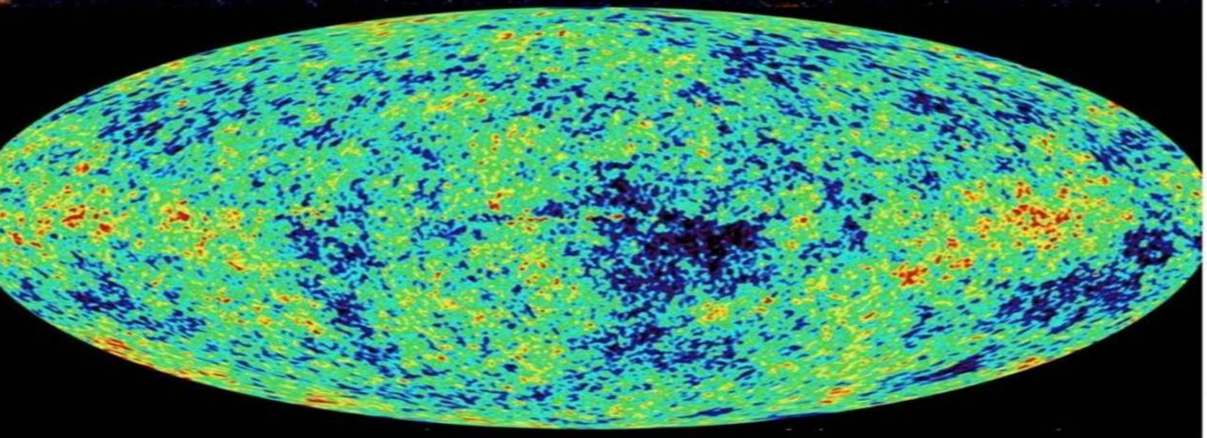
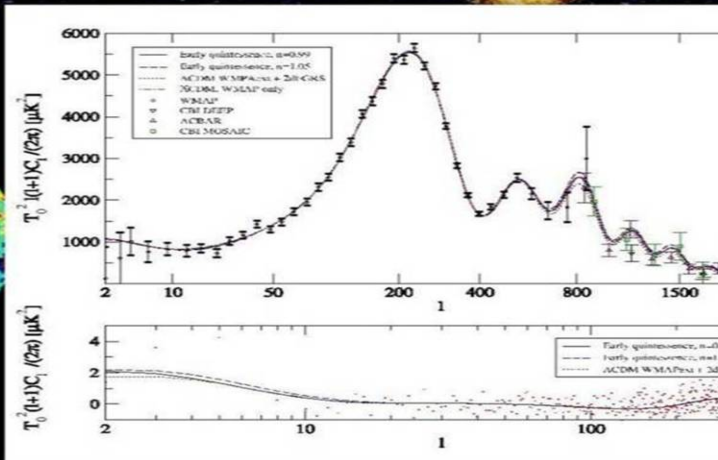
Quintessence





Quintessence

C.Wetterich

A.Hebecker,M.Doran,M.Lilley,J.Schwindt,
C.Müller,G.Schäfer,E.Thommes,
R.Caldwell

What is our Universe made of ?


Quintessence !
fire , air,
water, soil !

critical density

- $\varrho_c = 3 H^2 M^2$
  critical energy density of the universe
  (M : reduced Planck-mass , H : Hubble parameter)

- $\Omega_b = \varrho_b / \varrho_c$
  fraction in baryons
  energy density in baryons over critical energy density

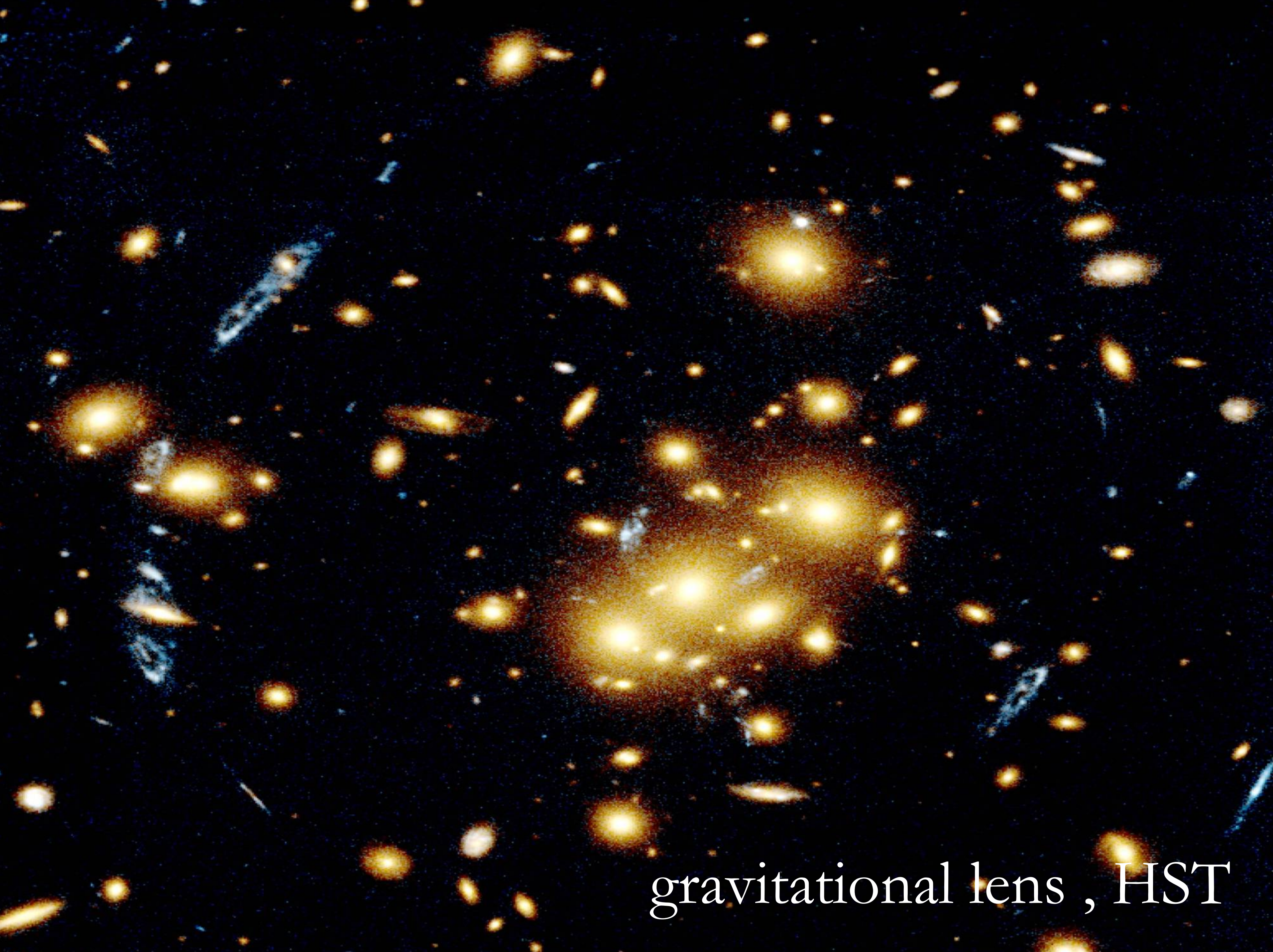
$$H = \dot{a}/a$$

Composition of the universe

$$\Omega_b = 0.045$$

$$\Omega_{dm} = 0.225$$

$$\Omega_h = 0.73$$



gravitational lens , HST

spatially flat universe

$$\Omega_{tot} = 1$$

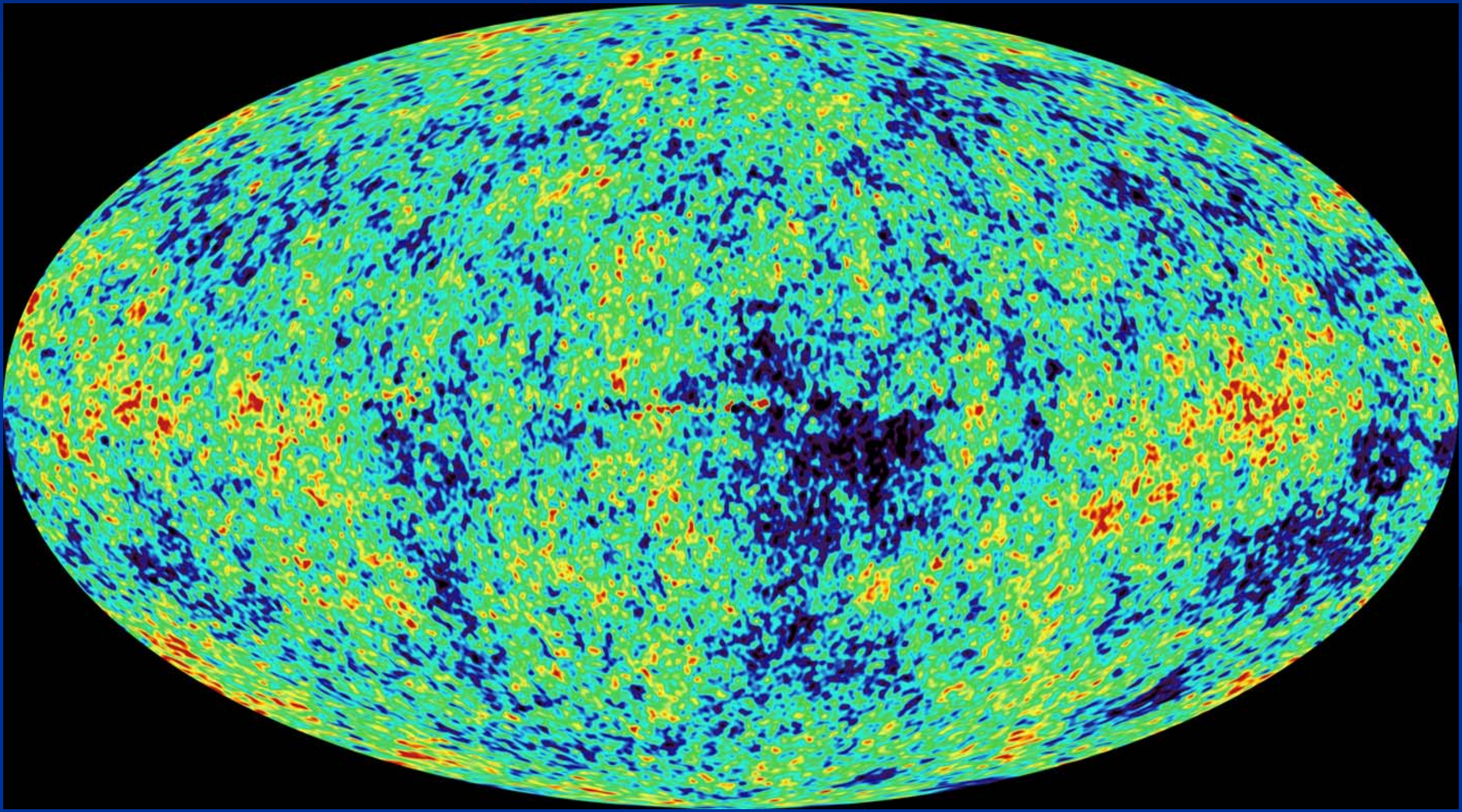
- theory (inflationary universe)

  Ω_{tot} =1.0000……….x

- observation (WMAP)

  Ω_{tot} =1.02 (0.02)

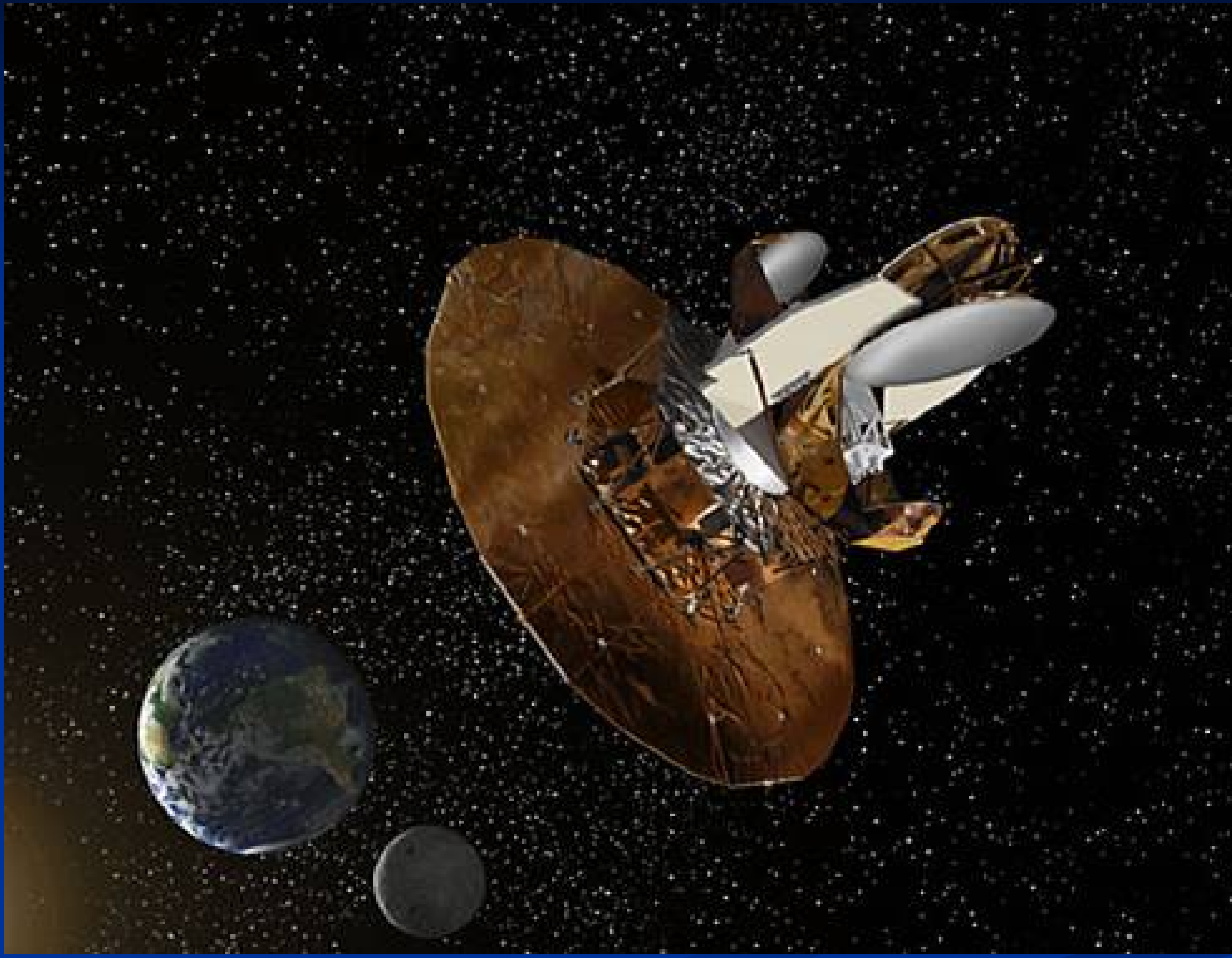
picture of the big bang



Wilkinson Microwave Anisotropy Probe

A partnership between NASA/GSFC and Princeton

Science Team:



NASA/GSFC

Chuck Bennett (**PI**)
Michael Greason
Bob Hill
Gary Hinshaw
Al Kogut
Michele Limon
Nils Odegard
Janet Weiland
Ed Wollack

Brown

Greg Tucker

UCLA

Ned Wright

UBC

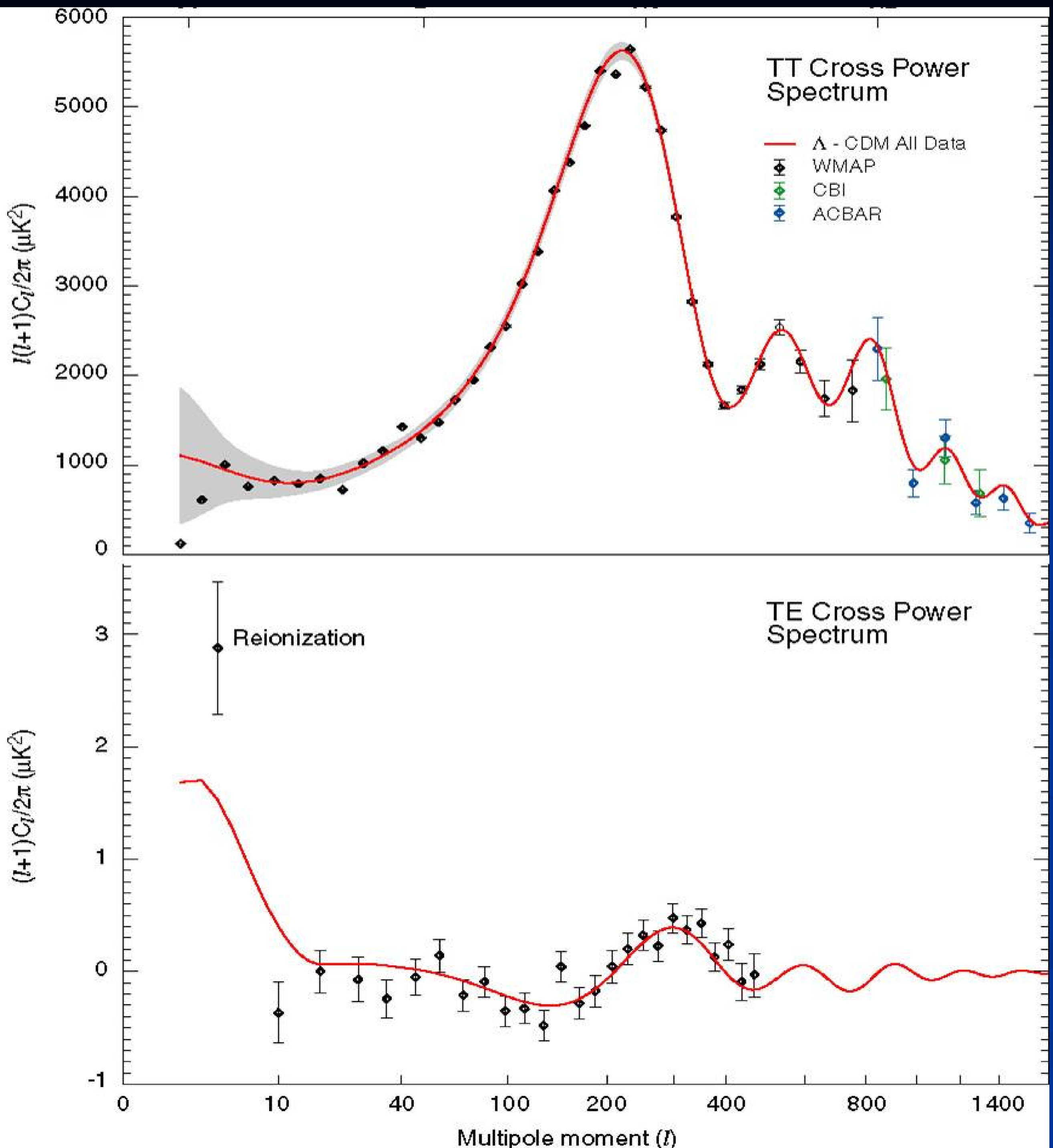
Mark Halpern

Chicago

Stephan Meyer

Princeton

Chris Barnes
Norm Jarosik
Eiichiro Komatsu
Michael Nolta
Lyman Page
Hiranya Peiris
David Spergel
Licia Verde



mean values

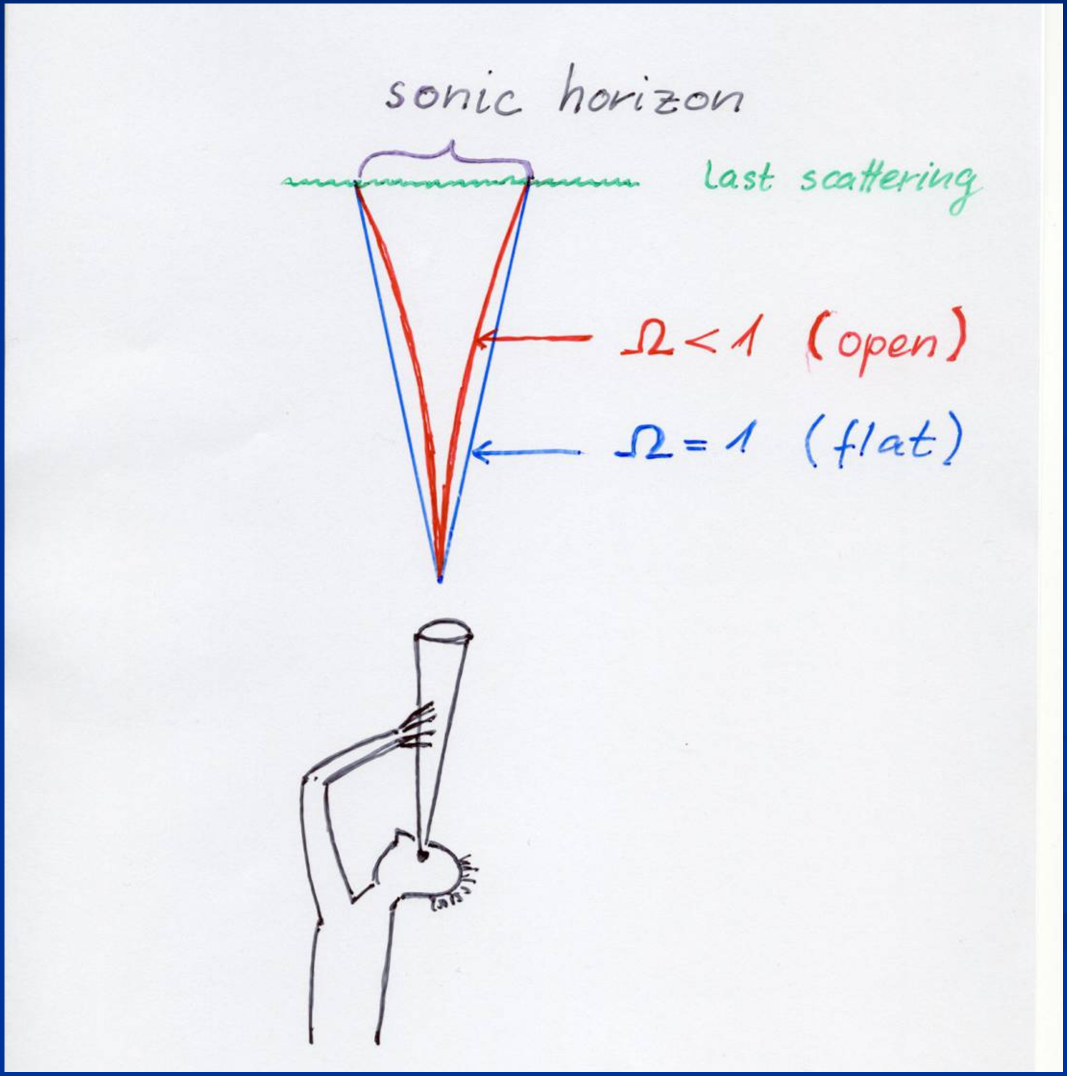
$\Omega_{tot} = 1.02$

$\Omega_{m} = 0.27$

$\Omega_{b} = 0.045$

$\Omega_{dm} = 0.225$

$\Omega_{tot}=1$



Dark Energy

$$\Omega_m + X = 1$$

$$\Omega_m \ : \ 30\%$$

$\Omega_h \ : \ 70\%$ Dark Energy

h : homogenous , often Ω_Λ instead of Ω_h

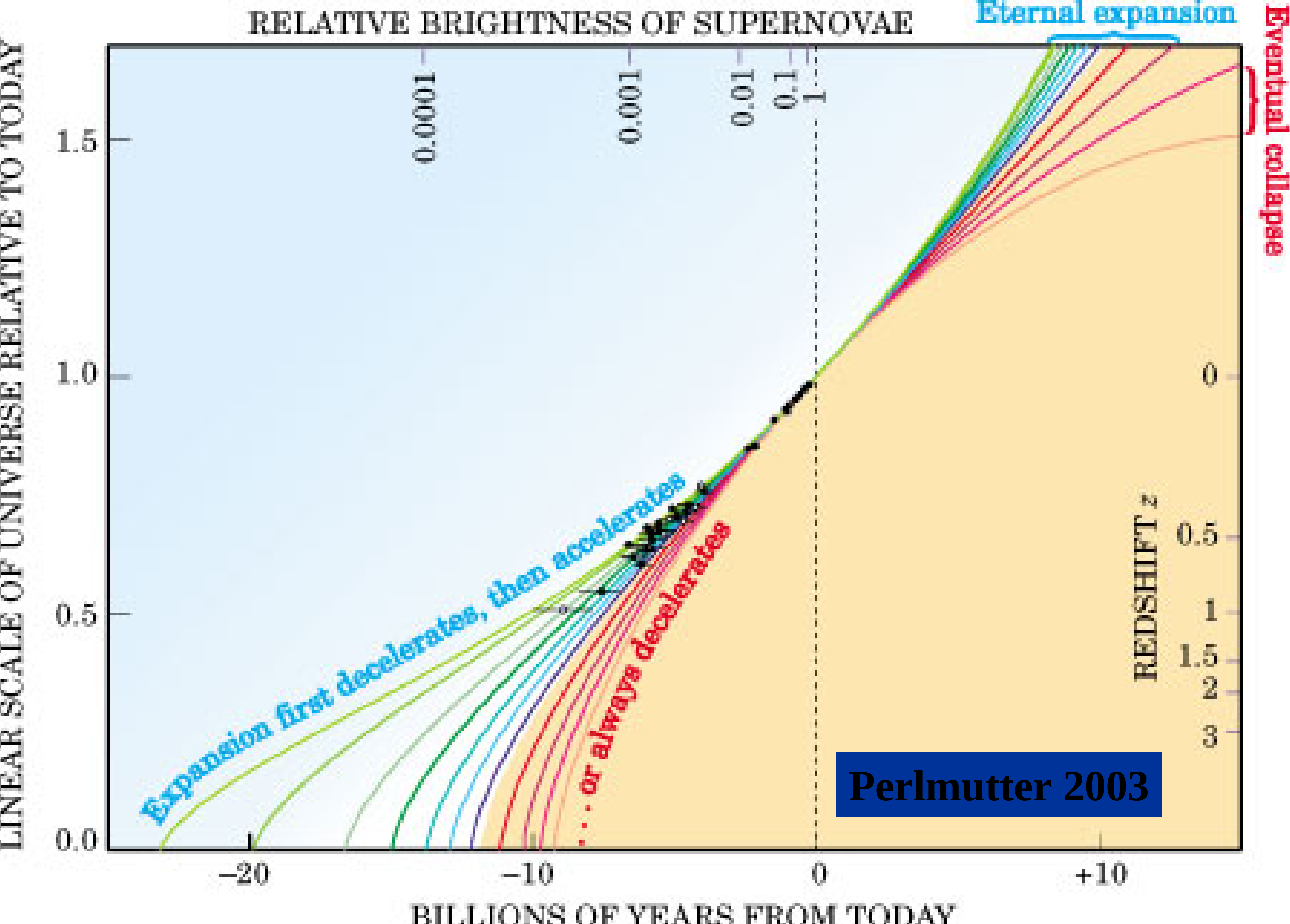
Dark Energy :

homogeneously distributed

Dark Energy :

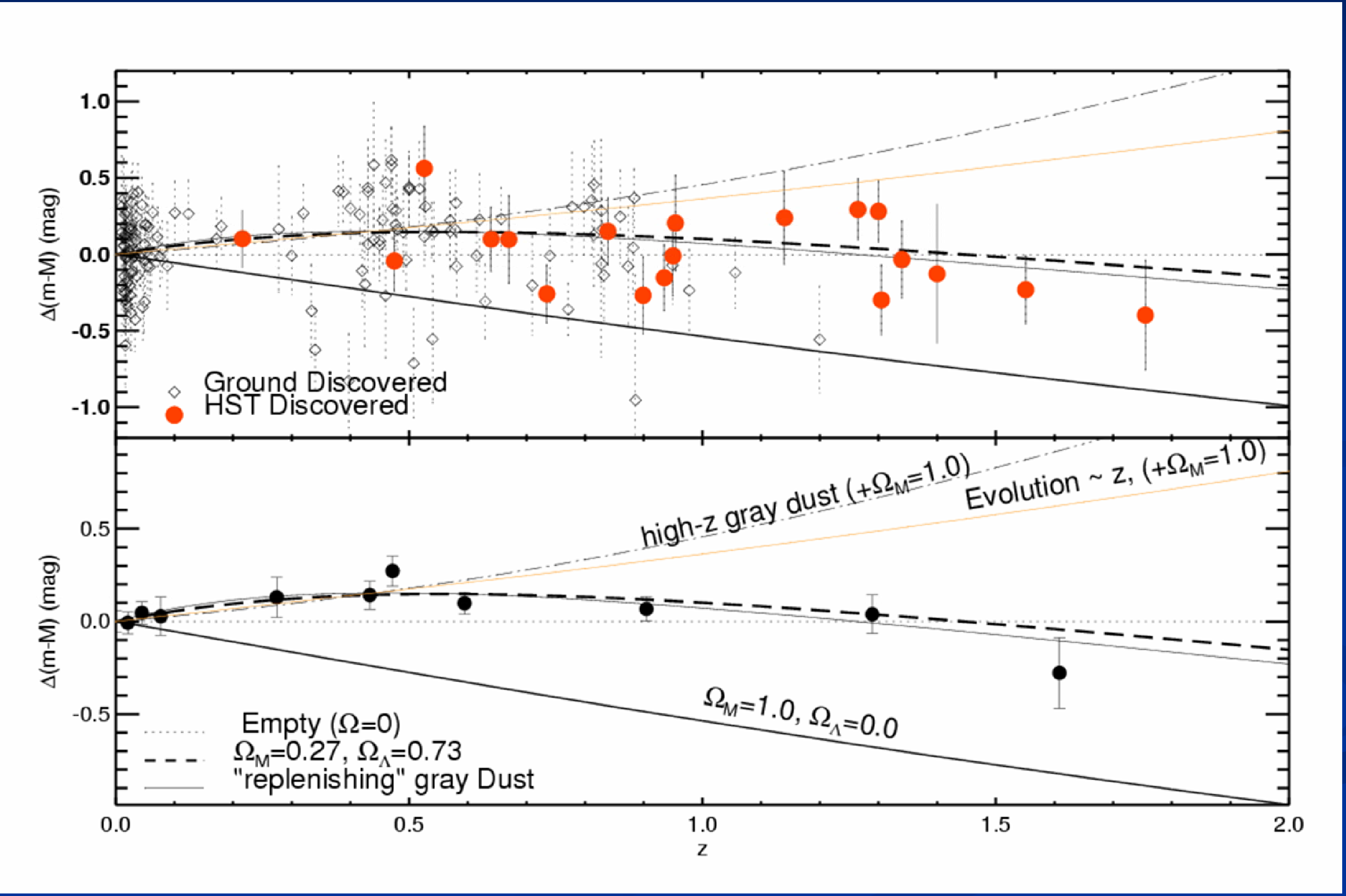
prediction:

The expansion of the Universe accelerates today !



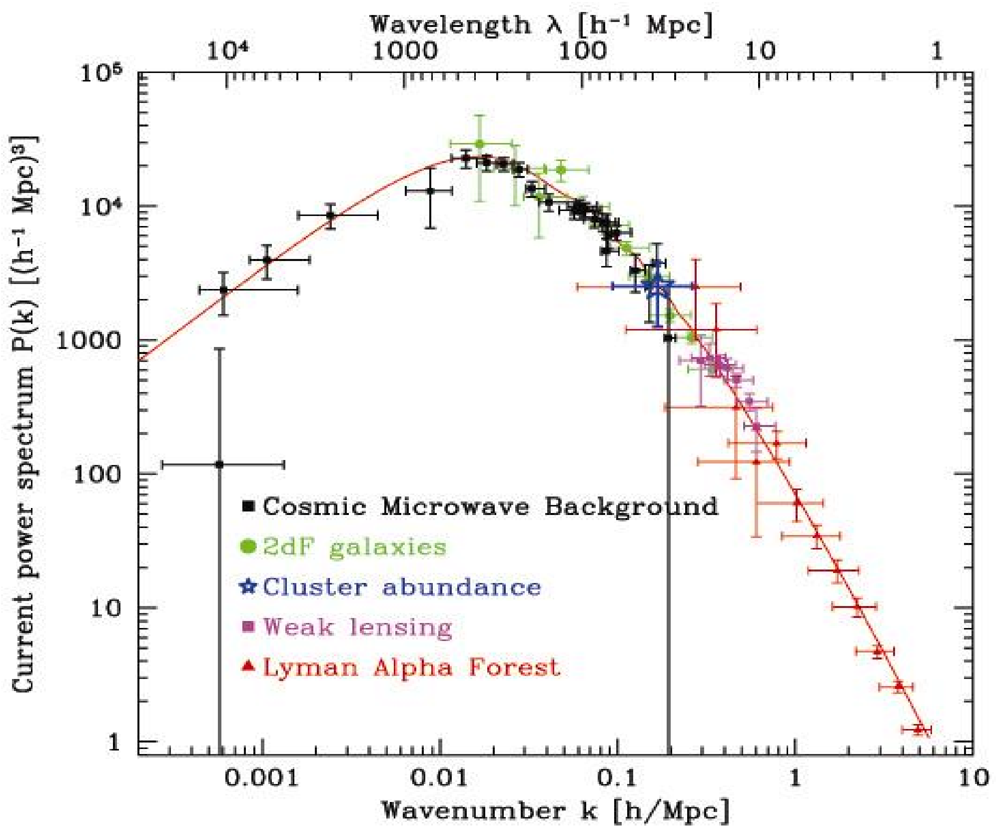
Perlmutter 2003

Supernova cosmology



Riess et al. 2004

Structure formation : fluctuation spectrum



Waerbeke

CMB agrees with

galaxy distribution

Lyman – α forest

and

gravitational lensing effect !

consistent cosmological model !

Composition of the Universe

$\Omega_b = 0.045$	visible	clumping
$\Omega_{dm} = 0.225$	invisible	clumping
$\Omega_h = 0.73$	invisible	homogeneous

What is Dark Energy ?

Cosmological Constant

or

Quintessence ?

Dynamics of Dark Energy

Cosmological Constant

- Constant λ compatible with all symmetries
- No time variation in contribution to energy density

- Why so small ? $\lambda/M^4 = 10^{-120}$

- Why important just today ?

Einstein equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = - \frac{8\pi}{M_p^2} T_{\mu\nu}$$

$$M_p = 1.22 \cdot 10^{19}\,\text{GeV} = G_N^{-1/2}$$

Energy-momentum-tensor

$$T_{\mu\nu} = T_{\mu\nu}^{(\text{baryon})} + T_{\mu\nu}^{(\text{radiation})}$$

$$+ T_{\mu\nu}^{(\text{dark matter})}$$

$$+ T_{\mu\nu}^{(\text{homogeneous})}$$

$$T_{\mu\nu}^{(\text{homogenous})} \stackrel{?}{=}$$

$$\left\{ \begin{array}{ll} \lambda\, g_{\mu\nu} & : \text{cosmological const.} \\ T_{\mu\nu}^{(q)} & : \text{quintessence} \end{array} \right.$$

scalar field ?

nonlocal gravity ?

$\vdots$

Gravitational action

$$S = + \int d^4x \, g^{1/2} \left(- \frac{M_p^2}{16\pi} R + \lambda \right)$$

λ : cosmological constant

Field equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi}{M_p^2} \left(T^M_{\mu\nu} - \lambda g_{\mu\nu} \right)$$

$M_p \approx 10^{19}$ GeV : Planck mass

$T^M_{\mu\nu}$: matter energy momentum tensor

accounts for nonvanishing entropy in the universe

without matter:

$$R = + \frac{32\pi}{M_p^2} \lambda$$

Cosmological constant

$$T_{\mu\nu} = T^{M}_{\mu\nu} - \lambda g_{\mu\nu}$$

$$\rho \rightarrow \rho + \lambda = \rho + \rho_\lambda$$

$$p \rightarrow p - \lambda = p + p_\lambda$$

$$\rho + p \rightarrow \rho + p$$

—

„Equation of state"

$$p_\lambda / \rho_\lambda = -1$$

Friedman universe ($\lambda = 0$)

Einstein equations $\rightarrow$

(i) $H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3M_p^2}\rho - \frac{k}{a^2}$

(evolution equation)

(ii) $\dot{\rho} + 3H(\rho + p) = 0$

(energy-momentum-conservation)

$\Leftrightarrow \frac{d}{dt}\left[a^3(\rho + p)\right] = a^3 \frac{d}{dt} p$

radiation ($p = \frac{1}{3}\rho$) $\quad \rho \sim a^{-4}$

matter ($p = 0$) $\quad \rho \sim a^{-3}$

$k = 0$ (always applicable for early universe)

Field equations involve only the Hubble-parameter $H = \dot{a}/a$

$\lambda \neq 0$

Einstein equations →

(i) $H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3M_P^2}(\varrho + \lambda) - \frac{k}{a^2}$

(evolution equation)

(ii) $\dot{\rho} + 3H(\rho + p) = 0$

(energy-momentum-conservation)

$\Leftrightarrow \frac{d}{dt}[a^3(\rho + p)] = a^3 \frac{d}{dt} p$

radiation $(p = \frac{1}{3}\rho)$ $\rho \sim a^{-4}$

matter $(p = 0)$ $\rho \sim a^{-3}$

<u>$k = 0$</u> (always applicable for early universe)

Field equations involve only the Hubble-parameter $H = \dot{a}/a$

Only minor modification for

$\lambda \ll \rho$

For $t \to \infty$: $\lambda \neq 0$ has always important effects !

asymptotic solution for cosmological constant (k=0)

$\lambda > 0$

$$H^2 \rightarrow \frac{8\pi}{3M_p^2}\lambda \qquad a \sim \exp H_0 t$$

$\lambda = 0$

$$H \rightarrow \eta t^{-1} \qquad a \sim t^{\eta}$$

$\lambda < 0$

$$H \rightarrow \left(\frac{8\pi}{3M_p^2}|\lambda|\right)^{1/2} tg\,(c_1 - c_2 t)$$

e.g. a expands to maximal a_0 and shrinks subsequently

For

$$|\lambda| \ll \frac{3 M_p^2 H^2}{8\pi} \quad :$$

only small corrections to

standard cosmology

$$\lambda \approx (0.6 - 0.7)\, \rho_c$$

good candidate for dark energy !

compatible with observation !

problems with small λ

- no symmetry explanation for $\lambda/M^4 = 10^{-120}$
- quantum fluctuations contribute

Zero point energies for normal modes of field with mass m, for wave numbers $|k| < \Lambda$ $\quad (m^2 \ll \Lambda^2)$

$$\langle \rho \rangle_{vac} = \int_0^{\Lambda} \frac{4\pi k^2 \, dk}{(2\pi)^3} \cdot \frac{1}{2}\sqrt{k^2+m^2} \simeq \frac{\Lambda^4}{16\pi^2}$$

In QED, zero point energies are measured by Casimir effect

—

detection of cosmological constant:

Link between quantum fluctuations and gravity !

Anthropic principle

For

$$\lambda \leq -\frac{1}{2}\rho_c$$

or

$$\lambda > (10-100)\rho_c$$

we simply would not exist!

Banks
Weinberg
Linde

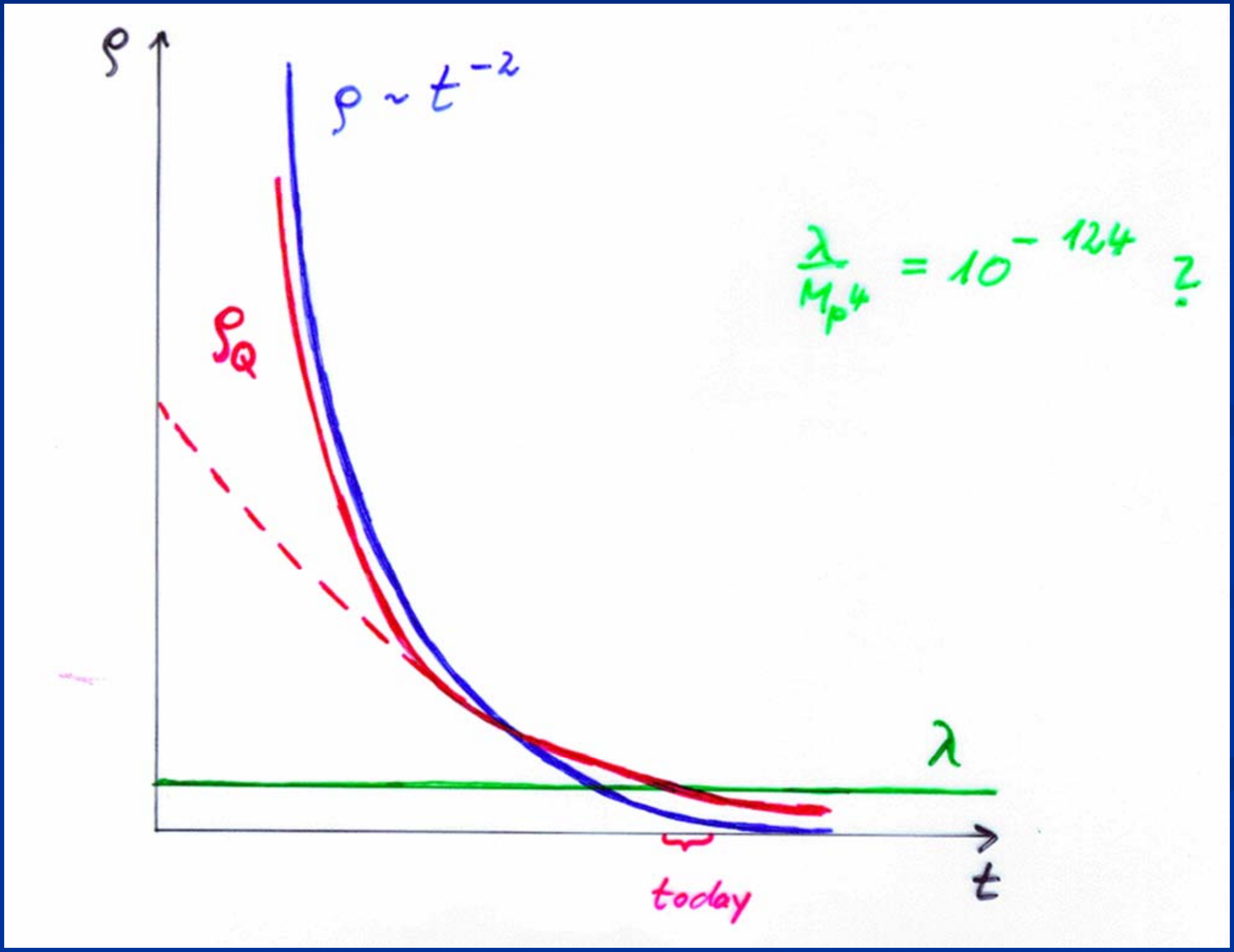
Cosmological Constant

- Constant λ compatible with all symmetries
- No time variation in contribution to energy density

- Why so small ? $\lambda/M^4 = 10^{-120}$

- Why important just today ?

Cosm. Const. | Quintessence
static | dynamical



Cosmological mass scales

- Energy density

  $\rho \sim (2.4\times10^{-3}\ \mathrm{eV})^{-4}$

- Reduced Planck mass

  $M=2.44\times10^{18}\mathrm{GeV}$

- Newton's constant

  $G_N=(8\pi M^2)$

Only ratios of mass scales are observable !

homogeneous dark energy: $\rho_h/M^4 = 6.5\ 10^{-121}$

matter: $\rho_m/M^4= 3.5\ 10^{-121}$

Time evolution

- $\rho_m/M^4 \sim a^{-3} \sim$
  - t^{-2} matter dominated universe
  - $t^{-3/2}$ radiation dominated universe
- $\rho_r/M^4 \sim a^{-4} \sim t^{-2}$ radiation dominated universe

Huge age ⇨ small ratio

Same explanation for small dark energy?

Quintessence

Dynamical dark energy ,

generated by scalar field

(cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87

P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Cosmon

- *Scalar field changes its value even in the present cosmological epoch*
- *Potential und kinetic energy of cosmon contribute to the energy density of the Universe*
- *Time - variable dark energy :*

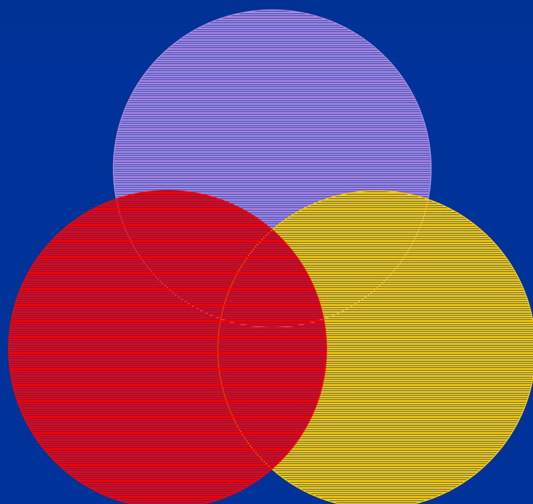
  $\rho_h(t)$ *decreases with time !*

Cosmon

- *Tiny mass*
- $m_c \sim H$
- *New long - range interaction*

“Fundamental” Interactions

Strong, electromagnetic, weak interactions



gravitation cosmodynamics

On astronomical length scales:

graviton

+

cosmon

Evolution of cosmon field

Field equations

$$\ddot{\phi} + 3H\dot{\phi} = -dV/d\phi$$

$$3M^2H^2 = V + \frac{1}{2}\dot{\phi}^2 + \rho$$

Potential $V(\varphi)$ determines details of the model

e.g. $V(\varphi) = M^4 \exp(-\varphi/M)$

for increasing φ the potential decreases towards zero !

Cosmological equations

$$\mathcal{L} = \sqrt{g}\left\{\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + V(\phi)\right\}$$

(homogeneous and isotropic Robertson-Walker metric , $k=0$)

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{\partial V}{\partial\varphi} = 0$$

matter

$$3M^2H^2 = V + \frac{1}{2}\dot{\phi}^2 + \rho$$

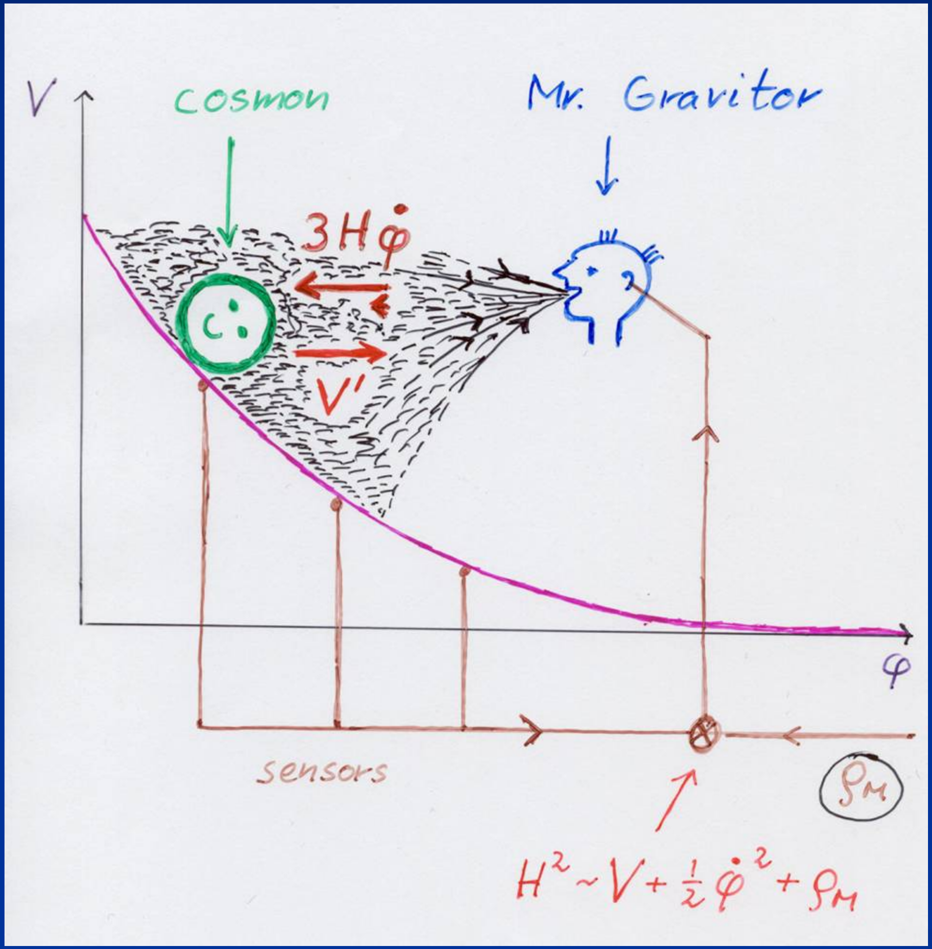
$$\dot{\rho}_M + 3H(\rho_M + p_M) = 0$$

$$V = \bar{V}_0 \exp\{-a\frac{\varphi}{\hat{M}}\}$$

$$(\bar{V}_0 = \hat{M}^4)$$

$$p_M = \frac{n-3}{3}\rho_M$$

Cosmological equations



$$\ddot{\phi} + 3H\dot{\phi} = -dV/d\phi$$

$$3M^2H^2 = V + \frac{1}{2}\dot{\phi}^2 + \rho$$

asymptotic solution for large time

Cosmological solutions with scalar field (cosmon)

for exponential potential $V \sim \exp(-a\frac{\varphi}{M})$

$$\Longrightarrow$$

asymptotic solution for $t \to \infty$:

$$V \sim t^{-2} \quad , \quad \dot{\varphi}^2 \sim t^{-2}$$

$$\varphi = \frac{2M}{a} \ln t$$

stable attractor!

independent of initial conditions
"tracker solution"

$$\Omega_{hom} = \frac{3}{2a^2}$$

fixed fraction in dark energy!

$$H^2 = \frac{1}{6M^2}(\rho_M + V(\varphi) + \frac{1}{2}\dot{\varphi}^2)$$

$$\dot{\rho}_M + 3H(\rho_M + p_M) = 0$$

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{\partial V}{\partial \varphi} = 0$$

$$p_M = \frac{n-3}{3}\rho_M$$

$$V = \bar{V}_0 \exp\{-a\frac{\varphi}{M}\}$$

$$\phi = \frac{2M}{\alpha}\ln(t/\bar{t}) \ , \quad \frac{1}{2}\dot{\phi}^2 = \frac{2M^2}{\alpha^2}t^{-2} \ , \quad V = \frac{2M^2}{\alpha^2}\frac{(6-n)}{n}t^{-2}$$

$$H = \frac{2}{n}t^{-1} \quad , \quad \rho \sim t^{-2}$$

$$\Omega_d = (V + \frac{1}{2}\dot{\phi}^2)/\rho_c = \rho_\phi/\rho_c = \frac{n}{2\alpha^2}$$

M → ^M

exponential potential → constant fraction in dark energy

$$\Omega_M = 1 - \frac{n}{2a^2}$$

$$\Omega_V = \frac{V}{\rho_c} = \frac{n(6-n)}{12a^2}$$

$$\Omega_{kin} = \frac{\dot{\varphi}^2}{2\rho_c} = \frac{n^2}{12a^2}$$

$$\Omega_h = \Omega_V + \Omega_{kin} = \frac{n}{2a^2}$$

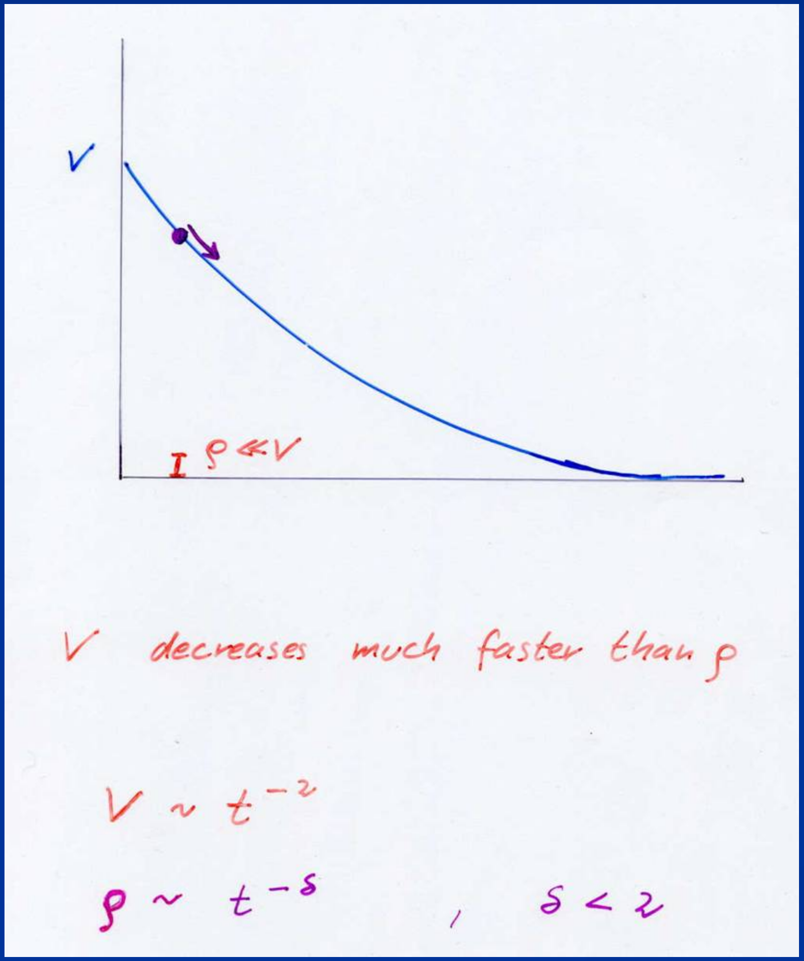
asymptotic solution

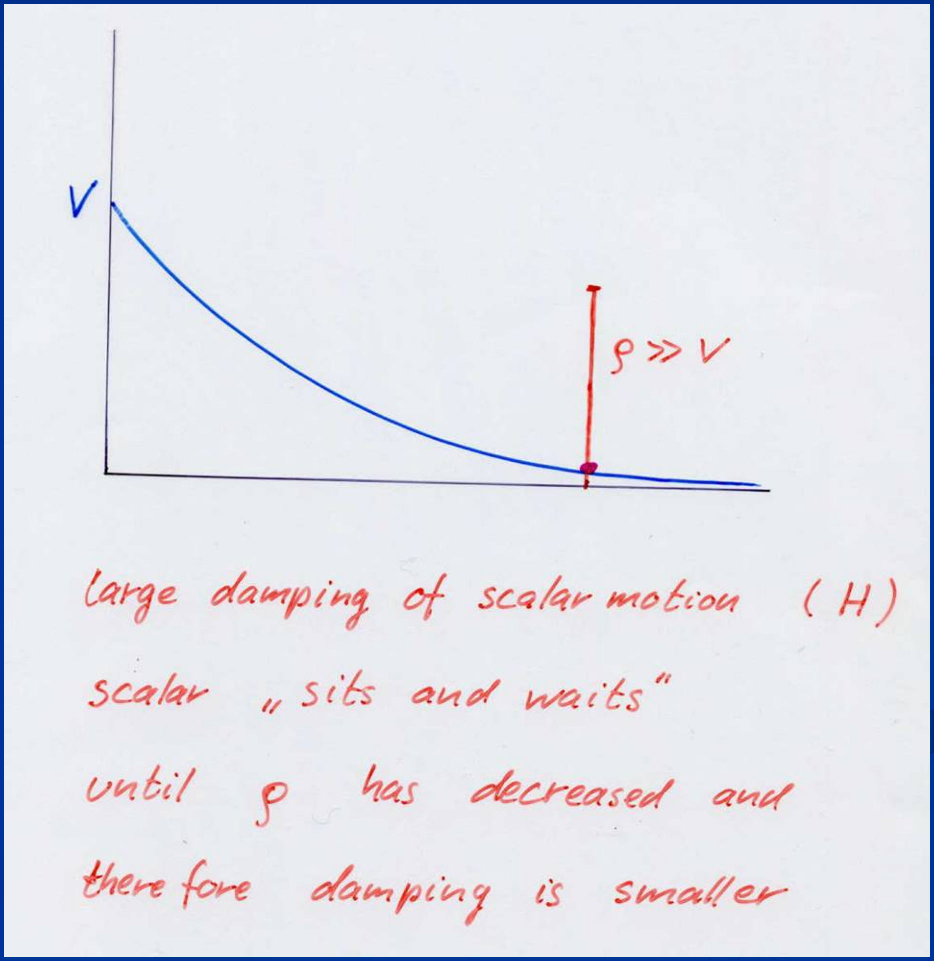
for $t \to \infty$

"attractor" for $a^2 > \frac{n}{2}$

$$\Omega_x = \frac{8\pi}{3} \frac{\rho_x}{M_p^2 H^2}$$

General mechanism for cosmic attractor





Cosmic Attractors

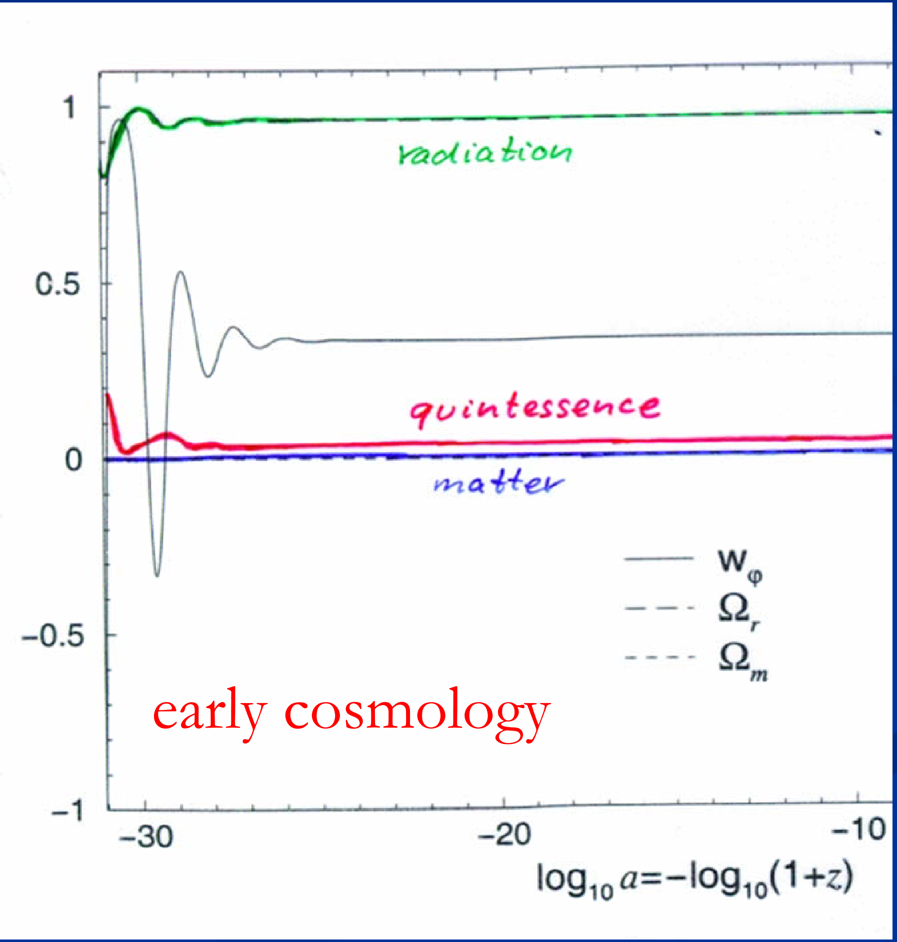
Solutions independent of initial conditions

typically $V \sim t^{-2}$

$\varphi \sim \ln(t)$

$\Omega_h \sim$ const.

details depend on $V(\varphi)$ or kinetic term



A few references

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P.Steinhardt,L.Wang,I.Zlatev , Phys.Rev.Lett.82,896(1999)

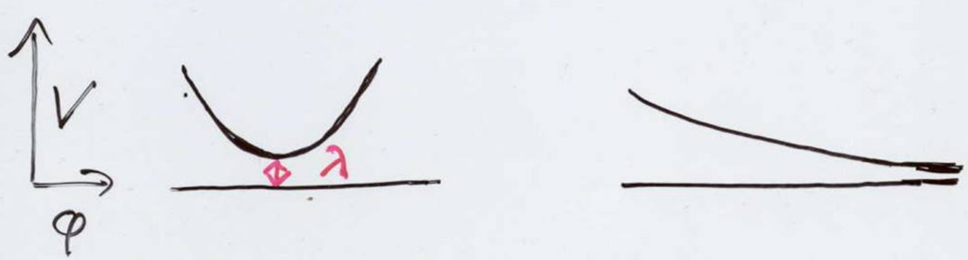
many models …

one unknown function $V(\varphi)$

$$\mathcal{L} = \frac{1}{2}\partial^{\mu}\varphi\,\partial_{\mu}\varphi + V(\varphi)$$

cosmological constant

only one constant $\lambda = V(\varphi_0)$



"kinetial"

choose field variable such that
potential has standard units

advantage:
φ acts as dark energy clock in cosmology

$$\mathcal{L}(\varphi) = \frac{1}{2}(\partial\varphi)^2\, k^2(\varphi) + \exp[-\varphi]$$

$$\phi = K(\varphi) \quad , \quad k(\varphi) = \frac{\partial K(\varphi)}{\partial \varphi}$$

M=1

$$\mathcal{L}(\phi) = \frac{1}{2}(\partial\phi)^2 + \exp[-K^{-1}(\phi)]$$

A.Hebecker,…

Dynamics of quintessence

- Cosmon φ : scalar singlet field
- Lagrange density $L = V + \frac{1}{2}\, \mathbf{k(\varphi)}\, \partial\varphi\, \partial\varphi$

  (units: reduced Planck mass M=1)
- Potential : $V=\exp[-\varphi]$
- "Natural initial value" in Planck era $\varphi=0$
- today: $\varphi=276$

Quintessence models

- Kinetic function $k(\varphi)$: parameterizes the details of the model - "kinetial"
  - $k(\varphi) = k=\text{const.}$ Exponential Q.
  - $k(\varphi) = \exp((\varphi - \varphi_1)/\alpha)$ Inverse power law Q.
  - $k^2(\varphi) =$ "$1/(2E(\varphi_c - \varphi))$" Crossover Q.
- possible naturalness criterion:

$k(\varphi=0)$: not tiny or huge !

- else: explanation needed -

crossover quintessence

k(φ) increase strongly for φ corresponding to present epoch

example:

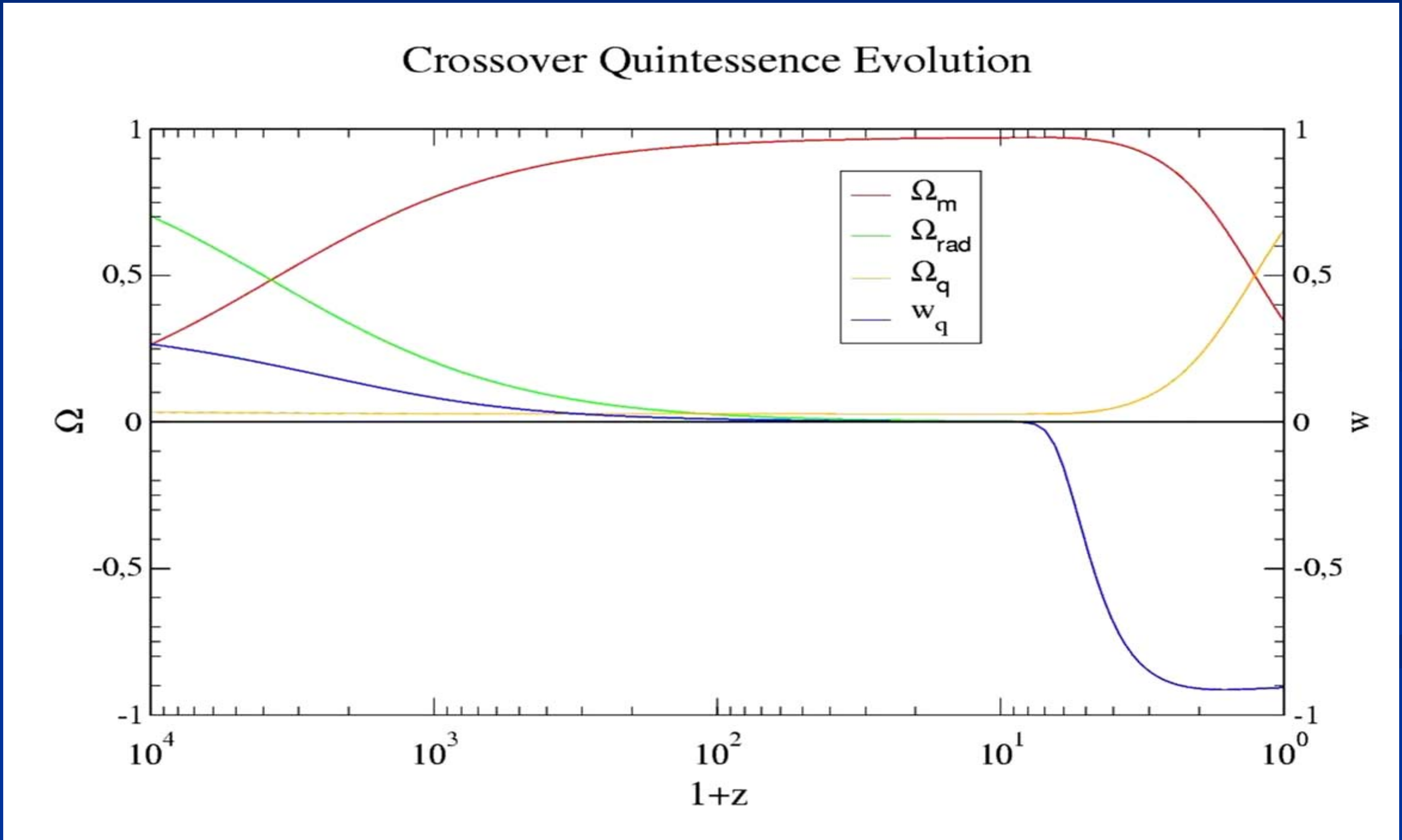
$$k(\varphi) = k_{min} + \tanh(\varphi - \varphi_1) + 1$$

$$(\text{with } k_{min} = 0.1\,, \quad \varphi_1 = 276.6\,)$$

exponential quintessence:

$$k = \frac{1}{\sqrt{2}\alpha}$$

Quintessence becomes important "today"



scale factor as "time"-variable

use scale factor a as time variable:

$$\text{matter}: \quad \frac{d \ln \rho_m}{d \ln a} = -3$$

$$\text{radiation}: \quad \frac{d \ln \rho_r}{d \ln a} = -4$$

$$\text{scalar}: \quad \frac{d \ln \rho_\phi}{d \ln a} = -6 \left(1 - \frac{V}{\rho_\varphi}\right)$$

$$V = e^{-\varphi}: \quad \frac{d \ln V}{d \ln a} = -\left(\frac{6(\rho_\varphi - V)}{k^2(-\ln V)(\rho_m + \rho_r + \rho_\varphi)}\right)$$

for $\mathrm{k}(\varphi) = k$ (constant)

$\mathrm{k}^2 \hat{=} \frac{1}{2\alpha^2}$ with potential $e^{-\alpha \frac{\varphi}{M}}$

determine kinetial k(φ) by observation !

end

cosmological equations

$$\frac{d\ln\rho_\varphi}{d\ln a} = -3(1+w_\varphi) \ , \qquad \frac{d\varphi}{d\ln a} = \sqrt{6\Omega_T/k^2(\varphi)}$$

$$\frac{d\ln\rho_m}{d\ln a} = -3\,(1+w_m) \ , \qquad \frac{d\ln\rho_r}{d\ln a} = -3\,(1+w_r) \ ,$$

$$\frac{d\ln\rho_\varphi}{d\ln a} = -6\left(1-\frac{V(\varphi)}{\rho_\varphi}\right) \ , \qquad \frac{d\varphi}{d\ln a} = \sqrt{\frac{6\,(\rho_\varphi - V(\varphi))}{k^2\,(\varphi)(\rho_m+\rho_r+\rho_\varphi)}}$$

$$\frac{d\ln V}{d\ln a} = -\sqrt{\frac{6\,(\rho_\varphi - V)}{k^2\,(-\ln V)(\rho_m+\rho_r+\rho_\varphi)}}$$