization

Flowing couplings

Couplings change with momentum scale due to quantum fluctuations.

Renormalization scale k : Only fluctuations with momenta larger k are included in functional integral

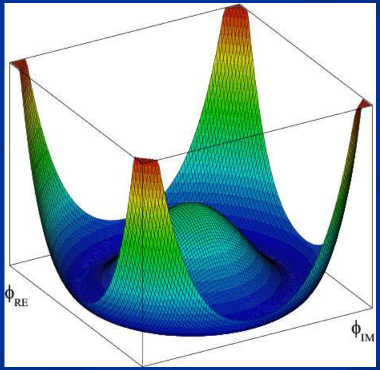
Flow of k to zero : all fluctuations included, IR
Flow of k to infinity : UV

Effective average action : couplings depend on k

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{M^2}{2} R + U_E(\varphi) + \frac{1}{2} Z \partial^\mu \varphi \partial_\mu \varphi \right\}$$

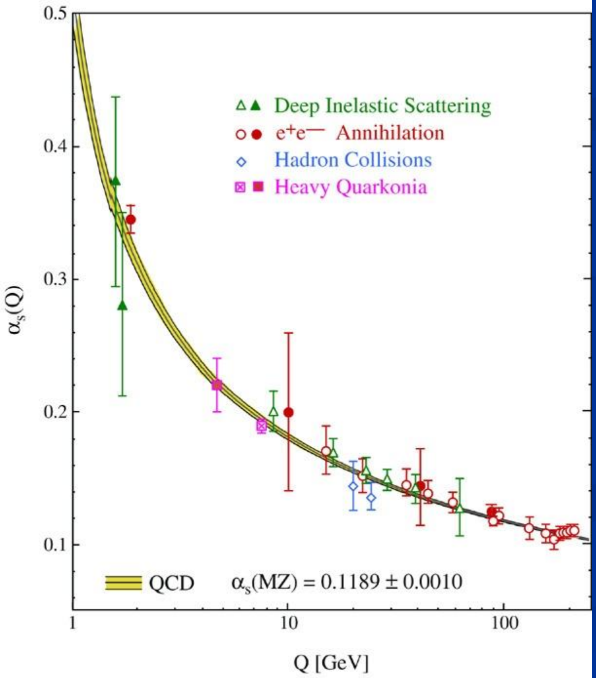
$$V(\varphi) = -\mu^2 \varphi^\dagger \varphi + \tfrac{1}{2}\lambda(\varphi^\dagger \varphi)^2$$

$$= \tfrac{1}{2}\lambda(\varphi^\dagger \varphi - \varphi_0^2)^2 + \text{const.})$$



Renormalization

How do couplings or physical laws change with scale k ?



Renormalizability

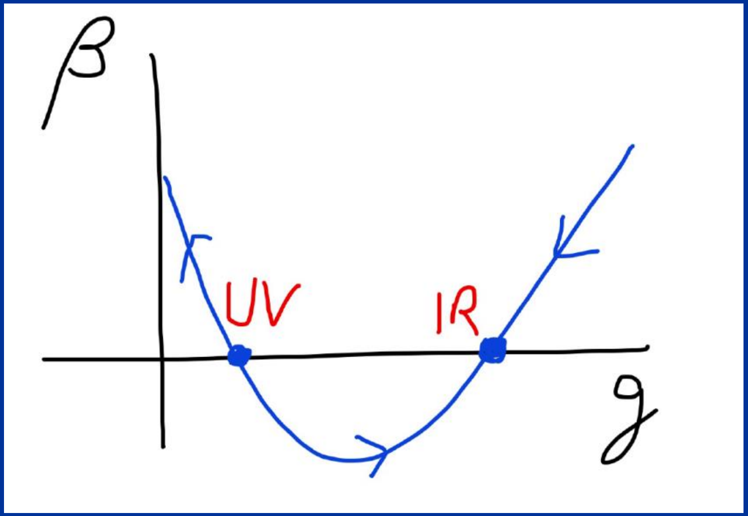
Theory can be extrapolated to
infinitely small distances or
infinitely large energies:
such a theory has no infinities -
it is renormalizable

Ultraviolet fixed point

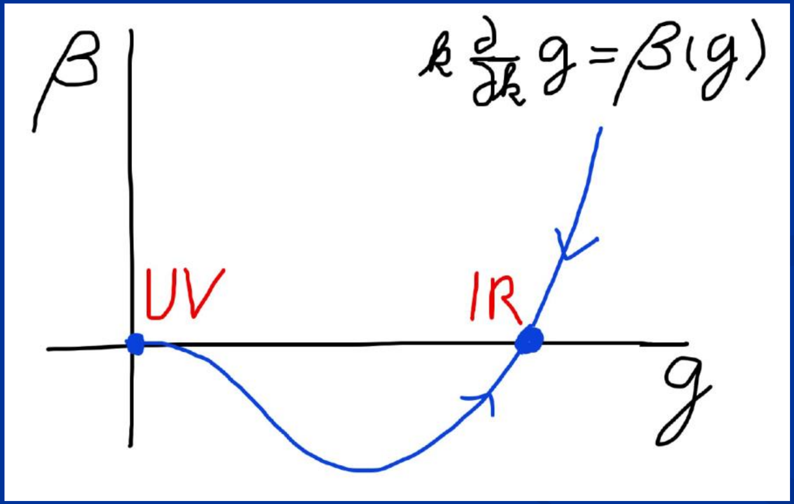
- Flow of dimensionless couplings stops as k increases towards infinity
- Renormalizable theories from ultraviolet fixed point in the scale dependence of couplings

  (renormalization flow)
- Theory can be extrapolated to arbitrarily short distances
- UV-Completeness

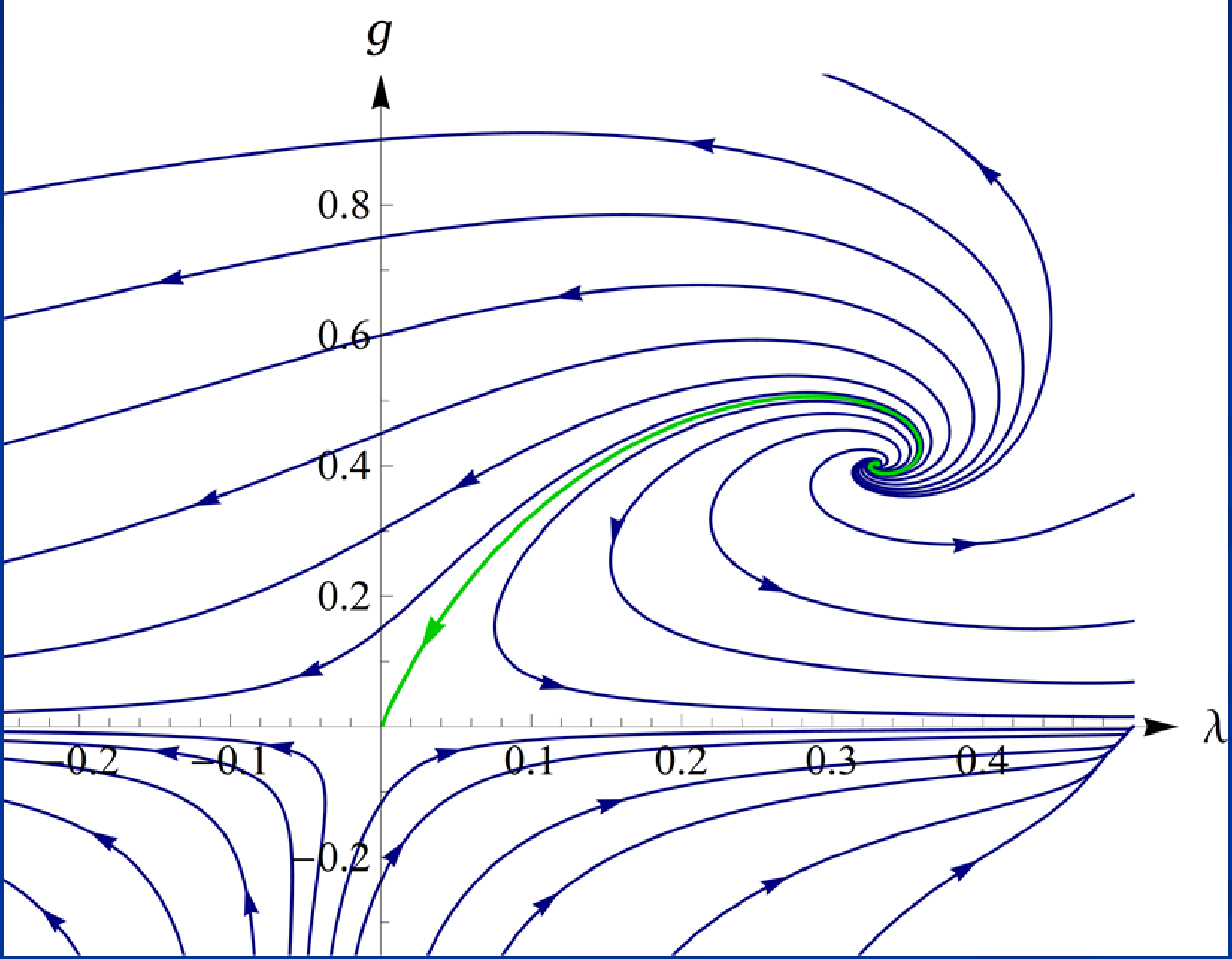
Asymptotic safety



Asymptotic freedom



UV- fixed point for quantum gravity



Wikipedia

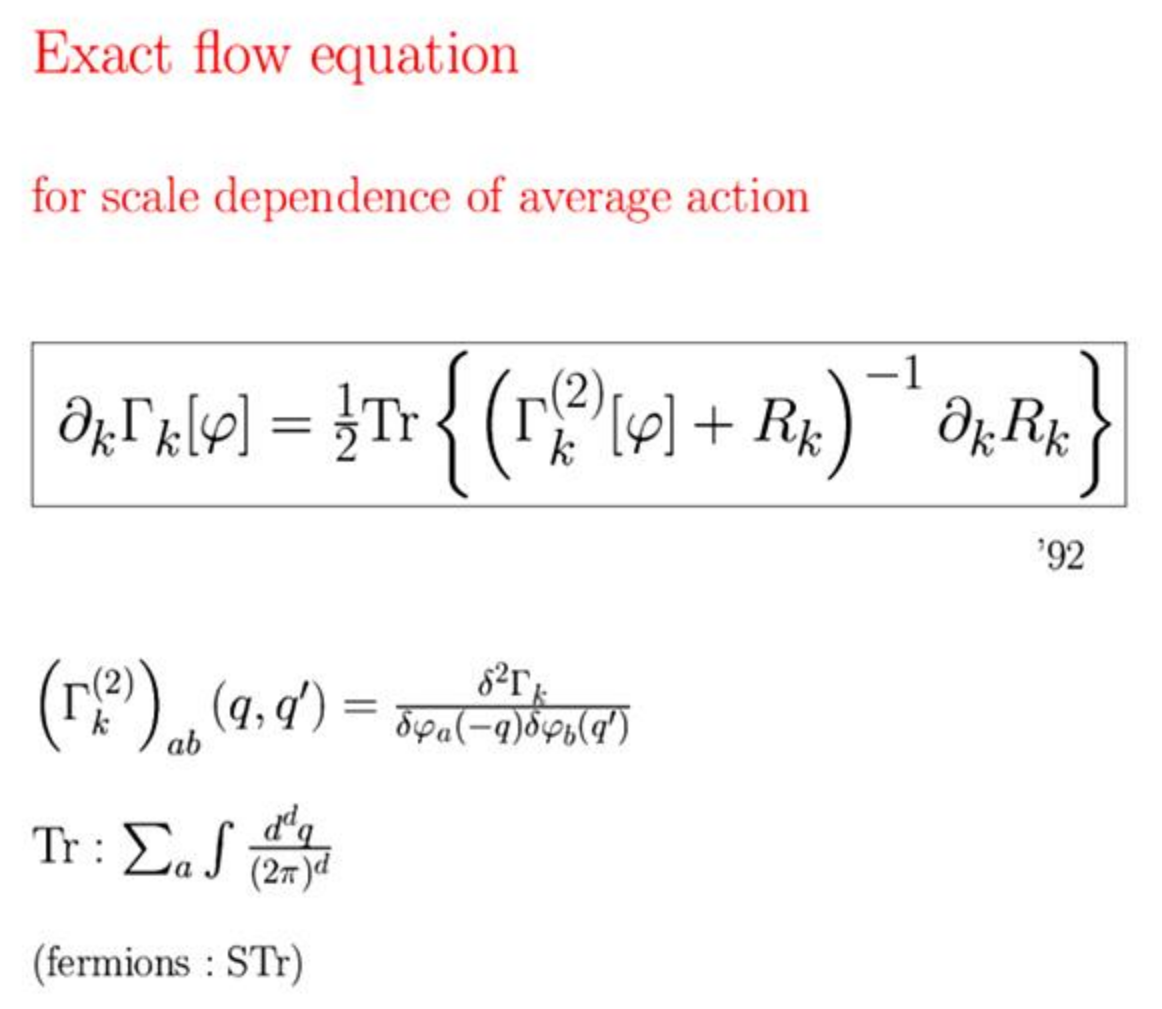
How to compute non-perturbative quantum gravity effects ?

Quantum gravity computation by functional renormalization

Introduce infrared cutoff with scale k, such that only fluctuations with (covariant) momenta larger than k are included.

Then lower k towards zero

Exact renormalization group equation



Exact flow equation

for scale dependence of average action

$$\boxed{\partial_k \Gamma_k[\varphi] = \tfrac{1}{2}\mathrm{Tr}\left\{\left(\Gamma_k^{(2)}[\varphi] + R_k\right)^{-1} \partial_k R_k\right\}}$$

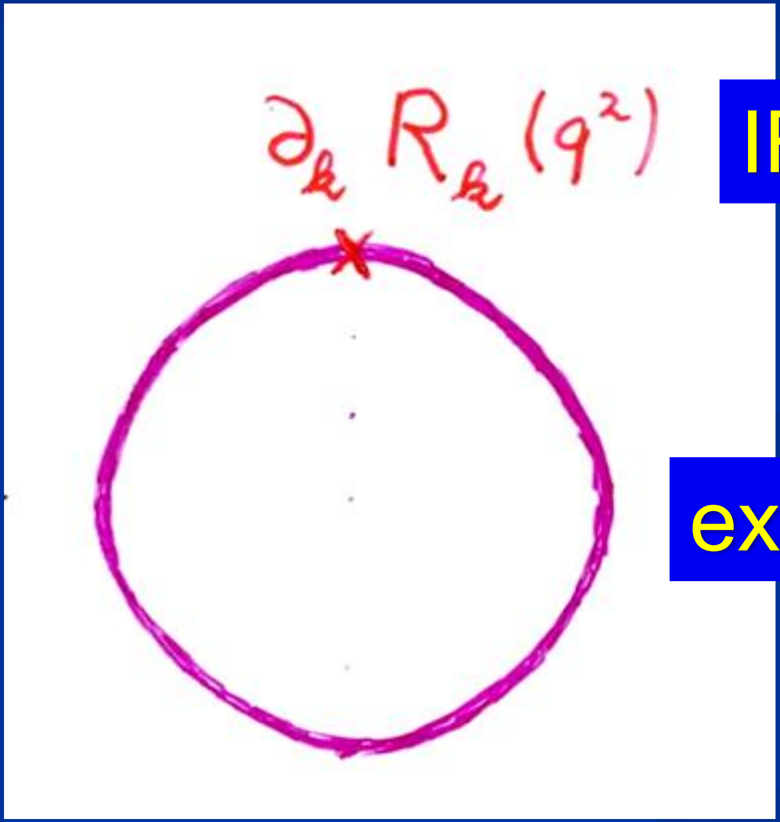
’92

$$\left(\Gamma_k^{(2)}\right)_{ab}(q,q') = \frac{\delta^2 \Gamma_k}{\delta\varphi_a(-q)\delta\varphi_b(q')}$$

$\mathrm{Tr} : \sum_a \int \frac{d^d q}{(2\pi)^d}$

(fermions : STr)

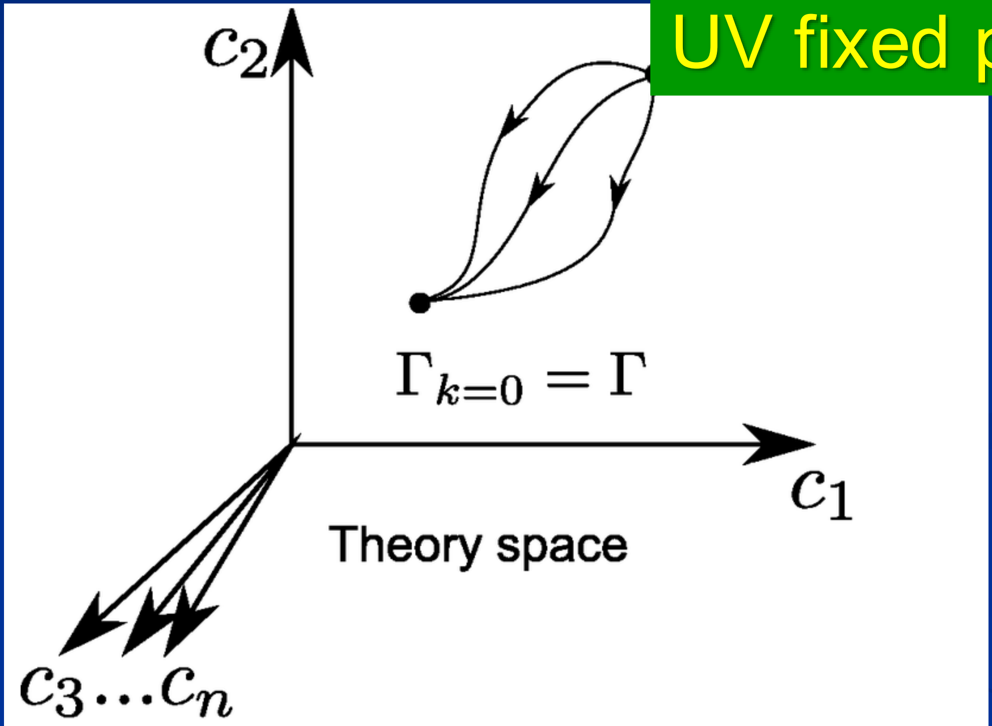
Functional flow equation for scale dependent effective action



IR cutoff

exact propagator

Ultraviolet fixed point



UV fixed point

Extrapolation of microscopic law to infinitely short distances is possible.

Complete theory

Asymptotic safety of quantum gravity

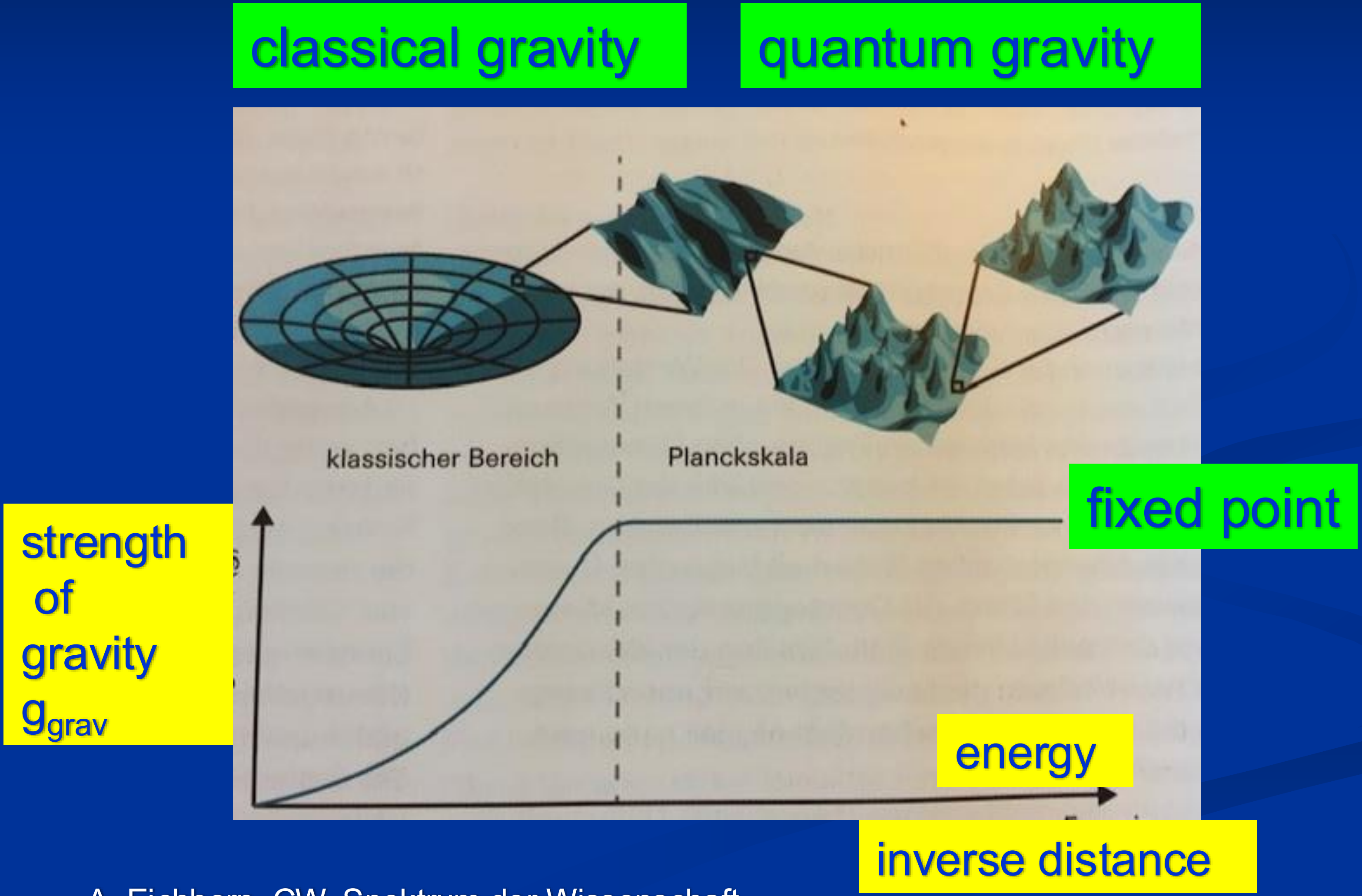
if UV fixed point exists :

quantum gravity is

non-perturbatively renormalizable !

S. Weinberg , M. Reuter

Strength of gravity



A. Eichhorn, CW, Spektrum der Wissenschaft

Quantum Gravity

Quantum Gravity is a

renormalisable quantum field theory

Standard model of particle physics (or extension) coupled to the metric is ultraviolet complete

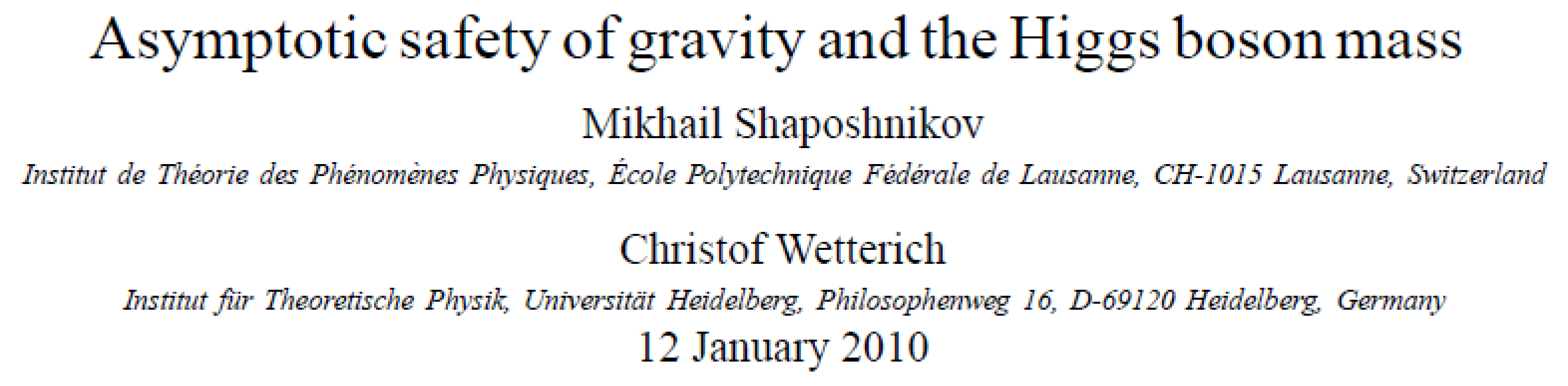
Nothing else is needed ?

One can construct an ultraviolet complete quantum field theory for gravity based on the metric,

but is it the correct one ?

Predictions of quantum gravity

Prediction of mass of Higgs boson



Asymptotic safety of gravity and the Higgs boson mass

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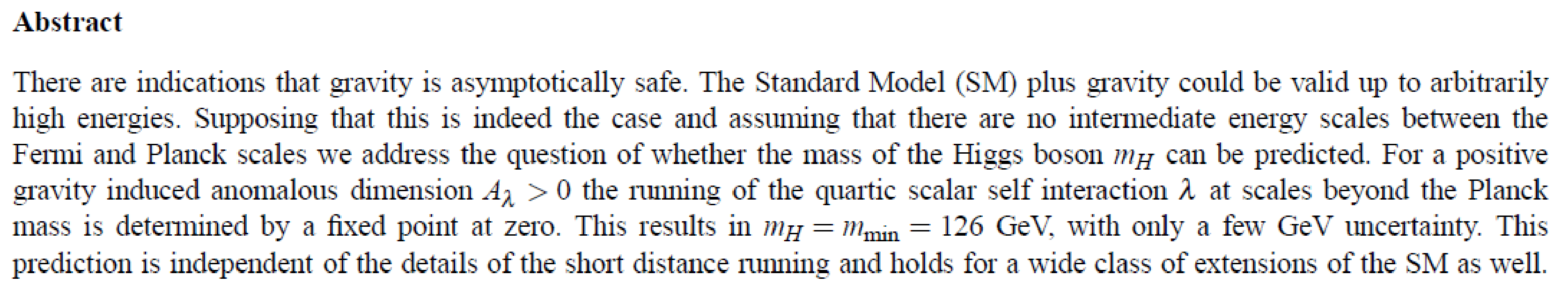
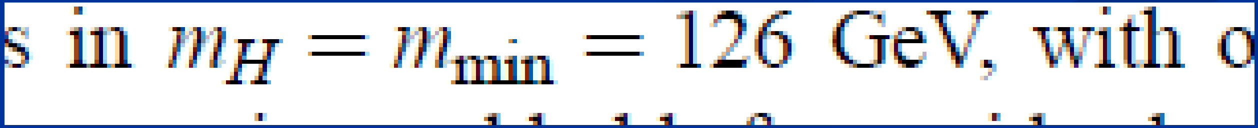
Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, D-69120 Heidelberg, Germany

12 January 2010



Abstract

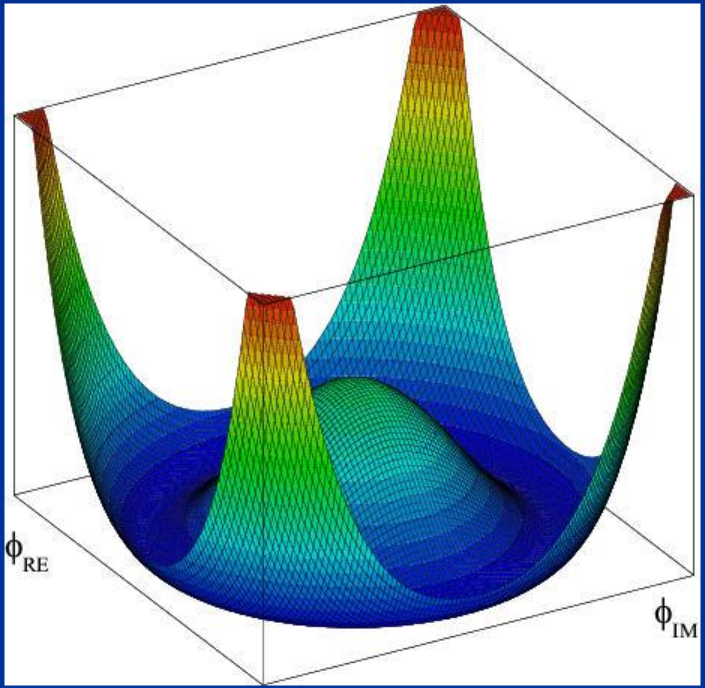
There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalous dimension $A_\lambda > 0$ the running of the quartic scalar self interaction λ at scales beyond the Planck mass is determined by a fixed point at zero. This results in $m_H = m_{\min} = 126$ GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well.



s in $m_H = m_{\min} = 126$ GeV, with o

Why can quantum gravity make predictions for particle physics ?

Effective potential for Higgs scalar



$$V(\varphi) = -\mu^2 \varphi^\dagger \varphi + \frac{1}{2}\lambda(\varphi^\dagger \varphi)^2$$

$$= \frac{1}{2}\lambda(\varphi^\dagger \varphi - \varphi_0^2)^2 + \text{const.})$$

Fermi scale

$$\varphi_0 = 175 \text{ GeV}$$

Quartic scalar coupling

prediction of mass of Higgs boson

=

prediction of value of quartic scalar coupling λ at Fermi scale

$$m^2 = 2\lambda\varphi_0^2$$

Why can quantum gravity make predictions for quartic scalar coupling ?

Mass scales

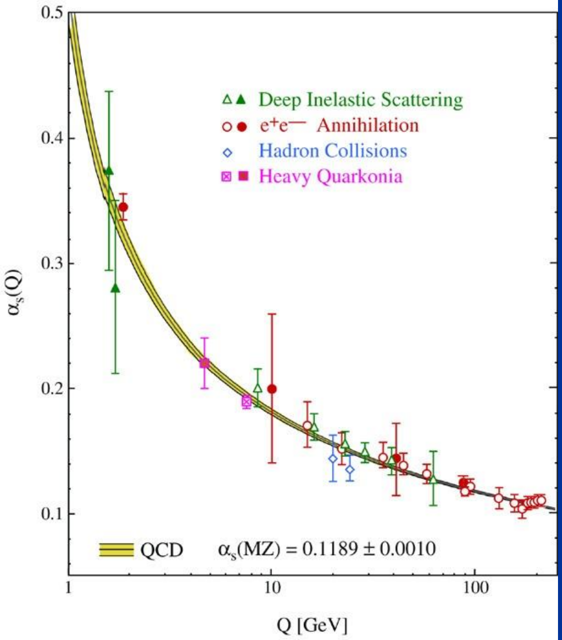
- Fermi scale φ_0 ~ 100 GeV
- Planck mass M ~ 10^{18} GeV
- Gravity at Fermi scale is very weak : How can it influence the effective potential for the Higgs scalar and the mass of the Higgs boson ?

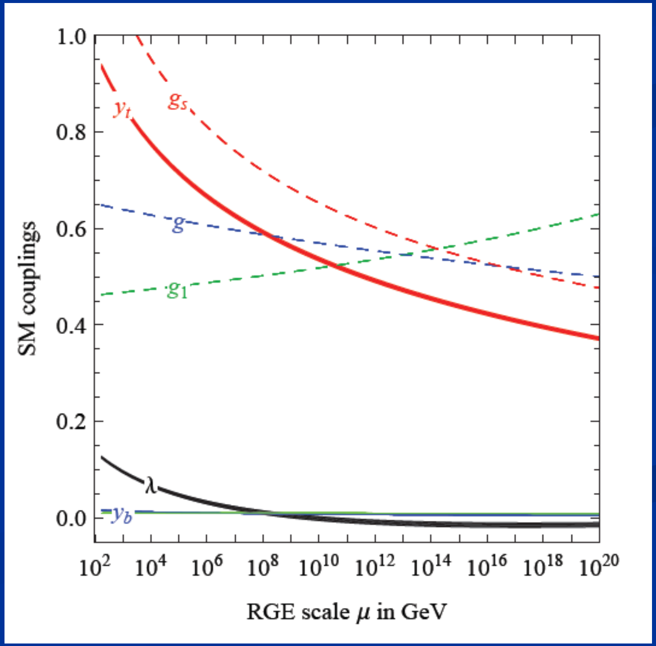
$$\varepsilon = \frac{\varphi_0^2}{M^2} = 5 \cdot 10^{-33}$$

Quantum fluctuations induce running couplings

- possible violation of scale symmetry
- well known in QCD or standard model

Bethke

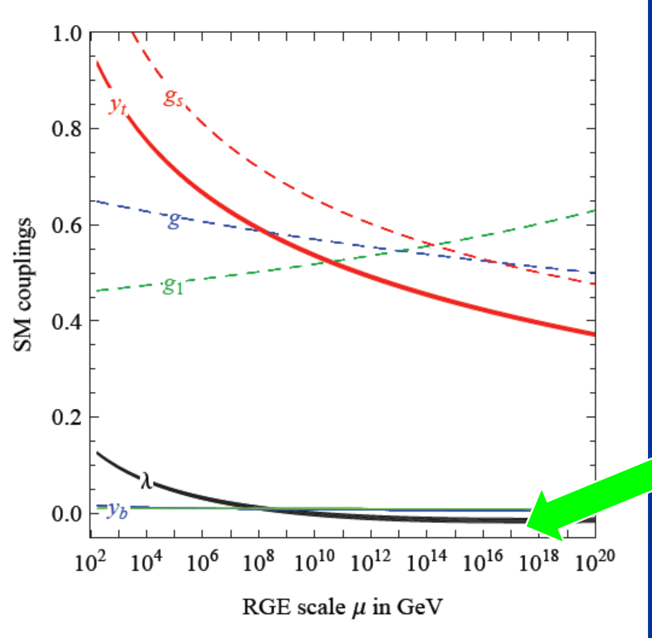




Degrassi et al

Quantum fluctuations induce running couplings

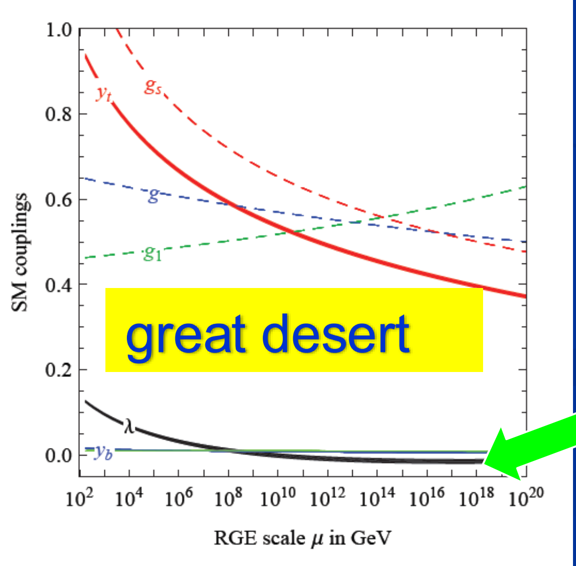
flow of couplings in standard model



prediction of quantum gravity

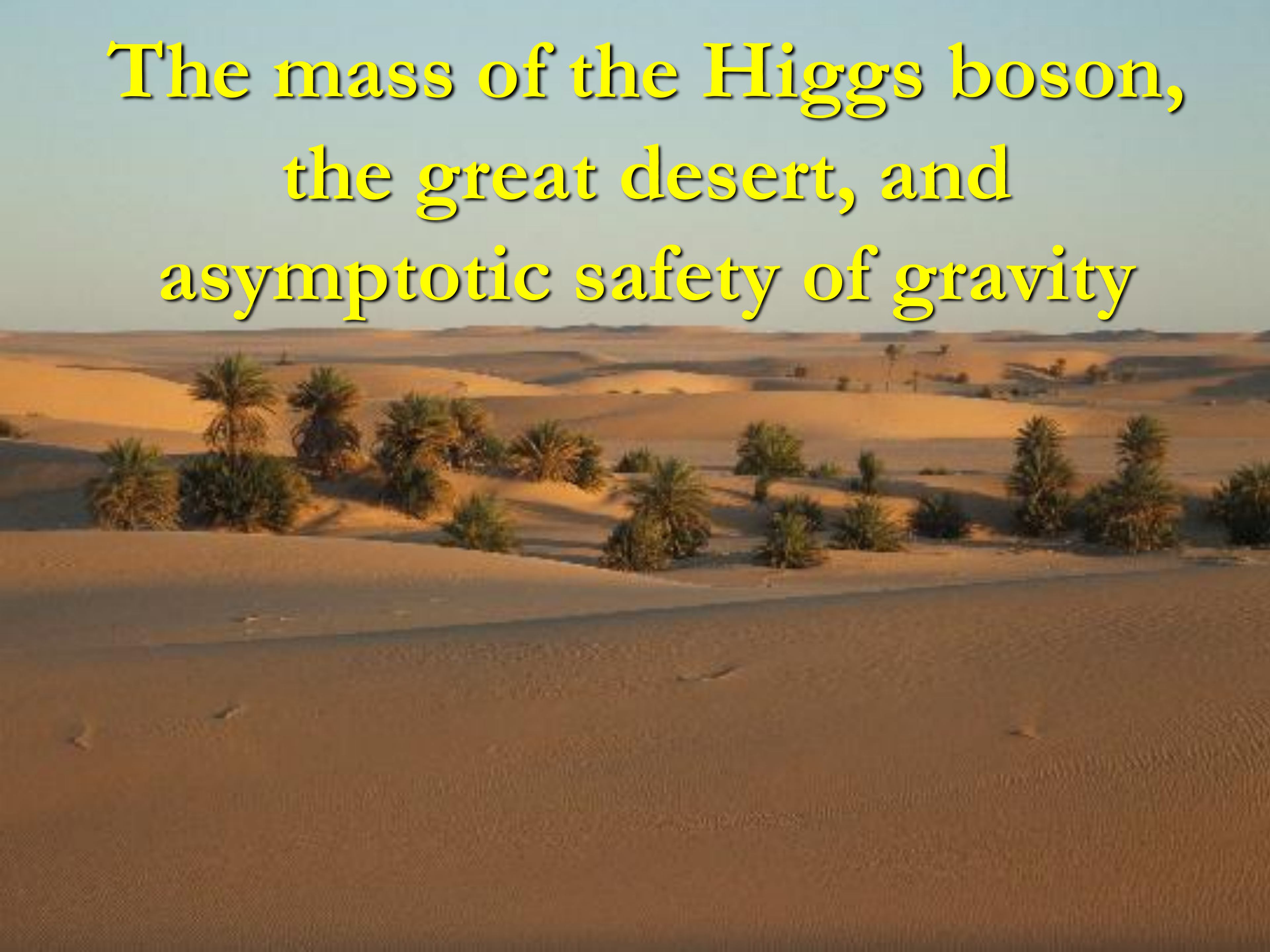
key points

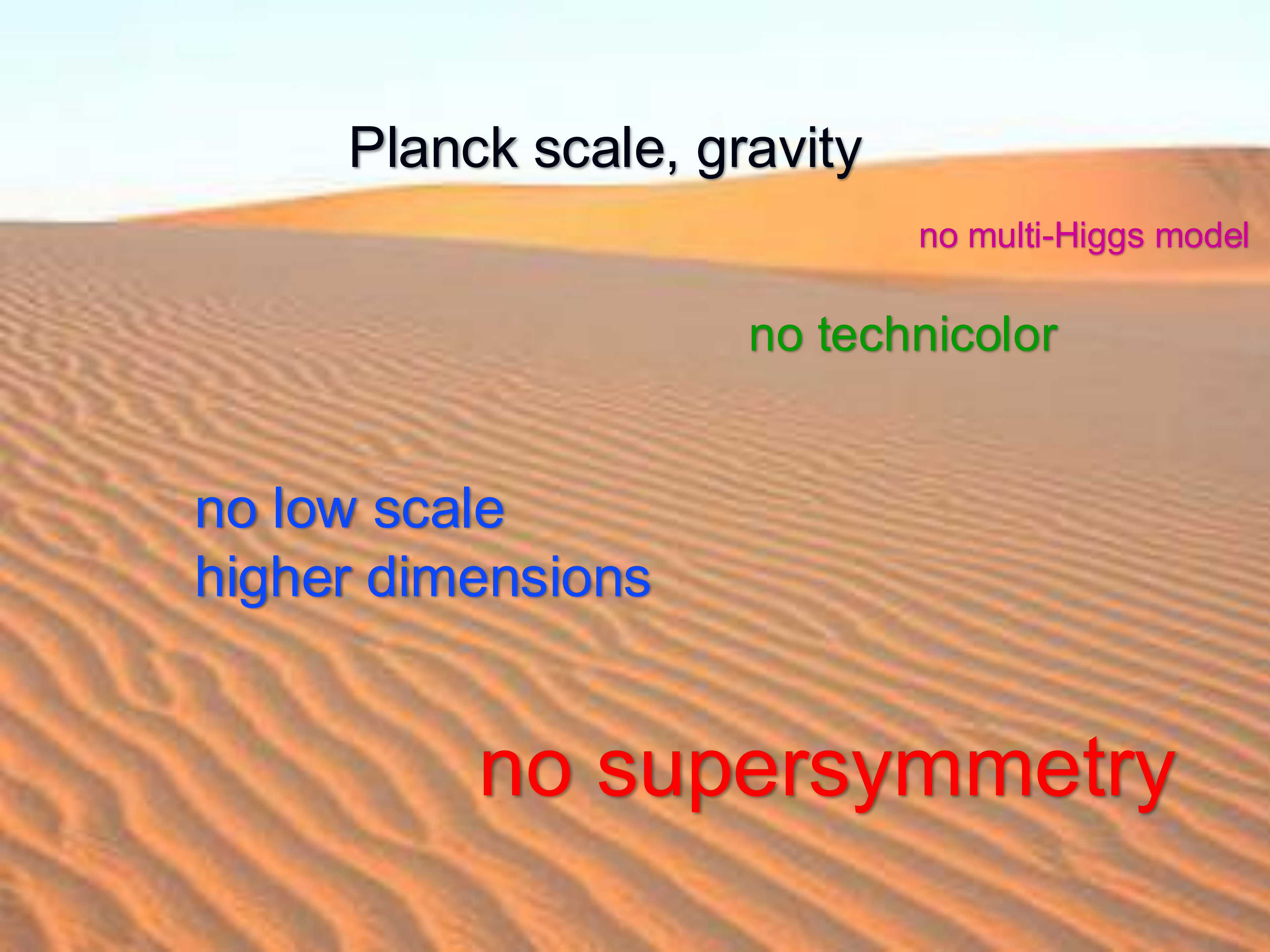
- great desert
- high scale fixed point
- vanishing scalar coupling at fixed point



fixed point

The mass of the Higgs boson, the great desert, and asymptotic safety of gravity





Planck scale, gravity

no multi-Higgs model

no technicolor

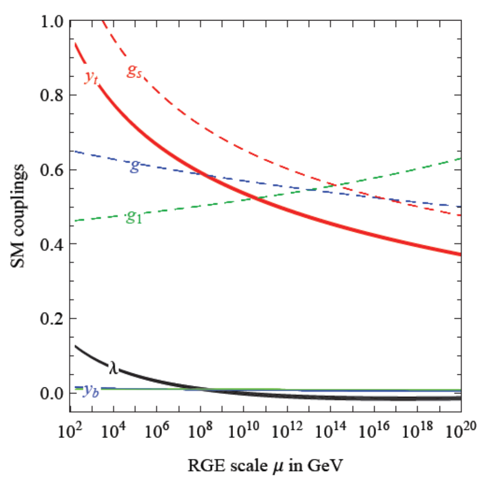
no low scale
higher dimensions

no supersymmetry

Essential point for prediction of Higgs boson mass:

Initial value of quartic scalar coupling near Planck mass is predicted by quantum gravity

Extrapolate perturbatively to Fermi scale



Results in prediction for ratio Higgs boson mass over W- boson mass, or Higgs boson mass over top quark mass

Near Planck mass gravity is not weak !

Predictive power !

Great desert

Big chance
for understanding of
quantum gravity !

Pregeometry

Microscopic degrees of freedom

Is the metric the fundamental microscopic degree of freedom ?

alternatives:

- superstrings
- triangles (lattice quantum gravity)
- fermions (spinor quantum gravity)

Composite metric

- Metric may be a collective or composite field
- At the end, every theory of quantum gravity has to construct a metric and compute the effective action for the metric

superstrings
triangles (lattice quantum gravity)
fermions (spinor quantum gravity)

Conclusions

- Quantum gravity is a renormalizable quantum field theory, realized by UV - fixed point of running couplings or flowing effective action

- Quantum gravity is predictive :

  Mass of the Higgs boson (and more …?)

  Properties of inflation

  Properties of dark energy

end

How does asymptotic safety predict the quartic scalar coupling ?

Graviton fluctuations erase quartic scalar coupling

Renormalization scale k : Only fluctuations with momenta larger k are included

$$k \frac{\partial \lambda}{\partial k} = A \lambda$$

gravity induced
anomalous dimension

$A > 0$

Graviton fluctuations erase quartic scalar coupling

for k beyond Planck scale :

$$k \frac{\partial \lambda}{\partial k} = A \lambda$$

gravity induced anomalous dimension

A : positive constant of order one

$$\lambda(k) = \lambda(\mu) \left(\frac{k}{\mu} \right)^{A}$$

$$k \to 0 \quad \Rightarrow \quad \lambda \to 0$$

Fixed point

$$k \frac{\partial \lambda}{\partial k} = A \lambda$$

$$\lambda(k) = \lambda(\mu) \left(\frac{k}{\mu} \right)^A$$

The quartic scalar coupling λ has a

fixed point at λ=0

It flows towards the fixed point as k is lowered :

irrelevant coupling

For a UV – complete theory it is predicted to assume the fixed point value

Gravitational contribution to running quartic coupling

$$\partial_t \lambda = A_\lambda \lambda$$

$$A = \frac{1}{48\pi^2 \tilde{M}_{\mathrm{p}}^2}\left[\frac{20}{(1-v_0)^2} + \frac{1}{(1-v_0/4)^2}\right]$$

$$\partial_t = k\partial_k$$

running Planck mass :

$$M_{\mathrm{p}}^2(k) = M^2 + \tilde{M}_{\mathrm{p}*}^2 k^2$$

dimensionless
squared Planck mass

$$\tilde{M}_{\mathrm{p}}^2 = \frac{M_{\mathrm{p}}^2}{k^2}$$

UV – fixed point for quartic coupling

Flow equation for λ :

$$\partial_t \lambda_H = A \lambda_H - C_H$$

$$C_H^{(p)} = -\beta_\lambda^{(SM)}$$

$$\approx -\frac{1}{16\pi^2}\left\{12\lambda_H^2 + 12y_t^2\lambda_H - 12y_t^4 + \frac{9}{4}g_2^2 + \frac{9}{10}g_2^2 g_1^2 + \frac{27}{100}g_1^4 - \left(9g_2^2 + \frac{9}{5}g_1^2\right)\lambda_H\right\}$$

Fixed point : λ = C / A

$$\lambda\left(k_{tr}\right) \approx 0\ ,\ \beta_\lambda\left(k_{tr}\right) \approx 0$$

Strength of gravity

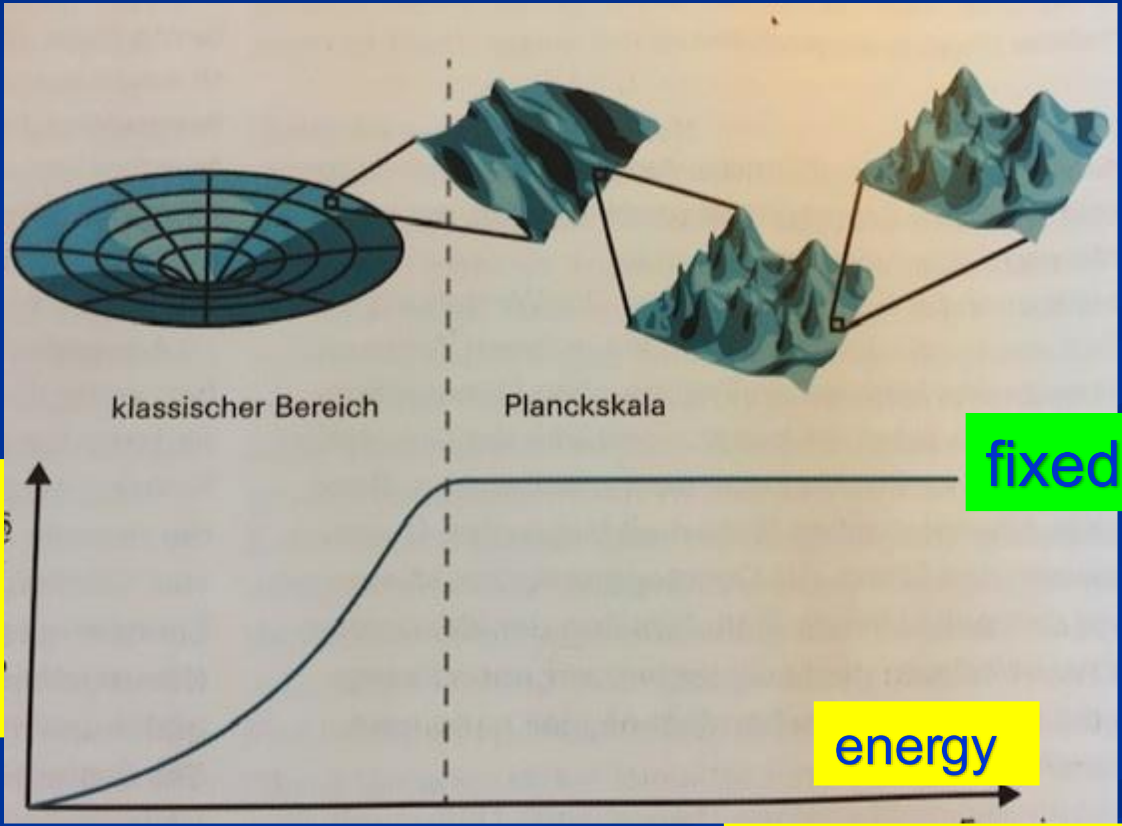
$g_{\text{grav}} = 1 / w$

running gravitational coupling

$$w = \frac{M^2}{2k^2}$$

M : Planck mass

Strength of gravity



classical gravity

quantum gravity

fixed point

strength of gravity g_{grav}

energy

inverse distance

A. Eichhorn, CW, Spektrum der Wissenschaft

Flowing dimensionless Planck mass

- Renormalization scale k : Only fluctuations with momenta larger k are included

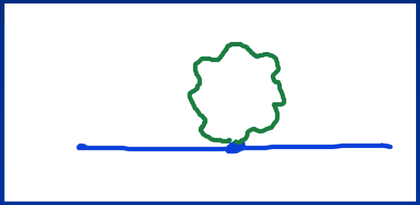
Flowing
Planck mass $M^2(k)$

$$\partial_t M^2 = 4ck^2$$

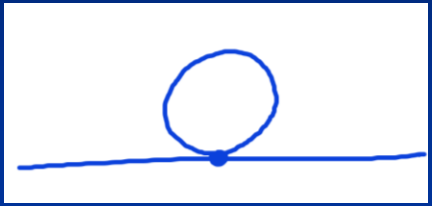
$$\partial_t = k\partial_k$$

Universality of gravity

scalar loop,
fermion loop



gauge boson
loop



graviton
loop

for massless particles :
c is independent of coupling constants

$$\partial_t M^2 = 4ck^2$$

Flowing dimensionless Planck mass

- Renormalization scale k : Only fluctuations with momenta larger k are included

Flowing
Planck mass $M^2(k)$

$$\partial_t M^2 = 4ck^2$$

$$\partial_t = k\partial_k$$

matter
contribution

$$c_M = \frac{\mathcal{N}_M}{192\pi^2}$$

$$\mathcal{N}_M = 4\,N_V - N_S - N_F$$

with graviton
contribution

$$c_M = \frac{1}{192\pi^2}\left(\mathcal{N}_M + \frac{43}{6} + \frac{75(1-\eta_g/6)}{2(1-v)}\right)$$

Flowing dimensionless Planck mass

Flowing
Planck mass $M^2(k)$

$$\partial_t M^2 = 4ck^2$$

$$\partial_t = k\partial_k$$

solution :

$$M^2(k) = M^2 + 2c_M k^2$$

Flowing dimensionless Planck mass

Flowing
Planck mass $M^2(k)$

$$\partial_t M^2 = 4ck^2$$

Dimensionless
squared Planck mass

$$w = \frac{M^2}{2k^2}$$

$$\partial_t w = -2w + 2c$$

solution :

$$w = c + \frac{\bar{M}^2}{2k^2}$$

$$M^2(k) = M^2 + 2c_M k^2$$

Fixed point and flow away from fixed point

UV - fixed point $w_* = c$

$$\partial_t w = -2w + 2c$$

approached
for k → ∞ $w = c + \frac{\bar{M}^2}{2k^2}$

$$M^2(k) = M^2 + 2c_M k^2$$

Near UV – fixed point : M ~ k

$$\tilde{M}^2_{p*} = 2c$$

Transition to constant M for small k,
gravity gets weak, w $^{-1}$ decreases to zero

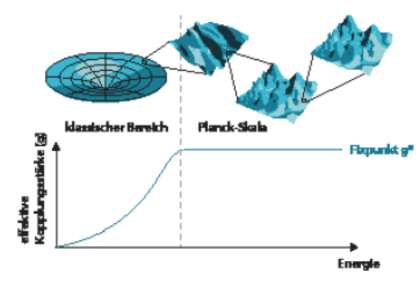
M is relevant parameter, cannot be predicted

Weak and constant gravity

$$M_{\rm p}^2(k) = \begin{cases} \tilde{M}_{\rm p*}^2\, k^2 & \text{for } k > k_t \\ M^2 & \text{for } k < k_t \end{cases}$$

$$k_t^2 = \frac{M^2}{\tilde{M}_{\rm p*}^2}$$

$$M_{\rm p}^2(k) = M^2 + \tilde{M}_{\rm p*}^2 k^2$$



Two regimes for the (inverse) strength of gravity

$$\tilde{M}_{\rm p}^2 = \frac{M_{\rm p}^2}{k^2} = \tilde{M}_{\rm p*}^2 \left[\left(\frac{k_t}{k} \right)^2 + 1 \right]$$

Gravitational contribution to running quartic coupling

$$\partial_t \lambda = A_\lambda \lambda$$

$$A = \frac{1}{48\pi^2 \tilde{M}_\mathrm{p}^2} \left[\frac{20}{(1-v_0)^2} + \frac{1}{(1-v_0/4)^2} \right]$$

$$\partial_t = k \partial_k$$

running Planck mass :

$$M_\mathrm{p}^2(k) = M^2 + \tilde{M}_{\mathrm{p}*}^2 k^2$$

dimensionless
squared Planck mass

$$\tilde{M}_\mathrm{p}^2 = \frac{M_\mathrm{p}^2}{k^2} = \tilde{M}_{\mathrm{p}*}^2 \left[\left(\frac{k_t}{k} \right)^2 + 1 \right]$$

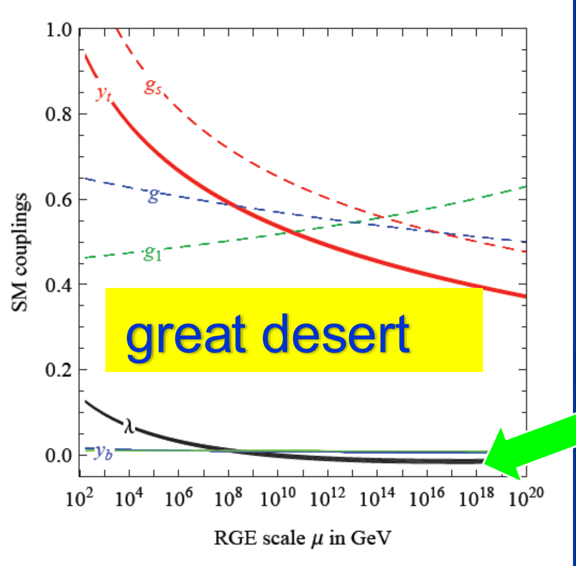
$$k_t^2 = \frac{M^2}{\tilde{M}_{\mathrm{p}*}^2}$$

large k : constant A
small k: A ~ k^2 / M^2

transition at
k_t ~ 10^{19} GeV

Prediction for quartic Higgs coupling

- high scale fixed point
- quartic scalar coupling predicted to be very small at transition scale where gravity decouples
- below Planck mass: quartic scalar coupling increases

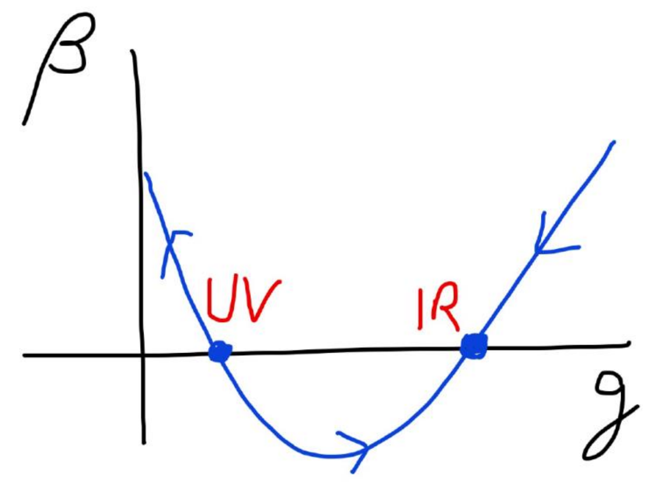


great desert

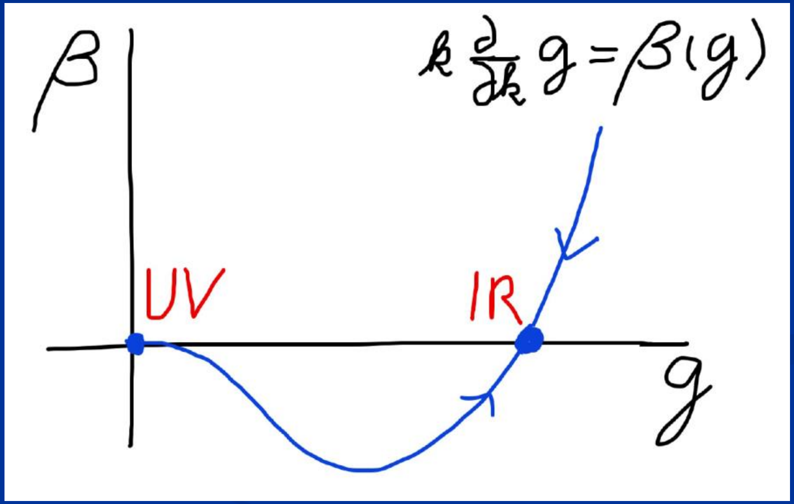
fixed point

Predictivity for asymptotically safe quantum gravity

Asymptotic safety



Asymptotic freedom



$k \frac{\partial}{\partial k} g = \beta(g)$

Relevant parameters yield undetermined couplings. Quartic scalar coupling is not relevant and can therefore be predicted.

Enhanced predictivity for UV – fixed point

- Free parameters of a theory correspond to relevant parameters for small deviations from fixed point.
- If the number of relevant parameters at the UV-fixed point is smaller than the number of free parameters (renormalizable couplings) in the standard model:
- Relations between standard model parameters become predictable !

Fixed points

$$g = \{g_1, ..., g_i, ...\}$$

couplings

$$\tilde{g}_i = g_i k^{-d_i}$$

dimensionless

Flow equation $$\partial_t \tilde{g}_i = \beta_i(\tilde{g})$$

Fixed points: zeros of beta-function

No running → No scale

Quantum scale symmetry

Stability matrix

$$g = \{g_1, ..., g_i, ...\}$$

$$\tilde{g}_i = g_i k^{-d_i}$$

couplings

dimensionless

Flow equation

$$\partial_t \tilde{g}_i = \beta_i(\tilde{g}) = -d_i \tilde{g}_i + f_i(\tilde{g})$$

Expand in vicinity of fixed point

$$\partial_t \tilde{g}_i = \sum_j \frac{\partial \beta_i}{\partial \tilde{g}_j}\Big|_{\tilde{g}=\tilde{g}_*} (\tilde{g}_j - \tilde{g}_{j*}) = -T_{ij}(\tilde{g}_j - \tilde{g}_{j*})$$

T : stability matrix

Critical exponents

$$\partial_t \tilde{g}_i = \sum_j \left.\frac{\partial \beta_i}{\partial \tilde{g}_j}\right|_{\tilde{g}=\tilde{g}_*} (\tilde{g}_j - \tilde{g}_{j*}) = -T_{ij}(\tilde{g}_j - \tilde{g}_{j*})$$

θ_l : Eigenvalues of stability matrix T

= Critical exponents

Linearized solution

$$\tilde{g}_i = \tilde{g}_{i*} + \sum_l C_l V_i^l \left(\frac{k}{\mu}\right)^{-\theta_l}$$

Irrelevant parameters: eigenvectors in coupling constant space with $\theta_l < 0$

flow towards fixed point values as k is lowered

Irrelevant parameters

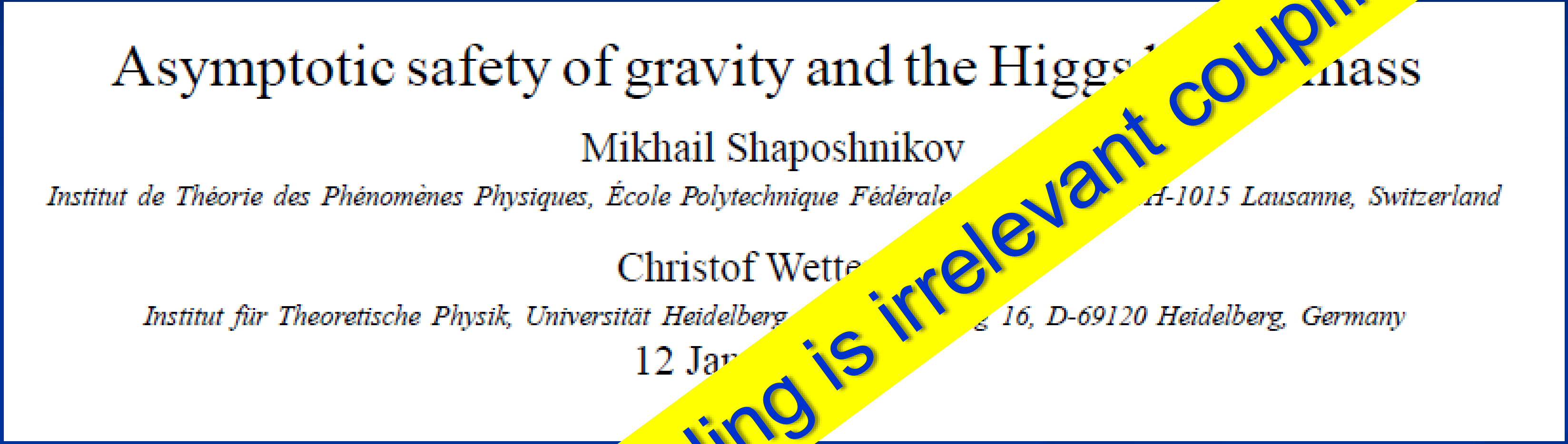
- “Forget” information about initial values
- Central ingredient for

  predictivity of quantum field theories
- For UV – complete theories : irrelevant parameters have to take precisely the fixed point values
- **Relevant parameters** flow away from fixed point as k is lowered – they are the only free parameters

Predictivity at fixed point

- Irrelevant parameters are predicted to take fixed point values
- Only relevant parameters are free
- Number of free parameters of a renormalizable quantum field theory = number of relevant parameters at the fixed point

a prediction…



Asymptotic safety of gravity and the Higgs [illegible] mass

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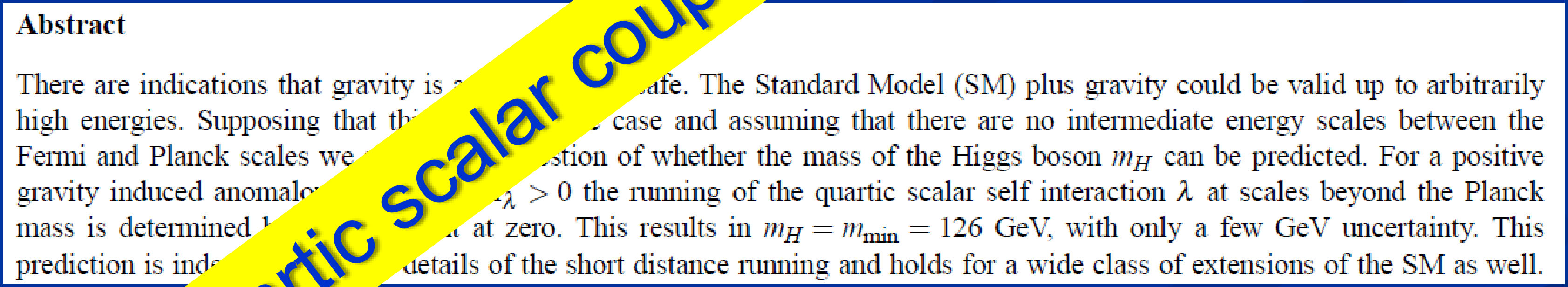
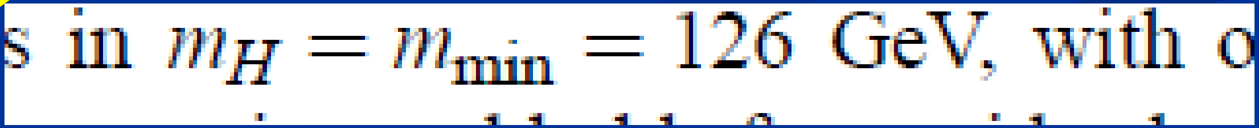


Abstract

There are indications that gravity is a[illegible] safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that thi[illegible] case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we [illegible]stion of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalo[illegible]$_\lambda > 0$ the running of the quartic scalar self interaction λ at scales beyond the Planck mass is determined [illegible] at zero. This results in $m_H = m_{\min} = 126$ GeV, with only a few GeV uncertainty. This prediction is inde[illegible] details of the short distance running and holds for a wide class of extensions of the SM as well.



s in $m_H = m_{\min} = 126$ GeV, with o

Quartic scalar coupling is irrelevant coupling

Quartic scalar couplings are
irrelevant couplings
for all models
(within range of validity of truncation)

Quartic scalar couplings are predicted for given quantum field theories with gravity