Dark Energy
from variation of the fundamental scale
What is our universe made of?

quintessence!

fire, air, water, soil!
Dark Energy dominates the Universe

Energy - density in the Universe

= 

Matter + Dark Energy

25 % + 75 %
Matter: Everything that clumps

Abell 2255 Cluster
~300 Mpc
Dark Energy density is the same at every point of space

“homogeneous“

No local force –

“In what direction should it draw?“

What is Dark Energy?

Cosmological Constant
or
Quintessence?
Quintessence and solution of cosmological constant problem should be related!
Cosmological Constant

- Einstein -

- Constant $\lambda$ compatible with all symmetries
- No time variation in contribution to energy density

- Why so small? $\lambda/M^4 = 10^{-120}$
- Why important just today?
Cosm. Const. | Quintessence
static | dynamical

\[ \frac{\lambda}{M_{pl}^4} = 10^{-124} \]

\[ \phi \sim t^{-2} \]

\( S_Q \)
challenge

- explain why Dark Energy goes to zero asymptotically,
- not to a constant!
\[ \Omega_m + X = 1 \]

\[ \Omega_m : 25\% \]

\[ \Omega_h : 75\% \]

Dark Energy
Time dependent Dark Energy: Quintessence

- What changes in time?
- Only dimensionless ratios of mass scales are observable!
- $V$: potential energy of scalar field or cosmological constant
- $V/M^4$ is observable
- Imagine the Planck mass $M$ increases …
Fundamental mass scale

- Unification fixes parameters with dimensions
- Special relativity: c
- Quantum theory: h
- Unification with gravity:
  fundamental mass scale
  (Planck mass, string tension, …)
<table>
<thead>
<tr>
<th>\textbf{Fundamental mass scale}</th>
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- **Fixed parameter or dynamical scale?**
- **Dynamical scale \leftrightarrow Field**
- **Dynamical scale compared to what?**

\textit{momentum versus mass}

\textit{(or other parameter with dimension)}
Cosmon and fundamental mass scale

- Assume all mass parameters are proportional to scalar field $\chi$ (GUTs, superstrings,...)
  - $M_p \sim \chi$, $m_{\text{proton}} \sim \chi$, $\Lambda_{\text{QCD}} \sim \chi$, $M_W \sim \chi$, ...

- $\chi$ may evolve with time: cosmon

- $m_n/M : (\text{almost})$ constant - observation!

Only ratios of mass scales are observable
Example:

Field $\chi$ is connected to scale of transition from higher dimensional physics to effective four dimensional description in theory without fundamental mass parameter (except for running of dimensionless couplings...)

theory without explicit mass scale

- Lagrange density:

\[
L = \sqrt{g} \left( -\frac{1}{2} \chi^2 R + \frac{1}{2} (\delta - 6) \partial^\mu \chi \partial_\mu \chi 
+ V(\chi) + h \chi \bar{\psi} \psi \right)
\]
realistic theory

- $\chi$ has no gauge interactions
- $\chi$ is effective scalar field after “integrating out” all other scalar fields
Dilatation symmetry

- **Lagrange density:**

\[
L = \sqrt{g} \left( -\frac{1}{2} \chi^2 R + \frac{1}{2} (\delta - 6) \partial^\mu \chi \partial_\mu \chi 
+ V(\chi) + h \chi \bar{\psi} \psi \right)
\]

- **Dilatation symmetry for**

\[
V = \lambda \chi^4, \quad \lambda = \text{const.}, \quad \delta = \text{const.}, \quad h = \text{const.}
\]

- **Conformal symmetry for** \(\delta=0\)
Dilatation anomaly

- Quantum fluctuations responsible for dilatation anomaly
- Running couplings: hypothesis
  \[ \frac{\partial \lambda}{\partial \ln \chi} = -A\lambda, \quad \frac{\partial \delta}{\partial \ln \chi} = E\delta^2 \]

- Renormalization scale \( \mu \): (momentum scale)
  - \( \lambda \sim (\chi/\mu)^{-A} \)
  - \( E > 0 \): crossover Quintessence
Asymptotic behavior of effective potential

- $\lambda \sim (\chi/\mu)^{-A}$
- $V \sim (\chi/\mu)^{-A} \chi^4$

$V \sim \chi^{4-A}$

crucial: behavior for large $\chi$!
Dilatation anomaly and quantum fluctuations

- Computation of running couplings (beta functions) needs unified theory!
- Dominant contribution from modes with momenta $\sim \chi$!
- No prejudice on “natural value “ of anomalous dimension should be inferred from tiny contributions at QCD- momentum scale!
Asymptotic behavior of effective potential

\[ V \sim \chi^{4-A} \]

e.g. \( V \sim \chi^2 \) or \( V \sim \text{const.} \)

crucial: behavior for large \( \chi \)!
Cosmology

Cosmology: $\chi$ increases with time!

( due to coupling of $\chi$ to curvature scalar )

for large $\chi$ the ratio $V/M^4$ decreases to zero

Effective cosmological constant vanishes asymptotically for large $t$!
Asymptotically vanishing effective “cosmological constant”

- Effective cosmological constant \( \sim V/M^4 \)

- \( \lambda \sim (\chi/\mu)^{-A} \)

- \( V \sim (\chi/\mu)^{-A} \chi^4 \)

- \( M = \chi \)

\[ V/M^4 \sim (\chi/\mu)^{-A} \]
Weyl scaling

Weyl scaling: \( g_{\mu\nu} \rightarrow (M/\chi)^2 g_{\mu\nu}, \)

\( \varphi/M = \ln (\chi^4/V(\chi)) \)

\[
L = \sqrt{g} \left( -\frac{1}{2} M^2 R + \frac{1}{2} k^2 (\phi) \partial^\mu \phi \partial_\mu \phi 
+ V(\phi) + m(\phi) \bar{\psi} \psi \right)
\]

Exponential potential: \( V = M^4 \exp(-\varphi/M) \)

No additional constant!
Without dilatation – anomaly :
\( V = \text{const.} \)
Massless Goldstone boson = dilaton

Dilatation – anomaly :
\( V (\varphi) \)
Scalar with tiny time dependent mass :
cosmon
quantum fluctuations and naturalness

- Jordan- and Einstein frame completely equivalent on level of effective action and field equations (after computation of quantum fluctuations!)
- Treatment of quantum fluctuations depends on frame: Jacobian for variable transformation in functional integral
- What is natural in one frame may look unnatural in another frame
quantum fluctuations and frames

- Einstein frame: quantum fluctuations make zero cosmological constant look unnatural
- Jordan frame: quantum fluctuations are at the origin of dilatation anomaly;
- key ingredient for solution of cosmological constant problem!
fixed points and fluctuation contributions of individual components

If running couplings influenced by fixed points:
individual fluctuation contribution can be huge overestimate!

here: fixed point at vanishing quartic coupling and anomalous dimension
\[ V \sim \chi^{4-A} \]

it makes no sense to use naïve scaling argument to infer individual contribution
\[ V \sim h \chi^4 \]
Exponential cosmon potential

\[ L = \sqrt{g} \left( -\frac{1}{2} M^2 R + \frac{1}{2} \kappa^2(\phi) \partial^{\mu} \phi \partial_\mu \phi \right. \]
\[ \left. + V(\phi) + m(\phi) \bar{\psi} \psi \right) \]

Exponential potential:

\[ V = M^4 \exp(-\phi/M) \]
Cosmic Attractors

Solutions independent of initial conditions

typically $V \sim t^{-2}$

$\varphi \sim \ln(t)$

$\Omega_h \sim \text{const.}$

details depend on $V(\varphi)$ or kinetic term
partial solution of cosmological constant problem

\[ \Omega_\text{h} \sim \text{const.} \]

Dark Energy and Matter of similar size!
Cosmological mass scales

- Energy density
  \[ \rho \sim (2.4 \times 10^{-3} \text{ eV})^{-4} \]

- Reduced Planck mass
  \[ M = 2.44 \times 10^{18} \text{ GeV} \]

- Newton’s constant
  \[ G_N = (8\pi M^2) \]

Only ratios of mass scales are observable!

- Homogeneous dark energy: \[ \rho_h/M^4 = 6.5 \times 10^{-121} \]
- Matter: \[ \rho_m/M^4 = 3.5 \times 10^{-121} \]
**Time evolution**

- $\rho_m / M^4 \sim a^{-3} \sim t^{-2}$  matter dominated universe
- $\rho_r / M^4 \sim a^{-4} \sim t^{-3/2}$  radiation dominated universe

Huge age $\Rightarrow$ small ratio

Same explanation for small dark energy?
Quintessence

Dynamical dark energy, generated by scalar field

(cosmon)

Prediction:

homogeneous dark energy influences recent cosmology
- of same order as dark matter -

Original models do not fit the present observations
.... modifications
realistic quintessence

fraction in dark energy has to increase in “recent time”!
Crossover Quintessence

\[ \frac{\partial \delta}{\partial \ln \chi} = E\delta^2 \] (like QCD gauge coupling)

critical \( \chi \) where \( \delta \) grows large

critical \( \varphi \) where \( k \) grows large

\[ k^2(\varphi) = \delta(\chi)/4 \]

\[ k^2(\varphi) = \frac{1}{2E(\varphi_c - \varphi)/M} \]

if \( \varphi_c \approx 276/M \) (tuning!):

this will be responsible for relative increase of dark energy in present cosmological epoch
Realistic cosmology

Hypothesis on running couplings yields realistic cosmology for suitable values of $A$, $E$, $\varphi_c$.
Quintessence becomes important “today”
many models...
the quintessence of Quintessence

**Cosmon – Field** $\varphi(x,y,z,t)$

similar to electric field, but no direction (scalar field)

may be fundamental or composite (effective) field

Homogeneous and isotropic Universe: $\varphi(x,y,z,t) = \varphi(t)$

Potential and kinetic energy of the cosmon-field contribute to a dynamical energy density of the Universe!
Cosmon

- Scalar field changes its value even in the present cosmological epoch
- Potential und kinetic energy of cosmon contribute to the energy density of the Universe
- Time - variable dark energy:
  \[ q_b(t) \text{ decreases with time!} \]
Cosmon

- Tiny (time varying) mass

- $m_c \sim H$

- New long-range interaction
“Fundamental” Interactions

Strong, electromagnetic, weak interactions

On astronomical length scales:

graviton

+ 

cosmon

gravitation  cosmodynamics
Dynamics of quintessence

- **Cosmon** $\varphi$: scalar singlet field

- Lagrange density $L = V + \frac{1}{2} k(\varphi) \partial \varphi \partial \varphi$
  
  (units: reduced Planck mass $M=1$)

- Potential: $V = \exp[-\varphi]

- “Natural initial value” in Planck era $\varphi=0$

- Today: $\varphi=276$
kinetial

\[ \mathcal{L}(\varphi) = \frac{1}{2} (\partial \varphi)^2 k^2(\varphi) + \exp[-\varphi] \]

Small almost constant \( k \):
- Small almost constant \( \Omega_h \)

Large \( k \):
- Cosmon dominated universe (like inflation)
Why has quintessence become important “now”? 

\[ w_h = \frac{1}{3\Omega_h (1-\Omega_h)} \frac{\partial \Omega_h}{\partial \ln(1+z)} \]
coincidence problem

What is responsible for increase of $\Omega_h$ for $z < 10$?
a) Properties of cosmon potential or kinetic term

Late quintessence
- $w$ close to -1
- $\Omega_h$ negligible in early cosmology
- Needs tiny parameter, similar to cosmological constant

Early quintessence
- $\Omega_h$ changes only modestly
- $w$ changes in time

Transition
- Special feature in cosmon potential or kinetic term becomes important “now”
- Tuning at $\%_0$ level
attractor solutions

Small almost constant $k$ :
- Small almost constant $\Omega_h$

$\rightarrow$ This can explain tiny value of Dark Energy!

Large $k$ :
- Cosmon dominated universe (like inflation)

\[ L(\varphi) = \frac{1}{2} (\partial \varphi)^2 k^2(\varphi) + \exp[-\varphi] \]
Transition to cosmon dominated universe

- Large value $k \gg 1$: universe is dominated by scalar field
- $k$ increases rapidly: evolution of scalar field essentially stops
- Realistic and natural quintessence:
  - $k$ changes from small to large values after structure formation
b) Quintessence reacts to some special event in cosmology

- **Onset of**
  - matter dominance

  - K-essence
    - Amendariz-Picon, Mukhanov, Steinhardt

  - needs higher derivative kinetic term

- **Appearance of**
  - non-linear structure

  - Back-reaction effect

  - needs coupling between Dark Matter and Dark Energy
Back-reaction effect

- Needs large inhomogeneities after structure has been formed
- Local cosmon field participates in structure

Scalar evolution equation:

\[ \langle \dddot{\phi} + 3H\dot{\phi} + V'(\phi) \rangle = 0 \]

\[ 0 = \dddot{\phi}_0 + 3H\dot{\phi}_0 + V'(\phi_0) + \frac{1}{2}V''(\phi_0)\langle \chi^2 \rangle + \frac{1}{2}V''(\phi_0)\langle \chi^2 \rangle \]

Fluctuation effect backreaction:

(In principle, same for metric, but small effect)
Time dependence of dark energy

$$\Omega_h \sim t^2 \sim (1+z)^{-3}$$

M.Doran,…
early dark energy

expected in models which explain same order of magnitude of dark energy and matter naturally
effects of early dark energy

- modifies cosmological evolution (CMB)
- slows down the growth of structure
Early quintessence slows down the growth of structure
Growth of density fluctuations

- Matter dominated universe with constant $\Omega_h$:

$$\Delta \rho \sim a^{1-\frac{\epsilon}{2}}, \quad \epsilon = \frac{5}{2} \left(1 - \sqrt{1 - \frac{24}{25} \Omega_h}\right)$$

P. Ferreira, M. Joyce

- Dark energy slows down structure formation
  $$\Rightarrow \Omega_h < 10\% \text{ during structure formation}$$
bounds on
Early Dark Energy
after WMAP’06
G.Robbers,M.Doran,…
interpolation of $\Omega_h$
Little Early Dark Energy can make large effect!
Non-linear enhancement

Two models with 4% Dark Energy during structure formation

Fixed $\sigma_8$
( normalization dependence!)

More clusters at high redshift!

Bartelmann, Doran, …
Quintessence from higher dimensions - an instructive example -

work with J. Schwindt

hep-th/0501049
It is not difficult to obtain quintessence potentials from higher dimensional or string theories.

Exponential form rather generic (after Weyl scaling)

But most models show too strong time dependence of constants!
Quintessence from higher dimensions

An instructive example:

Einstein – Maxwell theory in six dimensions

\[
S = \int d^6 x \sqrt{-g} \left\{ -\frac{M_6^4}{2} R + \lambda_6 + \frac{1}{4} F^{AB} F_{AB} \right\}
\]

Warning: not scale-free!
Dilatation anomaly replaced by explicit mass scales.
Field equations

\[ R_{AB} - \frac{1}{2} R g_{AB} = M_6^{-4} (T_{AB}^{(F)} + T_{AB}^{(M)} - \lambda_6 g_{AB}), \]

\[ \partial_A (\sqrt{-g} F^{AB}) = 0. \]
Energy momentum tensor

\[ T^{(F)}_{AB} = F_{AC}F_B^C - \frac{1}{4}F_{CD}F^{CD}g_{AB} \]

\[ R_{AB} - \frac{1}{2}Rg_{AB} = M_6^{-4}(T^{(F)}_{AB} + T^{(M)}_{AB} - \lambda_6 g_{AB}), \]

\[ \partial_A(\sqrt{-g}F^{AB}) = 0. \]
Metric

Ansatz with particular metric (not most general!) which is consistent with
d=4 homogeneous and isotropic Universe and internal $U(1) \times Z_2$ isometry

$$ds^2 = \exp\left(-\frac{\phi(t)}{M}\right) \{ -dt^2 + a^2(t) d\vec{x} d\vec{x} \}$$

$$+ \exp\left(\frac{\phi(t)}{M}\right) r_0^2 \{ d\rho^2 + B^2 \sin^2 \rho \, d\theta^2 \}$$

$r_0^2 = \frac{M^2}{4\pi B M_6^4}$

$B \neq 1$: football shaped internal geometry
Exact solution

\[ A_\vartheta = \frac{m}{2e_6}(1 - \cos \varrho) \]

\[ m : \text{monopole number (integer)} \]

\[ H^2 = \frac{1}{3M^2} \left( \frac{1}{2} \dot{\phi}^2 + V(\phi) \right) \]

\[ \ddot{\phi} + 3H \dot{\phi} + \frac{\partial V}{\partial \phi} = 0 \]

\[ V(\phi) = M^4 \left\{ \frac{\lambda_6}{M_6^4 M^2} e^{-\frac{\phi}{M}} - 4\pi B \frac{M_6^4}{M^4} e^{-\frac{2\phi}{M}} + 2\pi^2 m^2 \frac{M_6^4}{e_6^2 M^6} e^{-\frac{3\phi}{M}} \right\} \]
Free integration constants

\[ M, B, \Phi(t=0), (d\Phi/dt)(t=0) : \text{continuous} \]

\[ m : \text{discrete} \]
Conical singularities

deficit angle

\[ \Delta = 2\pi(1 - B) \]

singularities can be included with energy momentum tensor on brane

\[ (T^{(B)})^\nu_\mu = \frac{B - 1}{B\tau_0^2 e^{\phi/M}} M_6^4 \left( \frac{\delta(\rho)}{\rho} + \frac{\delta(\rho - \pi)}{\pi - \rho} \right) \delta^\nu_\mu \]

bulk point of view:
describe everything in terms of bulk geometry
( not possible for modes on brane without tail in bulk )
Warped branes

- model is similar to first co-dimension two brane model: C.W. Nucl.Phys.B255,480(1985);
  see also B253,366(1985)
- first realistic warped model
- see Rubakov and Shaposhnikov for earlier work
  (no stable solutions, infinitely many chiral fermions)
- see Randjbar-Daemi, C.W. for arbitrary dimensions
Asymptotic solution for large $t$

$$H = 2t^{-1}, \quad \phi = 2\tilde{M}\ln\frac{t}{\sqrt{10M_6^2\lambda_6^{-1/2}}}$$

$$\Omega_h = \frac{V + \frac{1}{2}\dot{\phi}^2}{3\tilde{M}^2H^2} \to 1$$

$$V + \frac{1}{2}\dot{\phi}^2 \propto t^{-2}$$
Naturalness

- No tuning of parameters or integration constants
- Radiation and matter can be implemented
- Asymptotic solution depends on details of model, e.g. solutions with constant $\Omega_h \neq 1$
problem:

time variation of fundamental constants
primordial abundances for three GUT models

present observations: $1\sigma$

T.Dent,
S.Stern,…
three GUT models

- unification scale \( \sim \) Planck scale
- 1) All particle physics scales \( \sim \Lambda_{QCD} \)
- 2) Fermi scale and fermion masses \( \sim \) unification scale
- 3) Fermi scale varies more rapidly than \( \Lambda_{QCD} \)

\[ \Delta \alpha / \alpha \approx 4 \times 10^{-4} \text{ allowed for GUT 1 and 3, larger for GUT 2} \]

\[ \Delta \ln(M_n/M_p) \approx 40 \Delta \alpha / \alpha \approx 0.015 \text{ allowed} \]
Dimensional reduction

\[ L^{(4)} = -\frac{\tilde{M}^2}{2}R + \frac{Z_1(\phi)}{4} F^{(1)}_{\mu\nu} F^{\mu\nu(1)} \]

\[ + \frac{Z_2(\phi)}{4} F^{(2)}_{\mu\nu} F^{\mu\nu(2)} \]

\[ + i \sum_j \bar{\psi}_j \gamma^\mu (\partial_\mu - i Q_j^{(1)} \bar{c}_1 A_\mu^{(1)} - i Q_j^{(2)} \bar{c}_2 A_\mu^{(2)}) \psi_j \]

\[ + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) \]
Time dependent gauge coupling

\[ e_{1(2)} = \frac{\bar{e}_{1(2)}}{\sqrt{Z_{1(2)}}} \]

\[ Z_1 = e^{\phi/M}, \quad Z_2 = e^{2\phi/M} \]
stabilizing the couplings...

gauge couplings go to zero as volume of internal space increases

two ways to solve this problem:

- irrelevant for modes on branes
- possible stabilization by fixed points in scale free models
Why becomes Quintessence dominant in the present cosmological epoch?

Are dark energy and dark matter related?

Can Quintessence be explained in a fundamental unified theory?
How to distinguish $Q$ from $\Lambda$?

A) Measurement $\Omega_h(z) \leftrightarrow H(z)$
   i) $\Omega_h(z)$ at the time of structure formation, CMB - emission or nucleosynthesis
   ii) equation of state $w_h(\text{today}) > -1$

B) Time variation of fundamental “constants”

C) Apparent violation of equivalence principle

D) Possible coupling between Dark Energy and Dark Matter
Cosmodynamics

**Cosmon mediates new long-range interaction**

- **Range:** size of the Universe – horizon
- **Strength:** weaker than gravity

- photon, electrodynamics
- graviton, gravity
- cosmon, cosmodynamics

**Small correction to Newton’s law**
“Fifth Force”

- Mediated by scalar field


- Coupling strength: weaker than gravity

  (nonrenormalizable interactions ~ $M^{-2}$)

- Composition dependence

  violation of equivalence principle

- Quintessence: connected to time variation of fundamental couplings

Violation of equivalence principle

Different couplings of cosmon to proton and neutron

Differential acceleration

“Violation of equivalence principle”

only apparent : new “fifth force”!
Differential acceleration

Two bodies with equal mass experience a different acceleration!

\[ \eta = \frac{a_1 - a_2}{a_1 + a_2} \]

bound: \( \eta < 3 \times 10^{-14} \)
Cosmon coupling to atoms

- Tiny !!!
- Substantially weaker than gravity.
- Non-universal couplings bounded by tests of equivalence principle.
- Universal coupling bounded by tests of Brans-Dicke parameter $\omega$ in solar system.
- Only very small influence on cosmology.
Cosmon coupling to Dark Matter

- Only bounded by cosmology
- Substantial coupling possible
- Can modify scaling solution and late cosmology
- Role in clustering of extended objects?

L. Amendola
Quintessence and time variation of fundamental constants

Generic prediction

Strength unknown


Strong, electromagnetic, weak interactions

gravitation  cosmodynamics
Time varying constants

- It is not difficult to obtain quintessence potentials from higher dimensional or string theories.
- Exponential form rather generic (after Weyl scaling).
- But most models show too strong time dependence of constants!
Are fundamental “constants”
time dependent?

- Fine structure constant $\alpha$ (electric charge)
- Ratio electron mass to proton mass
- Ratio nucleon mass to Planck mass
Quintessence and Time dependence of “fundamental constants”

- Fine structure constant depends on value of cosmon field: $\alpha(\phi)$

(similar in standard model: couplings depend on value of Higgs scalar field)

- Time evolution of $\phi$ → Time evolution of $\alpha$

Jordan,...
Standard – Model of electroweak interactions: Higgs - mechanism

- The masses of all fermions and gauge bosons are proportional to the (vacuum expectation) value of a scalar field $\varphi_H$ (Higgs scalar).

- For electron, quarks, W- and Z- bosons:

$$m_{\text{electron}} = h_{\text{electron}} \times \varphi_H$$ etc.
Restoration of symmetry at high temperature in the early Universe

Low T
SSB
$\langle \phi_H \rangle = \phi_0 \neq 0$

High T
SYM
$\langle \phi_H \rangle = 0$

high T: less order
more symmetry

example:
magnets
In the hot plasma of the early Universe:

No difference in mass for electron and muon!
symmetrisches Vakuum

unser Vakuum
Quintessence:
Couplings are still varying now!

Strong bounds on the variation of couplings - interesting perspectives for observation!
baryons:
the matter of stars and humans

$\Omega_b = 0.045$
Abundancies of primordial light elements from nucleosynthesis

A.Coc’05
Allowed values for variation of fine structure constant:

\[ \Delta \alpha/\alpha \ (z=10^{10}) = -1.0 \times 10^{-3} \quad \text{GUT 1} \]
\[ \Delta \alpha/\alpha \ (z=10^{10}) = -2.7 \times 10^{-4} \quad \text{GUT 2} \]

C. Mueller, G. Schaefer, ...
primordial abundances for three GUT models

He
D
Li

present observations: \(1\sigma\)

T. Dent, S. Stern, …
three GUT models

- unification scale \( \sim \) Planck scale
- 1) All particle physics scales \( \sim \Lambda_{QCD} \)
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\Delta \alpha / \alpha \approx 4 \times 10^{-4} \quad \text{allowed for GUT 1 and 3, larger for GUT 2}
\]

\[
\Delta \ln (M_n / M_P) \approx 40 \; \Delta \alpha / \alpha \approx 0.015 \quad \text{allowed}
\]
Variation of fine structure constant as function of redshift

Three independent data sets from Keck/HIRES

\[ \Delta \alpha / \alpha = -0.54 \pm 0.12 \times 10^{-5} \]

Murphy, Webb, Flammbaum, June 2003

VLT

\[ \Delta \alpha / \alpha = -0.06 \pm 0.06 \times 10^{-5} \]

Srianand, Chand, Petitjean, Aracil, Feb. 2004

\[ z \approx 2 \]
Atomic clocks and OKLO

* Atomic clocks:

\[
\frac{\dot{\alpha}_{\text{em}}}{\alpha_{\text{em}}} = -5.4 \cdot 10^{-10} \frac{\Delta \alpha_{\text{em}} (z=0.13)}{\alpha_{\text{em}}} \text{ yr}^{-1}
\]

observation \[\frac{\dot{\alpha}_{\text{em}}}{\alpha_{\text{em}}} = (4.2 \pm 6.9) \cdot 10^{-15} \text{ yr}^{-1}\]

Sortais et al.

assumes that both effects are dominated by change of fine structure constant
Time variation of coupling constants must be tiny –

would be of very high significance!

Possible signal for Quintessence
Everything is flowing

Παντα ρει
Apparent violation of equivalence principle and time variation of fundamental couplings measure both the cosmon – coupling to ordinary matter
Differential acceleration $\eta$

For unified theories (GUT):

$$\eta = -1.75 \times 10^{-2} \Delta R_Z \left( \frac{\partial \ln \alpha}{\partial Z} \right)^2 \frac{1 + \tilde{Q}}{\Omega_h(1 + \omega_h)}$$

$$\Delta R_Z = \frac{\Delta Z}{Z + N} \approx 0.1$$

$\eta = \Delta a / 2a$

$Q$ : time dependence of other parameters
Link between time variation of $\alpha$

and violation of equivalence principle

typically: $\eta = 10^{-14}$

if time variation of $\alpha$

near Oklo upper bound

to be tested (MICROSCOPE, ...)

Summary

- $\Omega_h = 0.7$

- $Q/\Lambda$ : dynamical und static dark energy
  will be distinguishable

- $Q$ : time varying fundamental coupling “constants”
  violation of equivalence principle
End
Quintessence cosmology
- models -
Quintessence models

- Kinetic function $k(\phi)$: parameterizes the details of the model - “kinetial”

  - $k(\phi) = k=\text{const.}$ Exponential Q.
  - $k(\phi) = \exp ((\phi - \phi_1)/\alpha)$ Inverse power law Q.
  - $k^2(\phi) = \frac{1}{2E(\phi_c - \phi)}$ Crossover Q.

- possible naturalness criterion:

  $k(\phi=0)/ k(\phi_{\text{today}}$: not tiny or huge!

  - else: explanation needed -
More models ...

- **Phantom energy** (Caldwell)
  - negative kinetic term (\( w < -1 \))
  - consistent quantum theory?

- **K – essence** (Amendariz-Picon, Mukhanov, Steinhardt)
  - higher derivative kinetic terms
  - why derivative expansion not valid?

- **Coupling cosmon / (dark ) matter** (C.W.’95, Amendola)
  - why substantial coupling to dark matter and not to ordinary matter?

- **Non-minimal coupling to curvature scalar** – \( f(\varphi) \) \( R \) –
  - can be brought to standard form by Weyl scaling!

- **Non-local gravity** (C.W.’97, Reuter, Turner,..)
  - not obvious where non-local terms come from
Cosmon

- Tiny mass

- $m_c \sim H$

- New long-range interaction
cosmon mass changes with time!

for standard kinetic term

\[ m_c^2 = V'' \]

for standard exponential potential, \( k \approx \text{const.} \)

\[ m_c^2 = \frac{V''}{k^2} = \frac{V}{(k^2 M^2)} \]

\[ = 3 \Omega_h (1 - w_h) H^2 / (2 k^2) \]
Quintessence becomes important “today”
Equation of state

\[ p = T - V \]  \hspace{1cm} \text{pressure} \hspace{1cm} \text{kinetic energy} \hspace{1cm} T = \frac{1}{2} \dot{\phi}^2

\[ \rho = T + V \]  \hspace{1cm} \text{energy density} \hspace{1cm}

\[ w = \frac{p}{\rho} = \frac{T-V}{T+V} \]

Depends on specific evolution of the scalar field
Negative pressure

- $w < 0$ \quad $\Omega_h$ increases \quad (with decreasing $z$)

\begin{align*}
\text{late universe with small radiation component:} \quad w_h &= \frac{1}{3\Omega_h(1-\Omega_h)} \left( \frac{\partial \Omega_h}{\partial \ln(1+z)} \right)
\end{align*}

- $w < -1/3$ \quad expansion of the Universe is accelerating

- $w = -1$ \quad cosmological constant
Transition to cosmon dominated universe

- Large value $k >> 1$: universe is dominated by scalar field
- $k$ increases rapidly: evolution of scalar field essentially stops
- Realistic and natural quintessence:
  - $k$ changes from small to large values after structure formation
crossover quintessence

$k(\varphi)$ increase strongly for $\varphi$ corresponding to present epoch

Example (LKT) :

$$k(\varphi) = k_{\text{min}} + \tanh(\varphi - \varphi_1) + 1$$

(with $k_{\text{min}} = 0.1$, $\varphi_1 = 276.6$)

exponential quintessence:

$$k = \frac{1}{\sqrt{2\alpha}}$$
Cosmon dark matter?

- Can cosmon fluctuations account for dark matter?
- Cosmon can vary in space

\[
\varphi(\vec{x}, t) = \varphi_0(t) + \chi(\vec{x}, t)
\]
\[
\varphi_0(t) = \frac{1}{V} \int d^3x \varphi(x^2, t)
\]

**cosmological expectation value**

* similar to gravity
* different for gauge bosons, fermions

-energy density in cosmological fluctuations \(\rho_c\)

\[
\rho_c = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \left\{ |\dot{\chi}_k|^2 + \left( \frac{k^2}{a^2} + V''(\varphi_0) \right) |\chi_k|^2 \right\}
\]

+ higher order terms

-quintessence \(\rho_q\)

\[
\rho_q = \frac{1}{2} \dot{\varphi}_0^2 + V(\varphi_0) = T + V
\]
Different equation of state
for $p_c, \rho_c$ ?

$\omega = p/\rho$

Well possible!

e.g. $\rho_c$ dominated by modes inside the horizon, $\frac{k^2}{a^2} \gg H^2$

- neglect higher order terms
  a) $\frac{k^2}{a^2} \gg V'' \Rightarrow \frac{p_c}{\rho_c} = \frac{1}{3}$, radiation
  b) $\frac{k^2}{a^2} \ll V'' \Rightarrow \frac{p_c}{\rho_c} = 0$, matter

but

$$\frac{p_q}{\rho_q} = \frac{T-V}{T+V}, \text{ can be negative!}$$
most quintessence models:

\[ V'' \approx H^2 \]

\[ \Rightarrow \frac{P_m}{P_c} = \frac{1}{3} \quad \text{or} \quad \text{nonlinear terms play a role!} \]

one can construct models with \( V'' \gg H^2 \) (Matos et al.)

\[ \Rightarrow \text{cosmon dark matter} \]

(\( H \approx 10^{-33}\text{eV} \))

Can nonlinear effects induce an effective dynamical mass term?