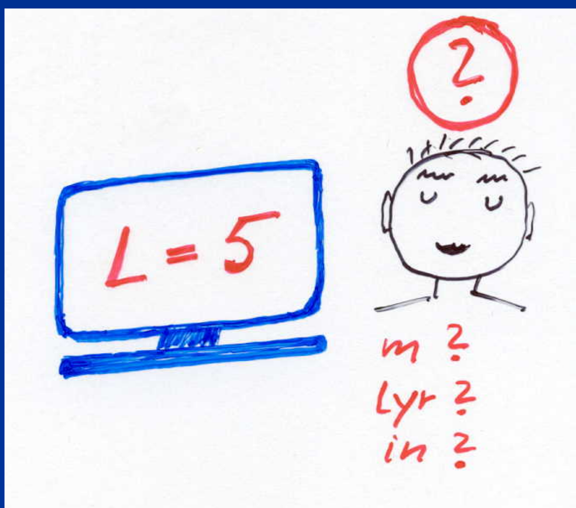
Quintessence
from time evolution of
fundamental mass scale

Quintessence and solution of cosmological constant problem should be related !

$$\Omega_m + X = 1$$



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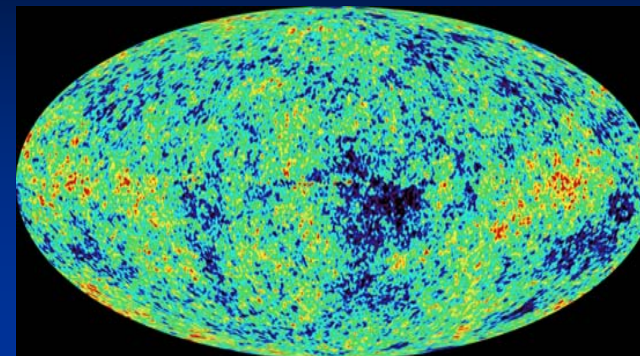


$$\Omega_m \; : \; 25\%$$

$$\Omega_h \; : \; 75\%$$

Dark Energy

Time dependent Dark Energy : Quintessence

- What changes in time ?

- **Only dimensionless ratios of mass scales are observable !**

- V : potential energy of scalar field or cosmological constant
- V/M^4 is observable

- **Imagine the Planck mass M increases …**

Fundamental mass scale

- Unification fixes parameters with dimensions

- Special relativity : c
- Quantum theory : h
- Unification with gravity :

fundamental mass scale

(Planck mass , string tension , …)

Fundamental mass scale

- Fixed parameter or dynamical scale ?
- Dynamical scale ⟷ Field
- Dynamical scale compared to what ?

momentum versus mass

(or other parameter with dimension)

Cosmon and fundamental mass scale

- Assume all mass parameters are proportional to scalar field χ (GUTs, superstrings,…)
- $M_p \sim \chi$, $m_{proton} \sim \chi$, $\Lambda_{QCD} \sim \chi$, $M_W \sim \chi$,…

- χ may evolve with time : **cosmon**
- m_n/M : (almost) constant - ***observation*** !

Only ratios of mass scales are observable

Example :

Field χ denotes scale of transition
from higher dimensional physics
to effective four dimensional description
in theory without fundamental mass parameter

(except for running of dimensionless couplings…)

Dilatation symmetry

- Lagrange density:

$$L = \sqrt{g}\left(-\frac{1}{2}\chi^2 R + \frac{1}{2}(\delta - 6)\partial^\mu \chi \partial_\mu \chi + V(\chi) + h\chi\overline{\psi}\psi\right)$$

- Dilatation symmetry for

$$V = \lambda\chi^4,\ \lambda = const.,\ \delta = const.,\ h = const.$$

- Conformal symmetry for $\delta=0$

Dilatation anomaly

- Quantum fluctuations responsible for dilatation anomaly
- Running couplings: **hypothesis**

$$\partial\lambda/\partial \ln\chi = -A\lambda \, , \, \partial\delta/\partial \ln\chi = E\delta^2$$

- Renormalization scale μ : (momentum scale)
- $\lambda \sim (\chi/\mu)^{-A}$
- E > 0 : crossover Quintessence

Dilatation anomaly and quantum fluctuations

- Computation of running couplings (beta functions) needs unified theory !
- Dominant contribution from modes with momenta ~χ !
- No prejudice on "natural value " of anomalous dimension should be inferred from tiny contributions at QCD- momentum scale !

Cosmology

Cosmology : χ increases with time !

(due to coupling of χ to curvature scalar)

for large χ the ratio V/M^4 decreases to zero

→

Effective cosmological constant vanishes asymptotically for large t !

Asymptotically vanishing effective "cosmological constant"

- Effective cosmological constant ~ V/M^4
- $\lambda \sim (\chi/\mu)^{-A}$
- $V \sim (\chi/\mu)^{-A}\,\chi^4$
- $M = \chi$

$$\mathbf{V/M^4 \sim (\chi/\mu)^{-A}}$$

Weyl scaling

Weyl scaling : $g_{\mu\nu} \rightarrow (M/\chi)^2 g_{\mu\nu}$,

$\varphi/M = \ln(\chi^4/V(\chi))$

$$L = \sqrt{g}\Big(-\frac{1}{2}M^2 R + \frac{1}{2}k^2(\phi)\partial^\mu\phi\partial_\mu\phi + V(\phi) + m(\phi)\overline{\psi}\psi\Big)$$

Exponential potential : $V = M^4 \exp(-\varphi/M)$

No additional constant !

Without dilatation – anomaly :

V= const.

Massless Goldstone boson = dilaton

Dilatation – anomaly :

V (φ)

Scalar with tiny time dependent mass :

cosmon

Crossover Quintessence

$$\partial\delta/\partial \ln \chi = E\delta^2$$

(like QCD gauge coupling)

critical χ where δ grows large
critical φ where k grows large

$k^2(\varphi) = \delta(\chi)/4$

$k^2(\varphi) =$ "$1/(2E(\varphi_c - \varphi)/M)$"

if $\varphi_c \approx 276/M$ (tuning !) :

this will be responsible for relative increase of dark energy in present cosmological epoch

Realistic cosmology

Hypothesis on running couplings
yields realistic cosmology
for suitable values of A , E , φ_c

Quintessence cosmology
- models -

Dynamics of quintessence

- Cosmon φ : scalar singlet field
- Lagrange density $L = V + ½\, k(\varphi)\, \partial\varphi\, \partial\varphi$

  (units: reduced Planck mass M=1)
- Potential : $V=\exp[-\varphi]$
- "Natural initial value" in Planck era $\varphi=0$
- today: $\varphi=276$

Quintessence models

- Kinetic function k(φ) : parameterizes the details of the model - "kinetial"
  - $k(\varphi) = k=\text{const.}$ Exponential Q.
  - $k(\varphi) = \exp((\varphi - \varphi_1)/\alpha)$ Inverse power law Q.
  - $k^2(\varphi) =$ "$1/(2E(\varphi_c - \varphi))$" Crossover Q.

- possible naturalness criterion:

$k(\varphi=0)/\, k(\varphi_{today})$: not tiny or huge !

- else: explanation needed -

More models …

- Phantom energy (Caldwell)
  negative kinetic term ($w < -1$)
  consistent quantum theory ?

- K – essence (Amendariz-Picon, Mukhanov, Steinhardt)
  higher derivative kinetic terms
  why derivative expansion not valid ?

- Coupling cosmon / (dark) matter (C.W., Amendola)
  why substantial coupling to dark matter and not to ordinary matter ?

- Non-minimal coupling to curvature scalar – $f(\varphi)\, R$ -
  can be brought to standard form by Weyl scaling !

kinetial

$$\mathcal{L}(\varphi) = \frac{1}{2}(\partial\varphi)^2\, k^2(\varphi) + \exp[-\varphi]$$

Small almost constant k :

- Small almost constant Ω_h

Large k :

- Cosmon dominated universe (like inflation)

Cosmon

- *Tiny mass*
- $m_c \sim H$
- *New long - range interaction*

cosmon mass changes with time !

for standard kinetic term

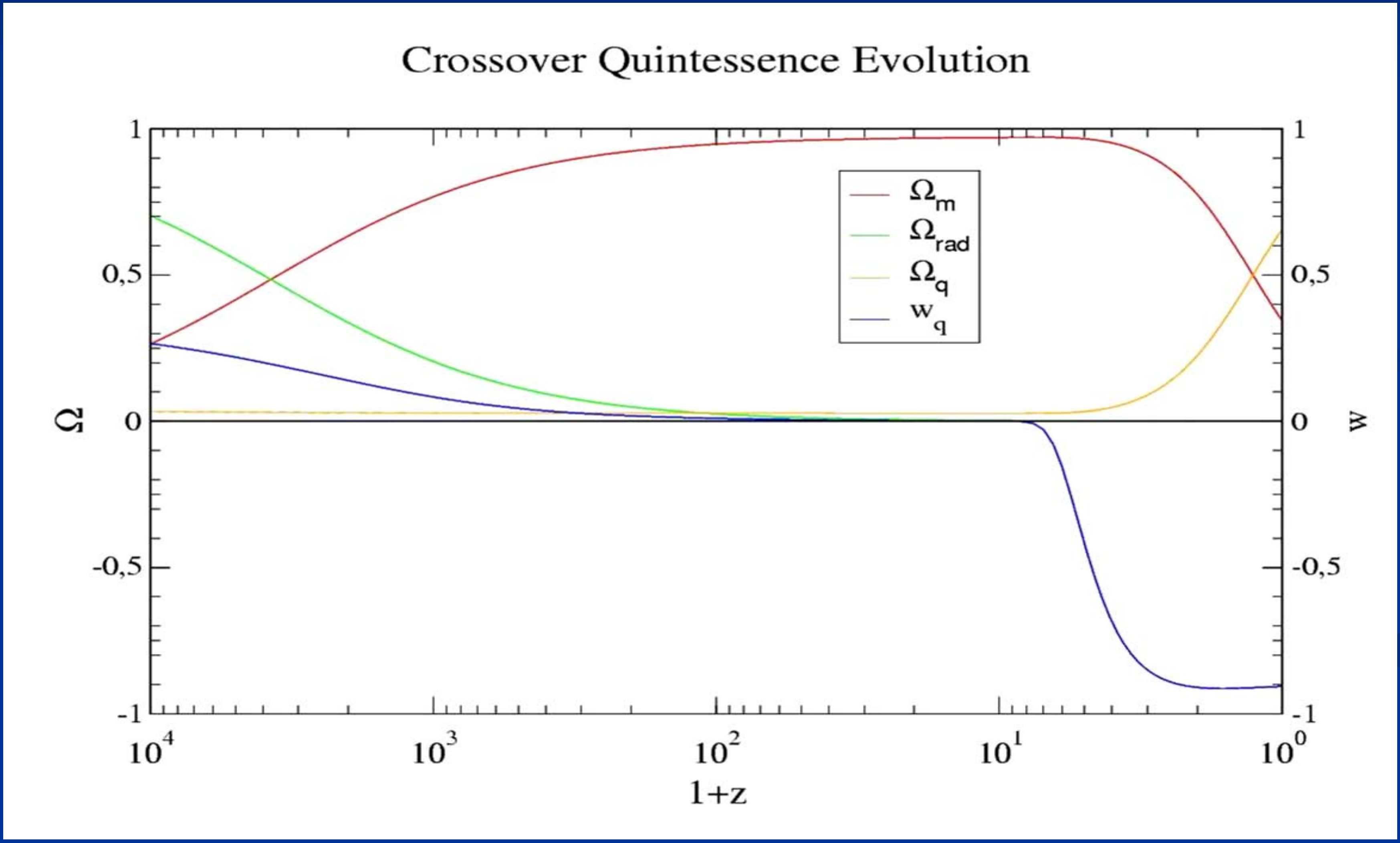
- $m_c^2 = V''$

for standard exponential potential , $k \approx$ const.

- $m_c^2 = V''/k^2 = V/(k^2 M^2)$

  $= 3\,\Omega_h\,(1 - w_h)\,H^2/(2\,k^2)$

Quintessence becomes important "today"



Transition to cosmon dominated universe

- Large value k >> 1 : universe is dominated by scalar field
- k increases rapidly : evolution of scalar fied essentially stops
- Realistic and natural quintessence:

  k changes from small to large values after structure formation

crossover quintessence

k(φ) increase strongly for φ corresponding to present epoch

Example (LKT) :

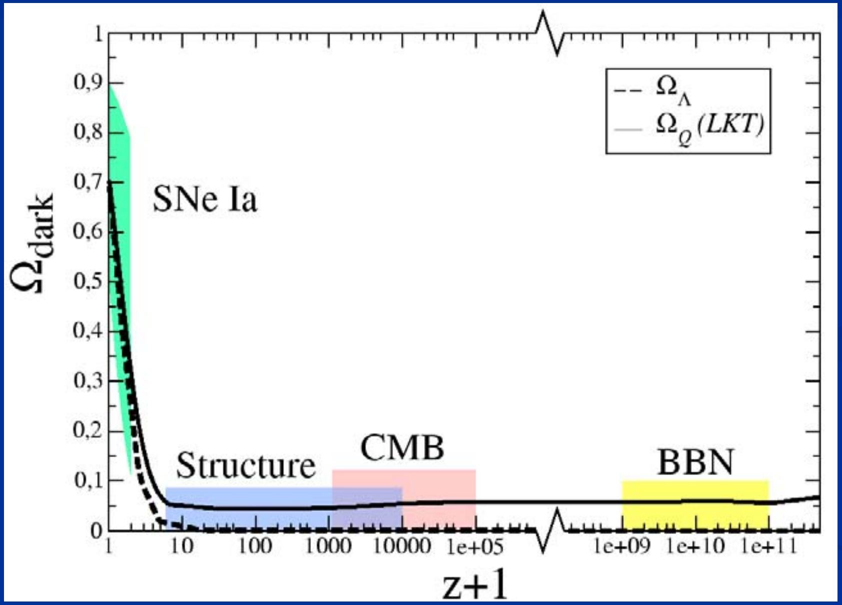
$$k(\varphi) = k_{min} + \tanh(\varphi - \varphi_1) + 1$$

$$(\text{with } k_{min} = 0.1\,,\quad \varphi_1 = 276.6\,)$$

exponential quintessence:

$$k = \frac{1}{\sqrt{2}\alpha}$$

Why has quintessence become important "now" ?



$$w_h = \frac{1}{3\Omega_h(1-\Omega_h)} \frac{\partial \Omega_h}{\partial \ln(1+z)}$$

coincidence problem

What is responsible for increase of Ω_h for $z < 10$?

a) Properties of cosmon potential or kinetic term

Late quintessence

- w close to -1
- Ω_h negligible in early cosmology
- needs tiny parameter, similar to cosmological constant

Early quintessence

- Ω_h changes only modestly
- w changes in time

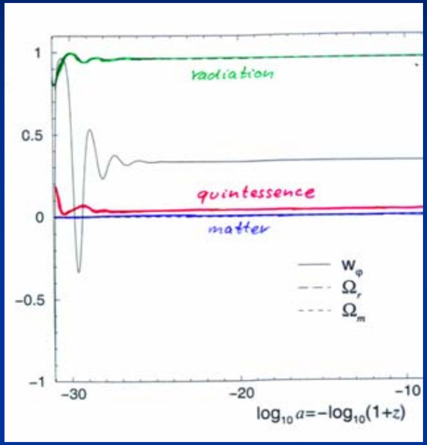
transition

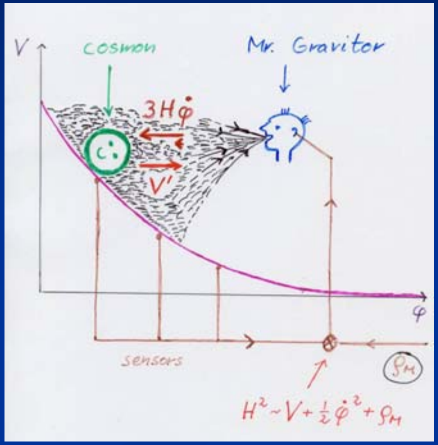
- special feature in cosmon potential or kinetic term becomes important "now"
- tuning at ‰ level

attractor solutions

Small almost constant k :

- Small almost constant Ω_h





⇨ This can explain tiny value of Dark Energy !

Large k :

- Cosmon dominated universe (like inflation)

$$\mathcal{L}(\varphi) = \frac{1}{2}(\partial\varphi)^2\, k^2(\varphi) + \exp[-\varphi]$$

Transition to cosmon dominated universe

- Large value k >> 1 : universe is dominated by scalar field
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b) Quintessence reacts to some special event in cosmology

- Onset of matter dominance

K- essence

Amendariz-Picon, Mukhanov, Steinhardt

needs higher derivative kinetic term

- Appearance of non-linear structure

Back-reaction effect

needs coupling between Dark Matter and Dark Energy

Back-reaction effect

scalar evolution equation

$$\langle \ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) \rangle = 0$$

$$0 = \ddot{\varphi}_0 + 3H\dot{\varphi}_0 + V'(\varphi_0) + V''(\varphi_0)\langle\chi\rangle + \tfrac{1}{2}V'''(\varphi_0)\langle\chi^2\rangle$$

fluctuation effect

backreaction

(In principle, same for metric, but small effect)

- Needs large inhomogeneities after structure has been formed
- Local cosmon field participates in structure

End

Quintessence from higher dimensions

work with J. Schwindt

hep-th/0501049

Time varying constants

- It is not difficult to obtain quintessence potentials from higher dimensional or string theories
- Exponential form rather generic

  (after Weyl scaling)
- But most models show too strong time dependence of constants !

Quintessence from higher dimensions

An instructive example:

Einstein – Maxwell theory in six dimensions

$$S = \int d^6x \sqrt{-g} \left\{ -\frac{M_6^4}{2} R + \lambda_6 + \frac{1}{4} F^{AB} F_{AB} \right\}$$

Warning : not scale - free !

Dilatation anomaly replaced by explicit mass scales.

Field equations

$$R_{AB} - \frac{1}{2} R g_{AB} = M_6^{-4} (T_{AB}^{(F)} + T_{AB}^{(M)} - \lambda_6 g_{AB}),$$

$$\partial_A (\sqrt{-g} F^{AB}) = 0.$$

Energy momentum tensor

$$T_{AB}^{(F)} = F_{AC}F_B{}^C - \frac{1}{4}F_{CD}F^{CD}g_{AB}$$

$$R_{AB} - \frac{1}{2}Rg_{AB} = M_6^{-4}(T_{AB}^{(F)} + T_{AB}^{(M)} - \lambda_6 g_{AB}),$$

$$\partial_A(\sqrt{-g}F^{AB}) = 0.$$

Metric

Ansatz with particular metric (not most general !)

which is consistent with

d=4 homogeneous and isotropic Universe

and internal U(1) x Z_2 isometry

$$ds^2 = \exp\left(-\frac{\phi(t)}{\bar{M}}\right)\{-dt^2 + a^2(t)d\vec{x}d\vec{x}\}$$

$$+\exp\left(\frac{\phi(t)}{\bar{M}}\right) r_0^2\{d\rho^2 + B^2 \sin^2\rho \, d\theta^2\}$$

$$r_0^2 = \frac{\bar{M}^2}{4\pi B M_6^4}$$

B ≠ 1 : football shaped internal geometry

Exact solution

$$A_\theta = \frac{m}{2e_6}(1-\cos\rho)$$

m : monopole number (integer)

$$H^2 = \frac{1}{3\bar{M}^2}(\frac{1}{2}\dot{\phi}^2 + V(\phi))$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

cosmology with scalar

and potential V :

$$V(\phi) = \bar{M}^4 \left\{ \frac{\lambda_6}{M_6^4 \bar{M}^2}\, e^{-\frac{\phi}{\bar{M}}} - 4\pi B \frac{M_6^4}{\bar{M}^4}\, e^{-\frac{2\phi}{\bar{M}}} + 2\pi^2 m^2 \frac{M_6^4}{e_6^2 \bar{M}^6}\, e^{-\frac{3\phi}{\bar{M}}} \right\}$$

Free integration constants

M , B , $\Phi(t=0)$, $(d\Phi/dt)(t=0)$: continuous

m : discrete

Conical singularities

deficit angle

$$\Delta = 2\pi(1 - B)$$

singularities can be included with
energy momentum tensor on brane

$$(T^{(B)})_\mu^\nu = \frac{B-1}{B r_0^2 e^{\phi/\bar{M}}} M_6^4 \left(\frac{\delta(\rho)}{\rho} + \frac{\delta(\rho - \pi)}{\pi - \rho} \right) \delta_\mu^\nu$$

bulk point of view : describe everything in terms of bulk geometry (no modes on brane without tail in bulk)

Asymptotic solution for large t

$$H = 2t^{-1}, \quad \phi = 2\bar{M} \ln \frac{t}{\sqrt{10} M_6^2 \lambda_6^{-1/2}}$$

$$\Omega_h = \frac{V + \frac{1}{2}\dot{\phi}^2}{3\bar{M}^2 H^2} \to 1$$

$$V + \frac{1}{2}\dot{\phi}^2 \propto t^{-2}$$

Naturalness

- No tuning of parameters or integration constants
- Radiation and matter can be implemented
- Asymptotic solution depends on details of model, e.g. solutions with constant $\Omega_h \neq 1$

problem :

time variation of fundamental constants

Dimensional reduction

$$L^{(4)} = -\frac{\bar{M}^2}{2} R + \frac{Z_1(\phi)}{4} F^{(1)}_{\mu\nu} F^{\mu\nu(1)}$$

$$+\frac{Z_2(\phi)}{4} F^{(2)}_{\mu\nu} F^{\mu\nu(2)}$$

$$+i \sum_j \bar{\psi}_j \gamma^\mu (\partial_\mu - i Q^{(1)}_j \bar{e}_1 A^{(1)}_\mu - i Q^{(2)}_j \bar{e}_2 A^{(2)}_\mu) \psi_j$$

$$+\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi)$$

Time dependent gauge coupling

$$e_{1(2)} = \frac{\bar{e}_{1(2)}}{\sqrt{Z_{1(2)}}}$$

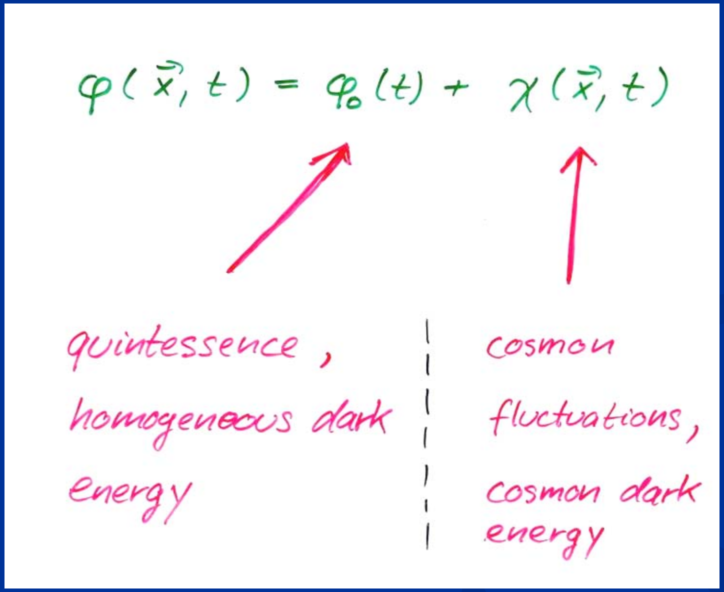
$$Z_1 = e^{\phi/\bar{M}}, \quad Z_2 = e^{2\phi/\bar{M}}$$

????????????????????????

- Why becomes Quintessence dominant in the present cosmological epoch ?
- Are dark energy and dark matter related ?
- Can Quintessence be explained in a fundamental unified theory ?

Cosmon dark matter ?

- Can cosmon fluctuations account for dark matter ?
- Cosmon can vary in space



$$\varphi_0(t) = \frac{1}{V}\int d^3x \, \varphi(\vec{x}, t)$$

cosmological expectation value

* similar to gravity
* different for gauge bosons, fermions

energy density in cosmon fluctuations ρ_c

$$\rho_c = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\left\{ |\dot{\chi}_k|^2 + \left(\frac{k^2}{a^2} + V''(\varphi_0)\right)|\chi_k|^2 \right.$$

$$\left. + \text{higher order terms} \right\}$$

quintessence ρ_q

$$\rho_q = \frac{1}{2}\dot{\varphi}_0^2 + V(\varphi_0) = T + V$$

Different equation of state

for ρ_c, ρ_q ?

$$w = p/\rho$$

Well possible !

e.g. • ρ_c dominated by modes inside the horizon, $\frac{k^2}{a^2} \gg H^2$

• neglect higher order terms

a) $\frac{k^2}{a^2} \gg V'' \Rightarrow \frac{p_c}{\rho_c} = \frac{1}{3}$, radiation

b) $\frac{k^2}{a^2} \ll V'' \Rightarrow \frac{p_c}{\rho_c} = 0$, matter

but

$$\frac{p_q}{\rho_q} = \frac{T-V}{T+V}$$, can be negativ !

- most quintessence models :

$$V'' \approx H^2$$

$$\Rightarrow \frac{p_c}{\rho_c} = \frac{1}{3} \qquad \text{or}$$

nonlinear terms play a role !

- one can construct models

with $V'' \gg H^2$ (Matos et al)

$\Rightarrow$ cosmon dark matter

$(H \approx 10^{-33} eV)$

Can nonlinear effects induce an

effective dynamical mass term ?