QCD – from the vacuum to high temperature

an analytical approach

Analytical description of phase transition

- Needs model that can account simultaneously for the correct degrees of freedom below and above the transition temperature.
- Partial aspects can be described by more limited models, e.g. chiral properties at small momenta.

Higgs picture of QCD

"spontaneous breaking of color "

in the QCD – vacuum

octet condensate

for $N_f = 3$ (u,d,s)

C.Wetterich, Phys.Rev.D64,036003(2001),hep-ph/0008150

Many pictures …

… of the QCD vacuum have been proposed

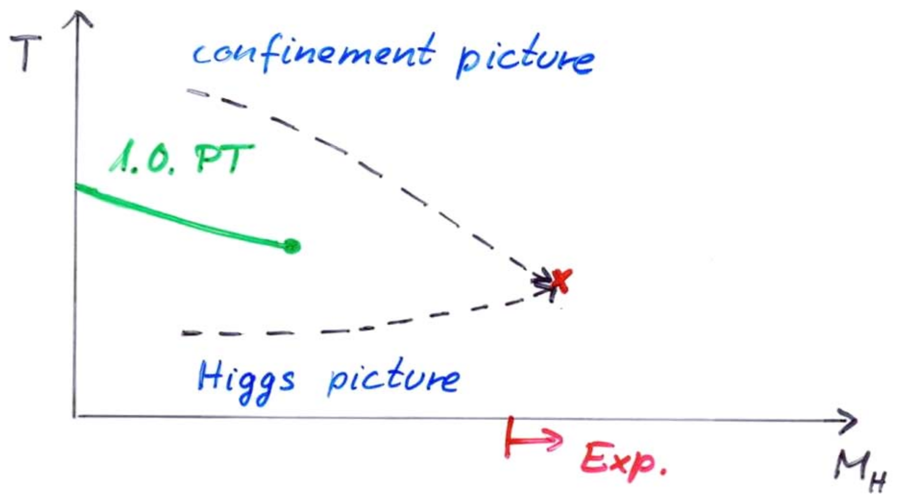
monopoles, instantons, vortices, spaghetti vacuum …

in principle, no contradiction – there may be more than one valid picture

most proposals say essentially nothing about the low mass excitations in **real** QCD, i.e mesons and baryons

different for Higgs picture !

Electroweak phase diagram



Reuter, W.

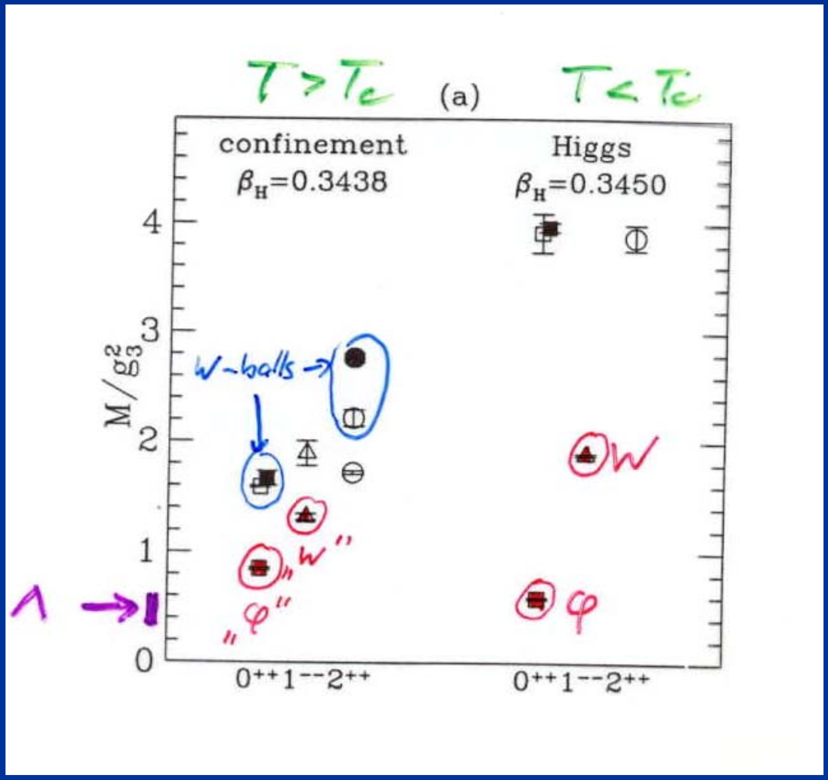
Philipsen, Buchmüller

Laine, Kajantie, Rummukainen, Shaposhnikov

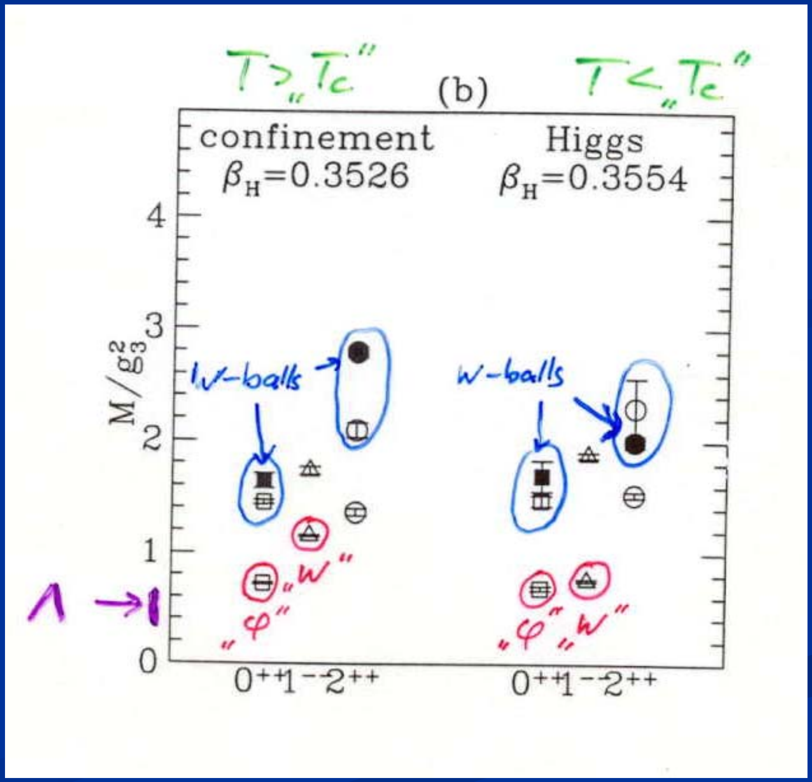
Crossover in the standard model !

Masses of excitations (d=3)

small M_H

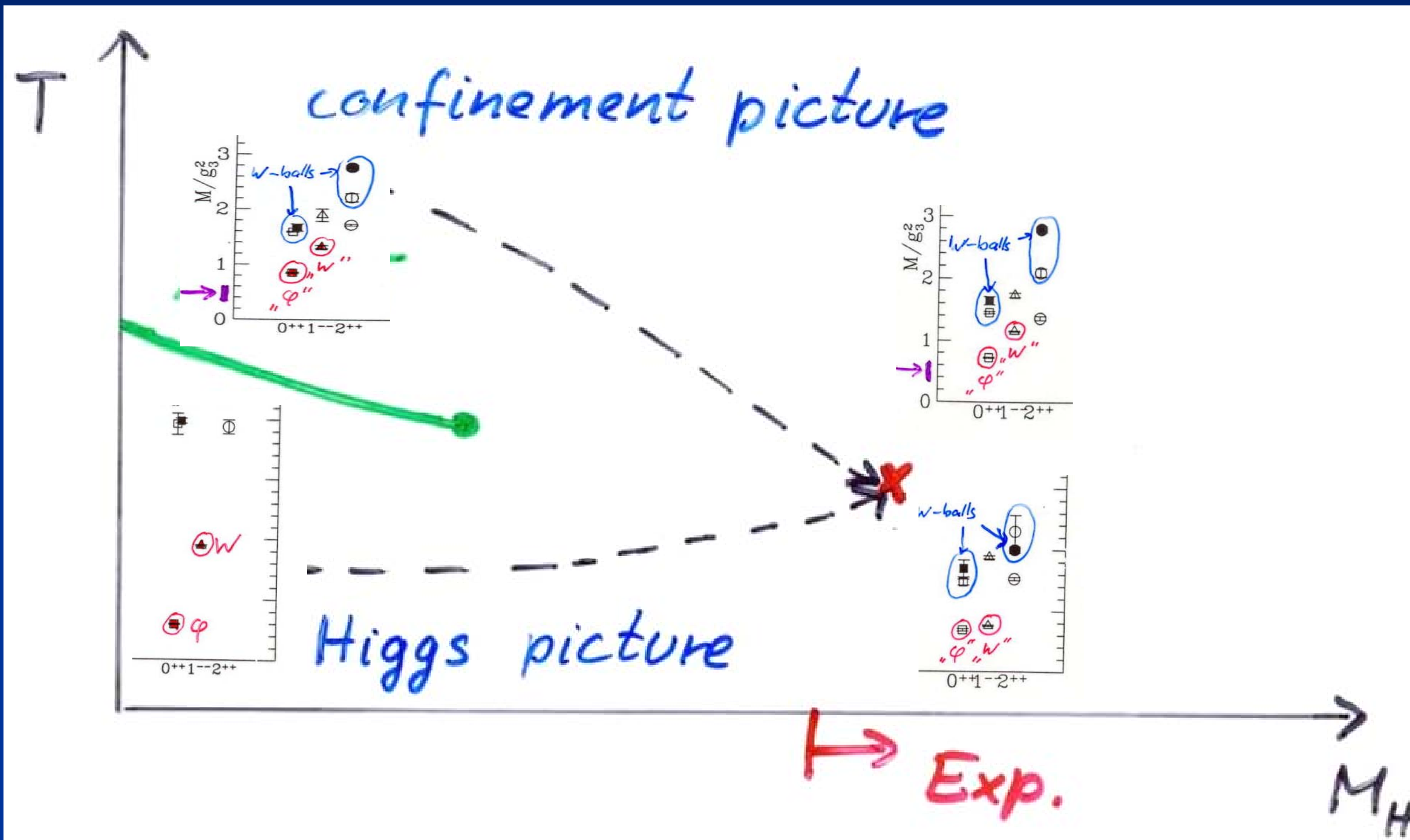


large M_H



O.Philipsen,M.Teper,H.Wittig '97

Continuity



T
confinement picture
M/g_3^2
W-balls
"W"
"φ"
$0^{++}1^{--}2^{++}$
W
φ
Higgs picture
Exp.
M_H

Higgs phase and confinement

can be equivalent –

then simply two different descriptions (pictures) of the same physical situation

Is this realized for QCD ?

Necessary condition : spectrum of excitations with the same quantum numbers in both pictures

- known for QCD : mesons + baryons -

Spontaneous breaking of color

- Condensate of colored scalar field
- Equivalence of Higgs and confinement description in real (N_f=3) QCD vacuum
- Gauge symmetries not spontaneously broken in formal sense (only for fixed gauge)

  Similar situation as in electroweak theory
- No "fundamental" scalars
- Symmetry breaking by quark-antiquark-condensate

Analogy between weak and strong interactions

Strong interactions

$\sim$

Weak interactions

* gauge symmetry „spontaneously broken"
* Higgs mechanism gives gauge bosons and fermions a mass

$\rho \sim W$

$M_\rho \approx 800\,\mathrm{MeV} \sim M_W \approx 80\,\mathrm{GeV}$

Quark –antiquark condensate

quarks : $\psi_{L,R\,a\,i}$

i — color

a — flavor

condensate in vacuum :

$$\langle \bar{\psi}_{L\,j\,b}\, \psi_{R\,a\,i} \rangle =$$

$$\frac{1}{\sqrt{6}}\, \bar{\xi}_0 \left(\delta_{ia}\, \delta_{jb} - \frac{1}{3}\, \delta_{ij}\, \delta_{ab} \right)$$

color octet

$$+\frac{1}{\sqrt{3}}\, \bar{\sigma}_0\, \delta_{ij}\, \delta_{ab}$$

color singlet

Octet condensate

$< \text{octet} > \neq 0$:

- "Spontaneous breaking of color"
- Higgs mechanism
- Massive Gluons – all masses equal
- Eight octets have vev
- Infrared regulator for QCD

Electric charge

$< \text{octet} > \neq 0$:

- Spontaneous breaking of electromagnetic U(1) symmetry

(some components of octet carry electric charge – similar to Higgs mechanism for hypercharge in electroweak theory)

- Combined U(1) symmetry survives

(cf. $Q = I_3 + \frac{1}{2} Y$ in e.w. standard model)

Electric charge of "quarks"

$$Q = \frac{1}{2}\left(\lambda_3^{(L)} + \lambda_3^{(R)} - \underline{\underline{\lambda_3^{(c)}}}\right) + \frac{1}{2\sqrt{3}}\left(\lambda_8^{(L)} + \lambda_8^{(R)} - \underline{\underline{\lambda_8^{(c)}}}\right)$$

	$-\frac{1}{2}\lambda_3^{(c)}$	$-\frac{1}{2\sqrt{3}}\lambda_8^{(c)}$	$\frac{1}{2}\lambda_3^{(v)}$	$\frac{1}{2\sqrt{3}}\lambda_8^{(v)}$	Q	
u_1	$-\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	0	Σ^0, Λ^0, S^0
u_2	$\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	1	Σ^+
u_3	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$	1	p
d_1	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{6}$	-1	Σ^-
d_2	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{6}$	0	Σ^0, Λ^0, S^0
d_3	0	$\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{6}$	0	n
s_1	$-\frac{1}{2}$	$-\frac{1}{6}$	0	$-\frac{1}{3}$	-1	X^-
s_2	$\frac{1}{2}$	$-\frac{1}{6}$	0	$-\frac{1}{3}$	0	X^0
s_3	0	$\frac{1}{3}$	0	$-\frac{1}{3}$	0	Λ^0, S^0

Flavor symmetry

for equal quark masses :

octet preserves global SU(3)-symmetry
"diagonal in color and flavor"
"color-flavor-locking"

(cf. Alford,Rajagopal,Wilczek ; Schaefer,Wilczek)

All particles fall into representations of the "eightfold way"

quarks : 8 + 1 , gluons : 8

Related earlier ideas:

K.Bardakci,M.Halpern; I.Bars '72
R.Mohapatra,J.Pati,A.Salam '76
A.De Rujula,R.Giles,R.Jaffe '78
T.Banks,E.Rabinovici '79
E.Fradkin,S.Shenker '79
G. t'Hooft '80
S.Dimopoulos,S.Raby,L.Susskind '80
T.Matsumoto '80
B.Iijima,R.Jaffe '81
M.Yasue '90
M.Alford,K.Rajagopal,F.Wilczek '99
T.Schaefer,F.Wilczek '99

Color-flavor-locking

Chiral symmetry breaking :

$$SU(3)_L \times SU(3)_R \Rightarrow SU(3)_V$$

Color symmetry breaking :

$$SU(3)_c \times SU(3)_V \Rightarrow SU(3)_{diagonal}$$

Quarks : $\bar{3} \times 3 \Rightarrow 8 + 1$

Gluons : $\underset{\text{color}\sim}{8} \times \underset{\text{flavor}}{1} \Rightarrow 8$

Similar to high density QCD :

Alford, Rajagopal, Wilczek ; Schaefer, Wilczek

Octet condensate

Color symmetry breaking :

$$SU(3)_c \times SU(3)_V \Rightarrow SU(3)_{diagonal}$$

$$< \chi > \; : \quad \underset{\text{color}\sim}{8} \quad \times \quad \underset{\text{flavor}}{8} \quad \Rightarrow \quad 1 + \ldots$$

$$\langle \bar{\psi}_{L\,jb}\, \psi_{R\,ai} \rangle = \frac{1}{\sqrt{6}}\, \bar{\xi}_0 \left(\delta_{ia}\, \delta_{jb} - \frac{1}{3}\, \delta_{ij}\, \delta_{ab} \right)$$

Quarks and gluons carry the observed quantum numbers of isospin and strangeness of the baryon and vector meson octets !

They are integer charged!

Duality

quarks = baryons

$p, n, \Sigma, \Lambda, \Xi$ (+5°)

quark - baryon duality

gluons = vector mesons

ρ, K^*, ω

gluon - meson duality

$\bar{M}_\rho$ = 850 MeV

„gluon mass"

gluons carry electric charge and strangeness

Quantum numbers match !

Of course , there are many more excitations (resonances).

Strong interactions $\Longrightarrow$ bound states

Higgs description seems possible - is it simple ?

Effective low energy model for QCD

- Composite scalars

  (quark-antiquark- bound states)
- Gauge invariance
- Approximation:

  renormalizable interactions

  for QCD with scalars
- Comparison with observation?

Low energy effective action

$$\mathcal{L} = Z_\psi \{ i \bar{\psi}_i \gamma^\mu \partial_\mu \psi_i + \underline{g}\, \bar{\psi}_i \gamma^\mu A_{ij,\mu} \psi_j \}$$

$$+ \frac{1}{2} G^{\mu\nu}_{ij} G_{ji,\mu\nu} \qquad \text{„QCD"}$$

$$+ \mathrm{Tr}\{ (D^\mu \gamma_{ij})^\dagger (D_\mu \gamma_{ij}) \} + \underline{U}(\gamma)$$

$$+ Z_\psi \bar{\psi}_i \left[(\underline{h}\, \varphi\, \delta_{ij} + \underline{\tilde{h}}\, \chi_{ij}) \frac{1+\gamma_5}{2} \right.$$

$$\left. - (\underline{h}\, \varphi^\dagger \delta_{ij} + \underline{\tilde{h}}\, \chi^\dagger_{ij}) \frac{1-\gamma_5}{2} \right] \psi_j$$

$$\gamma = \varphi + \chi$$

$$A_{ij,\mu} = \frac{1}{2} A^z_\mu (\lambda_z)_{ij}$$

Simplicity

This simple effective action will yield the masses and couplings of the baryons, pseudoscalars and vector mesons, (including electromagnetic couplings by covariant derivatives) !

(five parameters , to be later determined by QCD)

New scalar interactions

- Gauge covariant kinetic term
- Effective potential
- Yukawa coupling to quarks

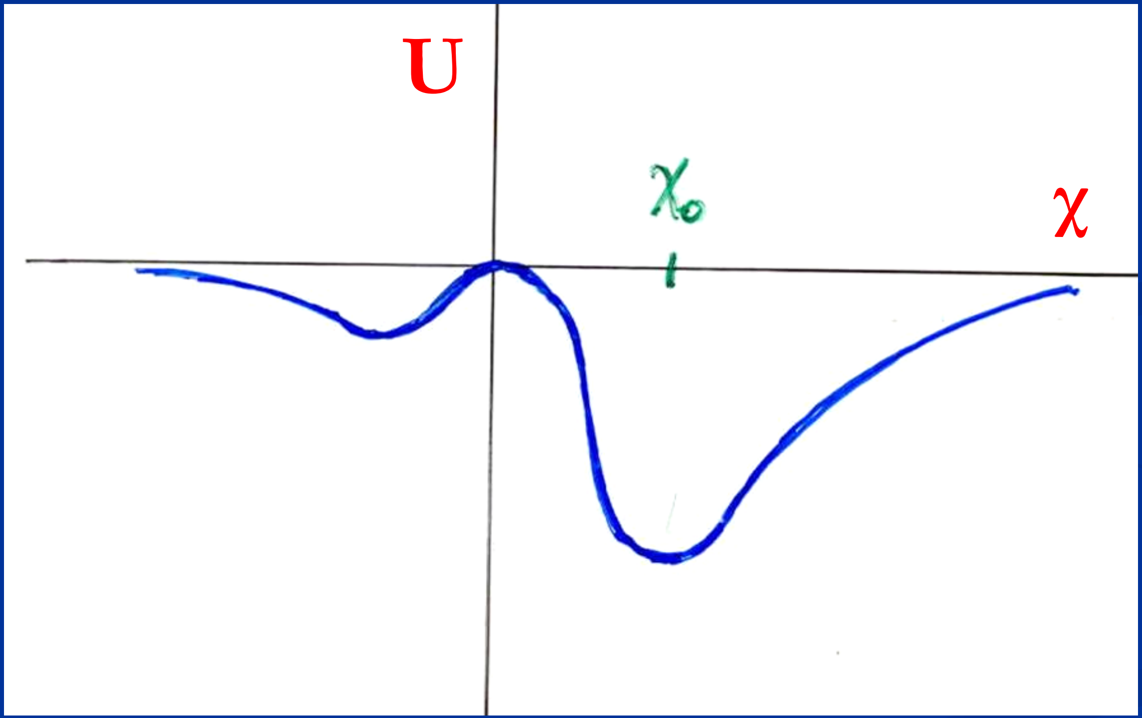
$$
\begin{aligned}
\mathcal{L} = \; & Z_\psi \left\{ i \bar{\psi}_i \gamma^\mu \partial_\mu \psi_i + \underline{g}\, \bar{\psi}_i \gamma^\mu A_{ij,\mu} \psi_j \right\} \\
& + \tfrac{1}{2} G^{\mu\nu}_{ij} G_{ji,\mu\nu} \qquad \text{„QCD"} \\
& + \mathrm{Tr}\left\{ (D^\mu \chi_{ij})^\dagger (D_\mu \chi_{ij}) \right\} + \underline{U}(\chi) \\
& + Z_\psi \bar{\psi}_i \Big[(\underline{h} \varphi \delta_{ij} + \underline{\tilde{h}} \chi_{ij}) \tfrac{1+\gamma_5}{2} \\
& \qquad - (\underline{h} \varphi^\dagger \delta_{ij} + \underline{\tilde{h}} \chi^\dagger_{ij}) \tfrac{1-\gamma_5}{2} \Big] \psi_j
\end{aligned}
$$

Calculability

- Remember : no fundamental scalars
- Effective couplings should be calculable from QCD – i.e. gauge coupling or confinement scale

Effective octet potential

simple instanton computation



$\chi_0 = 150$ MeV

$M_\varrho = 850$ MeV

Chiral anomaly !

Masses of physical particles

determine three phenomenological parameters

Average vector meson mass :

$$\bar{M}_\rho = g\chi_0 =: 850\ \mathrm{MeV}$$

Baryon masses :

$$M_8 = h\sigma_0 - \frac{\tilde{h}}{3\sqrt{6}}\chi_0 = 1.15\ \mathrm{GeV}$$

$$\pm M_1 = h\sigma_0 + \frac{8}{3}\frac{\tilde{h}}{\sqrt{6}}\chi_0 = 1.4\ \mathrm{GeV}$$

$$\frac{\tilde{h}}{g} = 0.24 \quad / \; 2.5$$

$$h\sigma_0 = 1.18\ \mathrm{GeV} \; / \; 0.87\ \mathrm{GeV}$$

$$(M_1 > 0) \qquad (M_1 < 0)$$

$M_1 < 0 \Rightarrow$ singlet has opposite parity of octet

J. Berges, ..

Phenomenological parameters

- 5 undetermined parameters

$\chi_0, \; \sigma_0, \; g, \; h, \; \tilde{h}$

fixed by 5 observable quantities

(for $m_q = 0$, averages over SU(3) multiplets)

$\bar{M}_\rho = 850$ MeV

$\bar{M}_N = 1150$ MeV

$M_1 = 1400$ MeV

$\bar{f} = 110$ MeV $\quad (\bar{f} = \frac{2}{3} f_K + \frac{1}{3} f_\pi)$

$\Gamma(\rho \to \mu^+\mu^-)$, $\Gamma(\rho \to e^+e^-) = 7$ keV

- predictions

* $\Gamma(\rho \to 2\pi) \approx 150$ MeV
* β-decay of neutrons: $g_A = 1$ (Exp: $g_A = 1.26$)
* vector dominance in electromagnetic interactions of pions, $g_{\gamma\pi\pi}/e = 0.04$

Chiral perturbation theory

+ all predictions of chiral perturbation theory
+ determination of parameters

		„Exp"
L_1	0.87	0.7 ± 0.3
L_2	1.74	1.7 ± 0.7
L_3	−5.2	$-(4.4 \pm 2.5)$

First conclusions

- Spontaneous color symmetry breaking plausible in QCD
- QCD - computation of effective vector mass needed
- Simple effective action can account for mass spectrum of light baryons and mesons as well as their couplings
- Gluon - Meson duality
- Quark - Baryon duality

Nonlinear formulation

- Use of nonlinear fields makes physical content of the effective action more transparent.
- Similar to nonlinear fields for pions
- Selection of nonlinear fields follows symmetry content of the theory

Gauge invariance

- Higgs picture is a guide for ideas and a way to compute gauge invariant quantities at the end
- Intuition can be misleading for certain questions
- Effective action, $U(\varphi,\chi)$: gauge invariant
- Nonlinear fields : gauge singlets

Only assumptions :

A) minimum of U preserves global SU(3)

B) minimum not for $\chi=0$

(for appropriate gauge and normalization of χ)

Nonlinear fields : π,K,η, η'

1) color singlet scalar

$$\phi = \sigma_0 U$$

U : 3×3 matrix , $U^\dagger U = 1$

$$U = \exp\left(-\frac{i}{3}\vartheta\right) \exp\left(i \frac{\Pi_z \lambda_z}{f}\right)$$

$z = 1 \ldots 8$

$\Pi_z : (\vec{\pi}, K^0, \bar{K}^0, K^\pm, \eta)$;

$\vartheta \to \eta'$

Nonlinear fields : diquark cloud

2) color octet scalar

$$\chi_{ij,ab} = \frac{1}{\sqrt{6}} \chi_0 \{ (W_R v)_{ai} (W_L^* v^*)_{bj} - \frac{1}{3} U_{ab} \delta_{ij} \}$$

$W_{L,R}$, v : unitary 3x3 matrices

$$U = W_R W_L^\dagger$$

- The product W•v transforms as an antidiquark
- B=-2/3
- v : color triplet

How quarks get dressed as baryons

$$\psi_L = Z_\psi^{-1/2}\, W_L\, \underline{\underline{N_L}}\, v$$

$$\psi_R = Z_\psi^{-1/2}\, W_R\, \underline{\underline{N_R}}\, v$$

$\psi = \psi_{ai}$: 3x3 matrix , quark field

$\underline{\underline{N}}$: " , baryon field

N: gauge singlet !

$$N = Z_\psi^{1/2}\, W^\dagger\, \psi\, v^\dagger$$

quark

cloud

baryons : $B = 1$!

Gauge bosons/vector mesons

$$A_\mu = - v^T \underline{\underline{V_\mu}}^T v^* - \frac{i}{g} \partial_\mu v^T v^*$$

V_μ : vector mesons $(\vec{\rho}, K^*, \omega)$

All fields except ν are gauge singlets

Effective action in terms of physical fields

express $\mathcal{L}$ in terms of

$W_{L,R}$, V_μ , $N_{L,R}$, v

$\mathcal{L}$ is independent of v

$\Rightarrow$ gauge invariant !

* extract physical propagators and vertices for π, V, N

(they only involve gauge invariant fields !)

π : pseudoscalars

V : vector mesons (ρ, K^*, ω)

N : baryons

Effective action in terms of physical fields

linear fields

$$\mathcal{L} = Z_\psi \{ i \bar{\psi}_i \gamma^\mu \partial_\mu \psi_i + \underline{g}\, \bar{\psi}_i \gamma^\mu A_{ij,\mu} \psi_j \}$$

$$+ \frac{1}{2} G^{\mu\nu}_{ij} G_{ji,\mu\nu} \qquad \text{„QCD"}$$

$$+ \mathrm{Tr}\{ (D^\mu \chi_{ij})^\dagger (D_\mu \chi_{ij}) \} + \underline{U}(\chi)$$

$$+ Z_\psi \bar{\psi}_i \Big[(\underline{h} \varphi \delta_{ij} + \underline{\tilde{h}} \chi_{ij}) \frac{1+\gamma_5}{2}$$

$$- (\underline{h} \varphi^\dagger \delta_{ij} + \underline{\tilde{h}} \chi^\dagger_{ij}) \frac{1-\gamma_5}{2} \Big] \psi_j$$

nonlinear fields

Insert expressions for ψ,A,χ,φ

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_U + \mathcal{L}_V$$

Nonlinear local symmetry

Has been investigated since long ago in the context of chiral theories, describes ϱ - bosons

Here :

- Not postulated
- Consequence of local color symmetry + "SSB"
- Gauge bosons = gluons = ϱ - bosons

Predictions correct !

Reparameterization symmetry

Decomposition into nonlinear fields is not unique. E.g.

$$\psi_L = Z_\psi^{-1/2} W_L \underline{N_L} v$$

N can be multiplied by unitary transformation from left, and W from right.

local U(3)

reparameterization symmetry

infinitesimal transformation

$\theta_p(x)$: hermitean 3x3 matrix

$$\delta N = i[\theta_p, N]$$
$$\delta V_\mu = i[\theta_p, V_\mu] + \frac{1}{g} \partial_\mu \theta_p$$
$$\delta W_L = -i W_L \theta_p$$
$$\delta W_R = -i W_R \theta_p$$
$$\delta U = 0$$
$$\delta v = i \theta_p v$$

gauge fixing : $W_L^\dagger = W_R = \xi$

$$U = \xi^2$$

Baryons

$$\mathcal{L}_N = i\,\mathrm{Tr}\{\bar{N}_8\,\gamma^\mu(\partial_\mu - i\gamma^5\tilde{a}_\mu + \frac{i}{6H_{\eta'}}\gamma^5\partial_\mu\eta')\,N_8\}$$

$$+ M_8\,\mathrm{Tr}\{\bar{N}_8\,\gamma^5 N_8\}$$

+ terms with baryon singlet

$$\tilde{a}_\mu = -\frac{i}{2}\xi^\dagger\partial_\mu U\xi^\dagger + \frac{1}{6H_{\eta'}}\partial_\mu\eta'$$

$$= \frac{1}{2f}\lambda_z\,\partial_\mu\pi^z + O(\pi^3)$$

$$H_{\eta'} = \frac{f}{\sqrt{6}}\left(\frac{1+\frac{16}{7}x}{1+x}\right)^{1/2} \quad , x = \frac{7}{36}\frac{\chi_0^2}{\sigma_0^2}$$

Pion nucleon coupling

$$\mathcal{L} = F\,\mathrm{Tr}\{\bar{N}_8\,\gamma^\mu\gamma^5\,[\tilde{a}_\mu, N_8]\}$$
$$+\, D\,\mathrm{Tr}\{\bar{N}_8\,\gamma^\mu\gamma^5\,\{\tilde{a}_\mu, N_8\}_+\}$$

prediction $F = D = 0.5$

(uses directly quark-baryon duality !)

observation

$g_A = F + D = 1.26$

(β-decay of neutron)

$D - F = 0.34$

Two more successful predictions

F,D are not fixed by chiral symmetry !

Pseudoscalar mesons

Kinetic term for pseudoscalar mesons as in chiral perturbation theory

$$\mathcal{L}_U = \frac{f_0^2}{4} \mathrm{Tr}\{ \partial^\mu \tilde{U}^\dagger \partial_\mu \tilde{U} \}$$

$$+ \frac{1}{2} \partial^\mu \eta' \partial_\mu \eta'$$

$$- M_{\eta'}^2 H_{\eta'}^2 \cos(\eta' / H_{\eta'})$$

$$\tilde{U} = \exp\left(i \frac{\pi_z \lambda_z}{f} \right)$$

meson decay constant

$$f_0 = 2\sigma_0 \sqrt{1+x} \quad =: 128\ \mathrm{MeV}$$

$$x = \frac{7}{36} \frac{\chi_0^2}{\sigma_0^2}$$ (octet : singlet ratio)

$$\Rightarrow \quad g = 5.9 \left(\frac{1+x}{x} \right)^{1/2}$$

(from M_ρ / f)

$$f_0 = \left(\frac{2}{3} f_K + \frac{1}{3} f_\pi \right) / \kappa_\rho$$

$\kappa_\rho \approx 0.83$: „partial Higgs effect"

(mixing between π

and $\partial_\mu (\bar{\psi} \gamma^\mu \gamma^5 \psi)$)

Vector mesons

$$\mathcal{L}_V = \frac{1}{2} \mathrm{Tr}\{ V^{\mu\nu} V_{\mu\nu} \} \qquad (\xi^2 = U)$$

$$+ \bar{M}_\rho^2 \, \mathrm{Tr}\{ V^\mu V_\mu \}$$

$$+ \frac{2}{g} \bar{M}_\rho^2 \, \mathrm{Tr}\{ V^\mu \tilde{v}_\mu \} \leftarrow \rho\pi\pi \text{ coupling}$$

$$+ \frac{1}{g^2} \bar{M}_\rho^2 \, \mathrm{Tr}\{ \tilde{v}^\mu \tilde{v}_\mu \}$$

$$- g \, \mathrm{Tr}\{ \bar{N}_8 \gamma^\mu N_8 V_\mu \}$$

$$- \mathrm{Tr}\{ \bar{N}_8 \gamma^\mu \tilde{v}_\mu N_8 \} + \ldots$$

$$\tilde{v}_\mu = -\frac{i}{2} \left(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger \right)$$

$$\Rightarrow \quad \tilde{v}_\mu = -\frac{i}{8 f^2} [\lambda_y, \lambda_z] \, \pi^y \partial_\mu \pi^z + \ldots$$

Electromagnetic interactions

include by covariant derivative

e.g. $\hat{v}_\mu = -\frac{i}{2}\left(\xi^\dagger D_\mu \xi + \xi D_\mu \xi^\dagger\right)$

$= v_\mu - \frac{e}{2} B_\mu \left(\xi^\dagger Q \xi + \xi Q \xi^\dagger - 2Q\right)$

transforms homogenously

$$\mathcal{L}_V = a f_\pi^2 \,\mathrm{Tr}\,\left(\underbrace{\hat{v}_\mu + \frac{1}{2} g \vec{\rho}_{V\mu}\vec{\tau} - \frac{1}{2} e B_\mu \tau_3}\right)^2 + \dots$$

dictated by local reparametrization symmetry and electromagnetic gauge invariance !

(restriction to ρ-mesons)

ϱ - couplings

$$\mathcal{L} = \frac{1}{2} M_\rho^2 \, \vec{\rho}_\nu^{\,\mu} \, \vec{\rho}_{\nu\mu}$$

$$- e \, g_{\rho\gamma} \, \rho^\mu_{\nu 3} \, B_\mu \qquad (\rho^0\text{-}\gamma\text{-mixing})$$

$$+ g_{\rho\pi\pi} \, \vec{\rho}_\nu^{\,\mu} \, (\vec{\pi} \times \partial_\mu \vec{\pi}) \qquad (\rho \to 2\pi)$$

$$+ g_{\gamma\pi\pi} \, B^\mu \, (\vec{\pi} \times \partial_\mu \vec{\pi})_3 \; + \dots$$

$$M_\rho^2 = a g^2 f_\pi^2 \quad , \quad g_{\rho\gamma} = a g f_\pi^2$$

$$g_{\rho\pi\pi} = \frac{1}{2} a g \quad ,$$

$$g_{\gamma\pi\pi} = e\left(1 - \frac{2 g_{\rho\pi\pi}^2 f_\pi^2}{M_\rho^2}\right) \approx 0 \;\checkmark$$

vector dominance!

$$a = \frac{\chi_0^2}{f_\pi^2} \approx 2.4 \, \frac{x}{1+x}$$

ϱ - couplings

experiment :

$$\Gamma(\rho \to \pi\pi) = \frac{g_{\rho\pi\pi}^2}{48\pi} \frac{(M_\rho^2 - 4M_\pi^2)^{3/2}}{M_\rho^2} \approx 150\,\mathrm{MeV}$$

$$\Rightarrow \quad g_{\rho\pi\pi} \approx 6$$

$$\Gamma(\rho_0 \to e^+e^-) = 6.62\ \mathrm{keV}$$

$$\Rightarrow \quad g_{\rho\gamma} = 0.12\ \mathrm{GeV}^2$$

prediction :

$$g_{\rho\gamma} = 2\, g_{\rho\pi\pi} f_\pi^2 \qquad \checkmark$$

KSFR - relation ⚠

$$M_\rho^2 = \frac{4}{a}\, g_{\rho\pi\pi}^2 f_\pi^2 \quad \Rightarrow \quad a \approx 2$$

Vector dominance is realized by Higgs picture of QCD

Connection to gauge invariant formulation for linear fields

- Vector channel : use singlet fields

$$\rho_L^\mu \sim \bar{\psi}_L \gamma^\mu \psi_L \; , \; \rho_R^\mu \sim \bar{\psi}_R \gamma^\mu \psi_R$$

(in addition to A,φ,χ ; fermions omitted here)

- Solve field equations for colored bosons

$$\Gamma[A_\mu, \varphi, \chi, \rho] \to \Gamma[\varphi, \rho]$$

- Γ[φ,ϱ] contains directly the information for gauge invariant correlation functions

A - ϱ mixing

$$(D_\mu\chi)^*_{ijab}\chi_{ijac}(\rho^\mu_L)_{cb} + \cdots \Rightarrow \chi_0^2\rho^\mu A_\mu$$

$$\mathcal{L}_{\rho A} = \frac{1}{2}M^2\rho^\mu\rho_\mu + \frac{1}{2}A^\mu(-\partial^2 + g^2\chi_0^2)A_\mu - \beta\chi_0^2\rho^\mu A_\mu + \ldots$$

$$\text{field equation for } A^\mu: \ (-\partial^2 + g^2\chi_0^2)A^\mu = \beta\chi_0^2\rho^\mu$$

Insert solution A[ϱ] ⇒

$$\mathcal{L}_\rho = \frac{1}{2}\rho^\mu\frac{M^2}{-\partial^2 + g^2\chi_0^2}(-\partial^2 + g^2\chi_0^2 - \beta^2\chi_0^4/M^2)\rho_\mu$$

$$\text{field equation for } \rho^\mu: \ (-\partial^2 + \bar{m}_\rho^2)\rho^\mu = 0$$

$$\bar{m}_\rho^2 = g^2\chi_0^2\left(1 - \frac{\beta^2\chi_0^2}{g^2M^2}\right)$$

Mixing produces mass shift

Conclusion (2)

Phenomenology
works well for
simple effective action

Chiral phase transition at high temperature

High temperature phase transition in QCD :

Melting of octet condensate

Lattice simulations :

Deconfinement temperature = critical temperature for restoration of chiral symmetry

Why ?

Simple explanation :

octet condensate

„confinement"

chiral symmetry breaking

melting of octet condensate

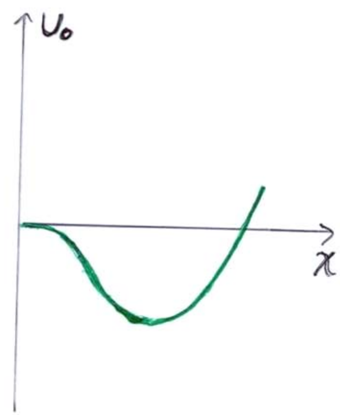
„deconfinement"

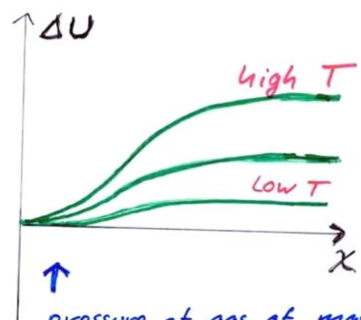
chiral symmetry restoration

„quarks and gluons become massless simultaneously"

Temperature dependent effective potential

$$U(\chi) = U_0(\chi) + \Delta U(\chi, T)$$





pressure of gas of massless or light particles

quarks, gluons, pseudoscalar mesons with χ - dependent mass

* $\mu_g^2 = g^2(\mu_g)\,\chi^2$ gluons
* $M_q^2 = h_\chi^2\,\chi^2$ quarks
* $M_G^2 = c_G \dfrac{\partial U}{\partial(\chi^2)}$ pseudoscalars

Temperature corrections to effective octet potential

$$\Delta U(\chi, T) = 24\, J_B(\mu_\rho^2) - 12 N_f\, J_F(M_q^2) + (N_f^2 - 1)\, J_G(M_G^2)$$

$$J_B(M^2; T) = T \int_0^\infty \frac{dq\, q^2}{2\pi^2} \ln\left(1 - \exp\left(-\frac{\sqrt{q^2+M^2}}{T}\right)\right)$$

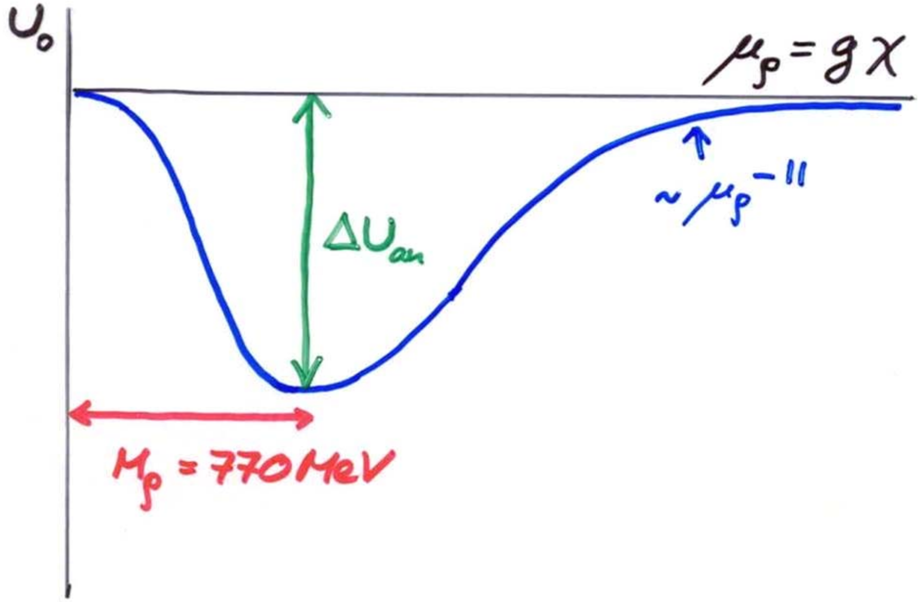
$$J_F(M^2; T) = T \int_0^\infty \frac{dq\, q^2}{2\pi^2} \ln\left(1 + \exp\left(-\frac{\sqrt{q^2+M^2}}{T}\right)\right)$$

$$J_G = J_B \quad \text{for } M^2 \geq 0$$

„particle" masses M depend on χ !

Vacuum effective potential (T=0)

instanton dominated



$$\Delta U_{an} = \frac{f_{\eta'}^2 M_{\eta'}^2}{2N_f^2} \approx 10^{-3}\ \mathrm{GeV}^4$$

Interesting relation between T_c and η' properties

$$T_c^4 \approx 10^{-2}\, M_{\eta'}^2\, f_{\eta'}^2$$

$(m_s = 0)$

$M_{\eta'} = 960\ \mathrm{MeV}$

$f_{\eta'} \approx 150\ \mathrm{MeV}$

A simple mean field calculation

	vanishing quark masses	equal $m_u = m_d = m_s \neq 0$ $2M_K^2 + M_\pi^2 = (390\,MeV)^2$	
$\bar{M}_\rho$	700 MeV	770 MeV	
f	68 MeV	116 MeV	
T_c	154 MeV	170 MeV	
$\bar{M}_\rho(T_c)$	290 MeV	290 MeV	screening masses
$\bar{M}_g(T_c)$	580 MeV	600 MeV	

equation of state : pion gas $\rightarrow$ QGP

$$\frac{\varepsilon - 3p}{\varepsilon + p} \approx \tau(T_c)\,\frac{T_c^4}{T^4} \qquad (T \geq T_c)$$

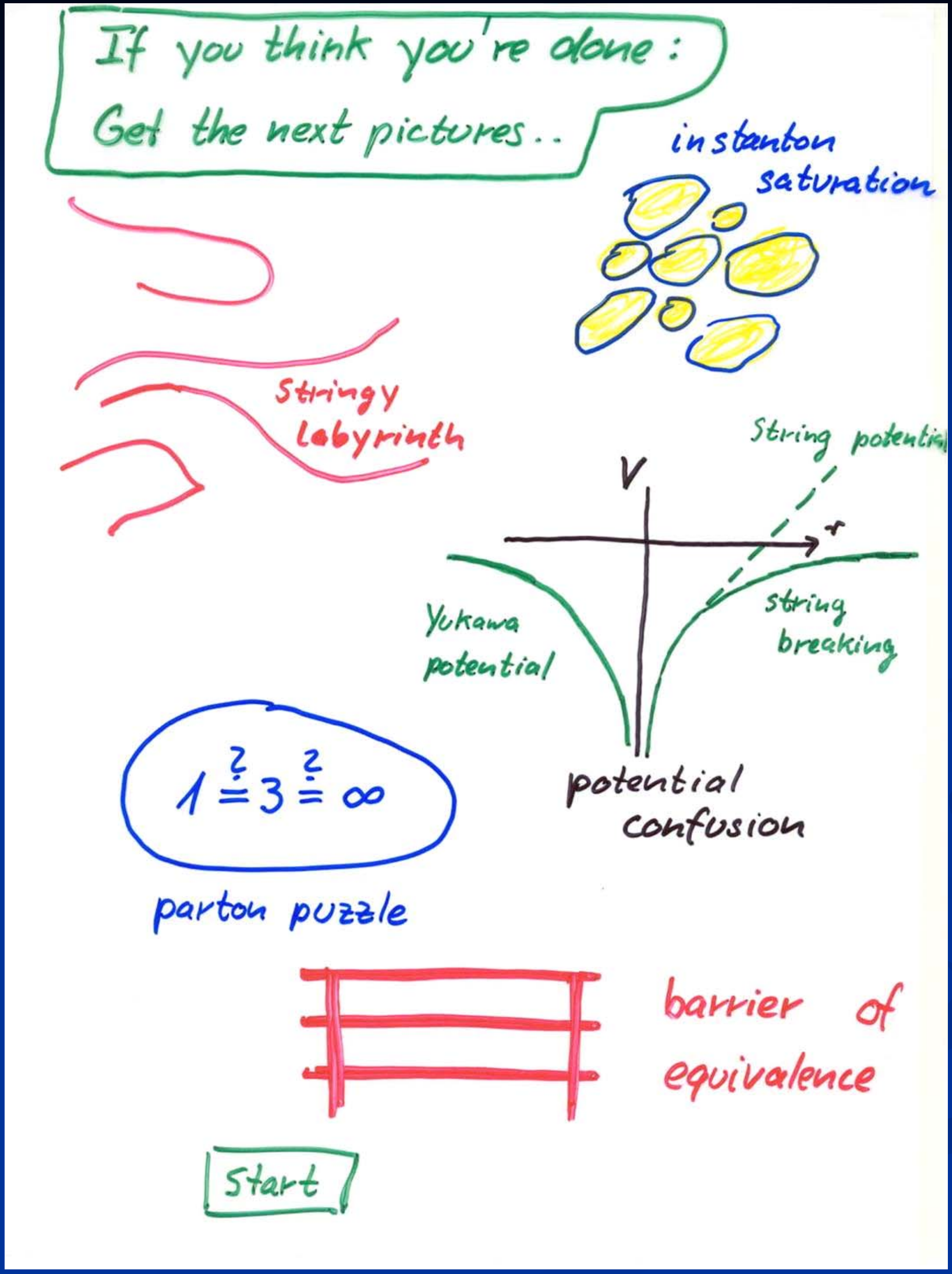
$\tau(T_c)$	0.37	0.53

Conclusions (3)

- Coherent picture for phase diagram of QCD is emerging
- Gluon meson duality allows for analytical calculations
- Quark-baryon duality :

  Direct contact to quantities of nuclear physics

Questions ?


If you think you're done:
Get the next pictures..
instanton saturation
Stringy labyrinth
String potential
V
r
Yukawa potential
string breaking
potential confusion
$1 \stackrel{?}{=} 3 \stackrel{?}{=} \infty$
parton puzzle
barrier of equivalence
Start

Lattice tests

a) Continuity

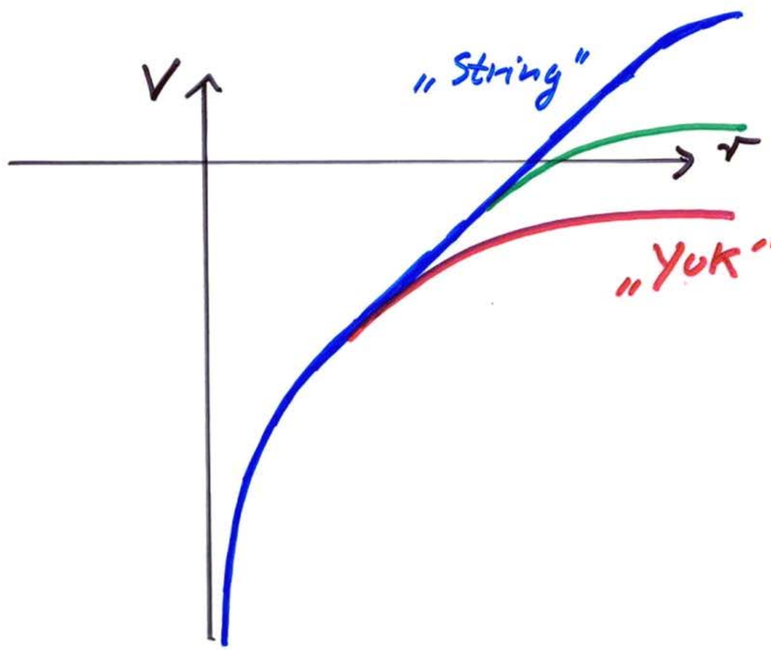
- Add “fundamental” scalar octets and start in perturbative Higgs phase

  (large negative mass term).
- Remove scalars continuously by increasing the mass term to large positive values

  Phase transition or analytical crossover ?

b) heavy quark potential

- start with large m_q :
  stringy potential
- lower m_q : continuous
  transition to Yukawa potential?



Challenges

- Instanton computation of $U(\varphi,\chi)$

  (improve by nonperturbative flow equation)
- Check continuity between Higgs and confinement description by lattice simulation
- Explicit construction of a local diquark operator with transformation Wv

  (nonvanishing expectation value)

end

Baryon number

quarks : $B = \frac{1}{3}$

baryons : $B = 1$

nonrelativistic quark model :

baryons composed from three quarks

$$N_{L,R} = Z_{\Psi}^{1/2} \, W_{L,R}^{\dagger} \, \Psi_{L,R} \, v^{\dagger}$$

baryons

quarks

pion cloud

diquark cloud

wave function renormalization

v^+ carries quantum numbers of

(qq) - diquark !

(1) $\bar{3}$ of $SU(3)_c$

(2) the fields in v^+ carry

fractional electric charges

$$\delta_{em}\, v = i\beta\, \tilde{Q}\, v$$

$$\tilde{Q} = \begin{pmatrix} 2/3 & & \\ & -1/3 & \\ & & -1/3 \end{pmatrix}$$

(3) v^+ has baryon number $B = \frac{2}{3}$

(triality !)

---

gauge fixing $v = 1$ not compatible with B

wave function renormalization

quarks :

quark number : $N_q = -i \int d^3x \, \mathrm{Tr} \, \bar{\Psi} \gamma^0 \psi$

coupling to baryon chemical potential :

$$\mathcal{L}^{(\mu)} = \frac{i}{3} \mu_B \, \mathrm{Tr} \, \bar{\Psi} \gamma^0 \psi$$

baryons :

baryon number : $N_B = -i \int d^3x \, \mathrm{Tr} \, \bar{N} \gamma^0 N$

coupling to baryon chemical potential :

$$\mathcal{L}^{(\mu)} = i \mu_B \, \mathrm{Tr} \, \bar{N} \gamma^0 N$$

$\Rightarrow \quad Z_\psi = 1/3$

Renormalization group:

Couplings and Z_ψ depend on scale k

our convention: $Z_\psi(k=0) = 1/3$

matching with linear quark-meson model and almost free quarks with $N_B = \frac{1}{3}$

if

$$Z_\psi(k \gtrsim k_B) \approx 1$$

$k_B \approx 250$ MeV : scale where „binding of three quarks to baryon" takes place relevant degrees of freedom at low momenta :

quarks $\quad \psi \quad \to$ baryons

gluons $\quad A_\mu \quad \to \rho, K^*, \omega$

composite scalars $(\sim \bar{\psi}\psi)$: χ

$$\left.\begin{array}{ll} \text{singlet} & \varphi \\ \text{octet} & \chi \end{array}\right\} \to \pi, K, \eta, \eta' + \dots$$

$$\{\varphi, \chi\}: \quad \chi_{ijab} \sim \bar{\psi}_{Ljb}\, \psi_{Rai}$$

parameters:

$U(\chi)$: effective scalar potential

here: only location of minimum needed

χ_0 : octet condensate

σ_0 : singlet condensate

five parameters : $\chi_0, \sigma_0, g, h, \tilde{h}$

+ explicit chiral symmetry breaking from current quark masses

$$\mathcal{L}_j = -\frac{1}{2} Z_\varphi^{-1/2} \mathrm{Tr}(j^\dagger \varphi + \varphi^\dagger j)$$

$$j = a_q \, \mathrm{diag}(\bar{m}_u, \bar{m}_d, \bar{m}_s)$$