Critical lines and points in the QCD phase diagram
Understanding the phase diagram

![Phase Diagram]

- Quarks + Gluons
- Hadrons

Variables:
- $T$ (Temperature)
- $\mu$ (Chemical Potential)
Phase diagram for $m_s > m_{u,d}$

- Quark-gluon plasma
  - "deconfinement"

- Quark matter: superfluid $B$ spontaneously broken

- Nuclear matter
  - $B$, isospin ($I_3$) spontaneously broken, $S$ conserved
Order parameters

- Nuclear matter and quark matter are separated from other phases by true critical lines
- Different realizations of global symmetries
- Quark matter: SSB of baryon number B
- Nuclear matter: SSB of combination of B and isospin $I_3$
  neutron-neutron condensate
“minimal” phase diagram
for equal nonzero quark masses
Endpoint of critical line?
How to find out?
Methods

• **Lattice**: You have to wait until chiral limit is properly implemented!

• **Models**: Quark meson models cannot work in the Higgs picture of QCD?

• **Experiment**: Has $T_c$ been measured?
  Indications for first order transition!
Lattice results

e.g. Karsch, Laermann, Peikert

Critical temperature in chiral limit:

\[
N_f = 3 : T_c = (154 \pm 8) \text{ MeV}
\]
\[
N_f = 2 : T_c = (173 \pm 8) \text{ MeV}
\]

Chiral symmetry restoration and deconfinement at same \( T_c \)
pressure

\[ \frac{p}{T^4} \]

\[ \frac{p_{SB}}{T^4} \]

3 flavour
2+1 flavour
2 flavour
pure gauge

T [MeV]
realistic QCD

- precise lattice results not yet available for first order transition vs. crossover
- also uncertainties in determination of critical temperature (chiral limit ...)
- extension to nonvanishing baryon number only for QCD with relatively heavy quarks
Models
Analytical description of phase transition

• Needs model that can account simultaneously for the correct degrees of freedom below and above the transition temperature.
• Partial aspects can be described by more limited models, e.g. chiral properties at small momenta.
Chiral quark meson model

- Limitation to chiral behavior
- Small up and down quark mass
  - large strange quark mass
- Particularly useful for critical behavior of second order phase transition or near endpoints of critical lines

(see N. Tetradis for possible QCD-endpoint)
Quark descriptions (NJL-model) fail to describe the high temperature and high density phase transitions correctly.

High T: chiral aspects could be ok, but glue ...
(pion gas to quark gas)

High density transition: different Fermi surface for quarks and baryons (T=0)
– in mean field theory factor 27 for density at given chemical potential –
Confinement is important: baryon enhancement

Chiral perturbation theory even less complete

Berges, Jungnickel,...
Universe cools below 170 MeV...

Both gluons and quarks disappear from thermal equilibrium: mass generation

Chiral symmetry breaking

mass for fermions

Gluons?

Analogous situation in electroweak phase transition understood by Higgs mechanism

Higgs description of QCD vacuum?
Higgs picture of QCD

“spontaneous breaking of color “
in the QCD – vacuum

octet condensate

for \( N_f = 3 \) (u,d,s)

Higgs phase and confinement can be equivalent – then simply two different descriptions (pictures) of the same physical situation. Is this realized for QCD? Necessary condition: spectrum of excitations with the same quantum numbers in both pictures. Known for QCD: mesons + baryons.
Quark–antiquark condensate

quarks: \( \psi_{L,R} a_i \)

\[ \langle \overline{\psi}_{L,b} \psi_{R,a} \rangle = \]

\[ \frac{1}{16} \bar{\xi}_0 \left( \delta_{ia} \delta_{jb} - \frac{1}{3} \delta_{ij} \delta_{ab} \right) \]

color octet

\[ + \frac{1}{13} \bar{\xi}_0 \delta_{ij} \delta_{ab} \]

color singlet
Octet condensate

\(< \text{octet} > \neq 0 : \)

- “Spontaneous breaking of color”
- Higgs mechanism
- Massive Gluons – all masses equal
- Eight octets have vev
- Infrared regulator for QCD
Flavor symmetry

for equal quark masses:

octet preserves global SU(3)-symmetry
“diagonal in color and flavor”
“color-flavor-locking”

(cf. Alford, Rajagopal, Wilczek; Schaefer, Wilczek)

All particles fall into representations of
the “eightfold way”

quarks : $8 + 1$ , gluons : 8
Quarks and gluons carry the observed quantum numbers of isospin and strangeness of the baryon and vector meson octets!

They are integer charged!
Low energy effective action

\[ L = \mathcal{Z}_r \left\{ i \bar{\psi}_i \gamma^\mu \partial_\mu \psi_i + g \bar{\psi}_i \gamma^\mu A_{i\mu} \psi_j \right\} \]

\[ + \frac{1}{2} G^{\mu\nu}_{ij} G_{ij}^{\mu\nu} \]

\[ + \mathcal{T} \{ (D^\mu \chi^\nu) (D_\mu \chi_{ij}) + U(\chi) \}
\]

\[ + \bar{\psi}_i \left[ \left( \frac{1+\gamma_5}{2} \right) \left( \frac{1+\gamma_5}{2} \right) \psi_j \right] \]

\[ - \left( \frac{1+\gamma_5}{2} \right) \frac{1+\gamma_5}{2} \chi^\nu \]

\[ A_{i\mu} = \frac{1}{2} A_\mu (\lambda_2)_{ij} \]

\[ \gamma = \phi + \chi \]
...accounts for masses and couplings of light pseudoscalars, vector-mesons and baryons!
Phenomenological parameters

- 5 undetermined parameters

\[ x_0, \bar{x}_0, g, h, \bar{h} \]

fixed by 5 observable quantities

(for \( m_q = 0 \), averages over SU(3) multiplets)

\[ M_p = 850 \text{ MeV} \]
\[ M_n = 1150 \text{ MeV} \]
\[ M_\Delta = 1400 \text{ MeV} \]
\[ \bar{p} = 110 \text{ MeV} \quad (\bar{p} = \frac{5}{4} \bar{p}_n + \frac{3}{4} \bar{p}_\pi) \]
\[ \Gamma(\rho \to \mu^+\mu^-), \quad \Gamma(\rho \to e^+e^-) = 7 \text{ keV} \]

- predictions

* \( \Gamma(\gamma \to 2\pi) = 150 \text{ MeV} \)
* \( \beta \)-decay of neutrons: \( g_A = 1 \) (Exp: \( g_A = 1.16 \))
* Vector dominance in electromagnetic interactions of pions, \( g_{\gamma\pi\pi}/e = 0.04 \)
Chiral perturbation theory
+ all predictions of chiral perturbation theory
+ determination of parameters

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Chiral phase transition at high temperature

High temperature phase transition in QCD:

Melting of octet condensate

Lattice simulations:

Deconfinement temperature = critical temperature for restoration of chiral symmetry

Why?
Simple explanation:

- Octet condensate
- Chiral symmetry breaking
- "Confinement"

- Melting of octet condensate
- Chiral symmetry restoration
- "Deconfinement"

"Quarks and gluons become massless simultaneously"
Higgs picture of the QCD-phase transition

A simple mean field calculation gives roughly reasonable description that should be improved.

\[ T_c = 170 \text{ MeV} \]

First order transition
Experiment
Has the critical temperature of the QCD phase transition been measured?
Heavy ion collision
Chemical freeze-out temperature

$T_{\text{ch}} = 176$ MeV

hadron abundancies
Exclusion argument

hadronic phase with sufficient production of $\Omega$ : excluded!!
Exclusion argument

Assume T is a meaningful concept - complex issue, to be discussed later

$T_{ch} < T_c$ : hadrochemical equilibrium

Exclude $T_{ch}$ much smaller than $T_c$ :
say $T_{ch} > 0.95 \, T_c$

$0.95 < T_{ch} / T_c < 1$
Has \( T_c \) been measured?

- Observation: statistical distribution of hadron species with "chemical freeze out temperature" \( T_{ch} = 176 \text{ MeV} \)

- \( T_{ch} \) cannot be much smaller than \( T_c \): hadronic rates for \( T < T_c \) are too small to produce multistrange hadrons (\( \Omega, \ldots \))

- Only near \( T_c \) multiparticle scattering becomes important (collective excitations \( \ldots \)) – proportional to high power of density

\[ T_{ch} \approx T_c \]

P. Braun-Munzinger, J. Stachel, CW
\[ T_{\text{ch}} \approx T_c \]
Phase diagram

\[ \langle \phi \rangle \approx 0 \]

\[ \langle \phi \rangle = 0 \neq 0 \]

RHIC
SPS
AGS
SIS

\[ T_{\gamma} \text{ [MeV]} \]

\[ \mu_B^r \text{ [GeV]} \]

R. Pisarski
Temperature dependence of chiral order parameter

Does experiment indicate a first order phase transition for $\mu = 0$?
Second order phase transition

\[
V = -\mu_0^2 \phi \phi + c T^2 \phi \phi + \frac{\lambda}{2} \phi \phi
\]

non analytical behaviour

critical exponents
Second order phase transition

for $T$ only somewhat below $T_c$:
the order parameter $\sigma$ is expected to be close to zero and deviate substantially from its vacuum value.

This seems to be disfavored by observation of chemical freeze out!
• Chiral order parameter $\sigma$ depends on $T$
• Particle masses depend on $\sigma$
• Chemical freeze out measures $m/T$ for many species
• Mass ratios at $T$ just below $T_c$ are close to vacuum ratios
Ratios of particle masses and chemical freeze out

at chemical freeze out:

- ratios of hadron masses seem to be close to vacuum values
- nucleon and meson masses have different characteristic dependence on $\sigma$
  - $m_{\text{nucleon}} \sim \sigma$, $m_{\pi} \sim \sigma^{-1/2}$
- $\Delta\sigma/\sigma < 0.1$ (conservative)
first order phase transition seems to be favored by chemical freeze out

...or extremely rapid crossover
How far has first order line been measured?
Exclusion argument for large density

hadronic phase with sufficient production of $\Omega$ : excluded !!
First order phase transition line

hadrons

quarks and gluons

$\mu = 923\text{MeV}$ transition to nuclear matter
Phase diagram for $m_s > m_{u,d}$

- Quark-gluon plasma: "deconfinement"
- Quark matter: superfluid $B$ spontaneously broken
- Nuclear matter: $B$, isospin ($I_3$) spontaneously broken, $S$ conserved
Is temperature defined?

Does comparison with equilibrium critical temperature make sense?
Prethermalization

J. Berges, Sz. Borsanyi, CW
Vastly different time scales
for “thermalization” of different quantities

here : scalar with mass $m$ coupled to fermions
(method : two particle irreducible non-equilibrium effective action (J. Berges et al))
Prethermalization

Equation of state $p/\varepsilon$

Similar for kinetic temperature
different “temperatures”
Mode temperature

\[ n_p(t) \equiv \frac{1}{\exp \left[ \omega_p(t)/T_p(t) \right] + 1} \]

\( \omega_{p(f,s)}(t) \) determined by peak of spectral function

\( n_p \): occupation number for momentum \( p \)

late time:
Bose-Einstein or
Fermi-Dirac distribution
Global kinetic temperature $T_{\text{kin}}$

Practical definition:
- association of temperature with average kinetic energy per d.o.f.
  \[ T_{\text{kin}}(t) = \frac{E_{\text{kin}}(t)}{c_{eq}} \]
- $c_{eq} = \frac{E_{\text{kin,eq}}}{T_{eq}}$ is given solely in terms of equilibrium quantities
  (E.g. relativistic plasma: $E_{\text{kin}}/N = \epsilon/n = \alpha T$)

**Kinetic equilibration:** \[ T_{\text{kin}}(t) = T_{eq} \]

Consider also chemical temperatures $T_{\text{ch}}^{(f, s)}$ from integrated number density of each species, $n_{\text{ch}}^{(f, s)}(t) = \int \frac{d^3p}{(2\pi)^3} n_p^{(f, s)}(t)$:

\[
n(t) = \frac{g}{2\pi^2} \int_0^\infty dp p^2 \left[ \exp \left( \frac{\omega_p(t)}{T_{\text{ch}}(t)} \right) \pm 1 \right]^{-1}
\]

**Chemical equilibration:** \[ T_{\text{ch}}^{(f)}(t) = T_{\text{ch}}^{(s)}(t) \]
Kinetic equilibration before chemical equilibration
Once a temperature becomes stationary it takes the value of the equilibrium temperature.

Once chemical equilibration has been reached the chemical temperature equals the kinetic temperature and can be associated with the overall equilibrium temperature.

Comparison of chemical freeze out temperature with critical temperature of phase transition makes sense.
Short and long distance degrees of freedom are different!

Short distances: quarks and gluons
Long distances: baryons and mesons

How to make the transition?
How to come from quarks and gluons to baryons and mesons?

Find effective description where relevant degrees of freedom depend on momentum scale or resolution in space.

Microscope with variable resolution:

- High resolution, small piece of volume: quarks and gluons
- Low resolution, large volume: hadrons
Functional Renormalization Group

from small to large scales
Exact flow equation

for scale dependence of average action

\[
\partial_k \Gamma_k[\varphi] = \frac{1}{2} \text{Tr} \left\{ \left( \Gamma_k^{(2)}[\varphi] + R_k \right)^{-1} \partial_k R_k \right\}
\]

\[
\left( \Gamma_k^{(2)} \right)_{ab}(q, q') = \frac{\delta^2 \Gamma_k}{\delta \varphi_a(-q) \delta \varphi_b(q')}
\]

\[
\text{Tr} : \sum_a \int \frac{d^d q}{(2\pi)^d}
\]

(fermions : STr)

\[
\partial_k R_k(q^\pm)
\]

\[
( Z_k q^2 + M_k^2 + R_k(q^\pm) )^{-d}
\]
Infrared cutoff

\[ R_k : \text{IR-cutoff} \]

e.g., \[ R_k = \frac{Z_k q^2}{e q^2/k^2 - 1} \]

or \[ R_k = Z_k(k^2 - q^2)\Theta(k^2 - q^2) \quad \text{(Litim)} \]

\[ \lim_{k \to 0} R_k = 0 \]

\[ \lim_{k \to \infty} R_k \to \infty \]
Nambu Jona-Lasinio model

\[ S = \int d^4x \{ i \bar{\psi}_a^i \gamma^\mu \partial_\mu \psi_a^i + 2 \lambda_c ( \bar{\psi}_{L_a}^i \psi_{R_a}^i ) ( \bar{\psi}_{R_a}^i \psi_{L_a}^i ) \} \]

\[ \psi_{L/R} = \frac{1 \pm \gamma^5}{2} \psi \]

\[ i, j = 1 \ldots N_c \quad \text{color} \quad (N_c = 3) \]

\[ a, b = 1 \ldots N_f \quad \text{flavor} \quad (N_f = 3, 2) \]

chiral flavor symmetry:

\[ SU(L(N_f)) \times SU(R(N_f)) \]

...and more general quark meson models
Chiral condensate

\[
\frac{\langle \bar{\psi} \psi \rangle}{T_c^3}
\]

\(\Uparrow\)

\(\chi PT\)

\(\rightarrow\) Explicit link between \(\chi PT\) domain of validity (4d) and critical (universal) domain near \(T_c\) (3d)

2nd order PT (expected for \(O(4)\) Heisenberg model)
Scaling form of equation of state

Berges, Tetradis, …
temperature dependent masses

pion mass

sigma mass

\[ m_\pi \]

MeV

\[ \tilde{m}_\sigma \]

MeV

\[ T/\text{MeV} \]

\[ m_\sigma < 2m_\pi \text{ for } T \gtrsim 100\text{ MeV} \]

No long pion correlation length in thermal equilibrium!
Conclusion

- Experimental determination of critical temperature may be more precise than lattice results
- Rather simple phase structure is suggested
- Analytical understanding is only at beginning
end
Cosmological phase transition...

...when the universe cools below 175 MeV

$10^{-5}$ seconds after the big bang
QCD at high density

Nuclear matter
Heavy nuclei
Neutron stars
Quark stars ...
QCD at high temperature

• Quark – gluon plasma
• Chiral symmetry restored
• Deconfinement (no linear heavy quark potential at large distances)
• Lattice simulations: both effects happen at the same temperature
“Solution” of QCD

Effective action (for suitable fields) contains all the relevant information of the solution of QCD

Gauge singlet fields, low momenta:
- Order parameters, meson-(baryon-) propagators

Gluon and quark fields, high momenta:
- Perturbative QCD

Aim: Computation of effective action
QCD – phase transition

Quark–gluon plasma  Hadron gas

- Gluons: $8 \times 2 = 16$
- Quarks: $9 \times \frac{7}{2} = 12.5$
- Dof: 28.5

- Light mesons: 8
- (pions: 3)
- Dof: 8

Chiral symmetry  Chiral sym. broken

Large difference in number of degrees of freedom!
Strong increase of density and energy density at $T_c$!
Spontaneous breaking of color

- Condensate of colored scalar field
- Equivalence of Higgs and confinement description in real \((N_f=3)\) QCD vacuum
- Gauge symmetries not spontaneously broken in formal sense (only for fixed gauge)
  - Similar situation as in electroweak theory
- No “fundamental” scalars
- Symmetry breaking by quark-antiquark-condensate
A simple mean field calculation

vanishing quark masses $\rightarrow$ equal $m_u=m_d=m_s \neq 0$

$2m_u^2 + m_s^2 = (390 \text{MeV})^2$

\[
\begin{array}{lll}
F_p & 700 \text{ MeV} & 770 \text{ MeV} \\
\theta & 68 \text{ MeV} & 66 \text{ MeV} \\
T_c & 154 \text{ MeV} & 170 \text{ MeV} \\
\bar{F}_p(T_c) & 290 \text{ MeV} & 290 \text{ MeV} \\
\bar{F}_\theta(T_c) & 580 \text{ MeV} & 600 \text{ MeV} \\
\end{array}
\]

\[
\left\{ \begin{array}{l}
\text{screening masses} \\
\end{array} \right. \\
\]

equation of state: pion gas $\rightarrow$ QGP

\[
\frac{\epsilon - 3p}{\epsilon + p} = \tau(T_c) \quad \frac{T_c^*}{T^*} \quad (T > T_c)
\]

$\tau(T_c) \quad 0.37 \quad 0.53$
Hadron abundancies

\[
\frac{p}{p} \quad \frac{\bar{\Lambda}/\Lambda}{\Lambda/\Lambda} \quad \frac{\bar{\Xi}/\Xi}{\Xi/\Xi} \quad \frac{\bar{\Omega}/\Omega}{\Omega/\Omega} \quad \frac{\pi^{-}/\pi^{+}}{K^{-}/K^{+}} \quad \frac{K^{0}/h^{-}}{\pi^{-}/\pi^{+}} \quad \frac{h^{-}}{\phi^{-}} \quad \frac{\Lambda/h^{-}}{\Xi/h^{-}} \quad \frac{\Omega/\pi^{-}}{X \times 10}
\]

\[
\sqrt{s_{NN}} = 130 \text{ GeV}
\]

Model re-fit with all data
\[T = 176 \text{ MeV}, \quad \mu_h = 41 \text{ MeV}\]

Bound for critical temperature

0.95 \( T_c < T_{ch} < T_c \)

- not: "I have a model where \( T_c \approx T_{ch} \)"
- not: "I use \( T_c \) as a free parameter and find that in a model simulation it is close to the lattice value (or \( T_{ch} \))"

\( T_{ch} \approx 176 \text{ MeV} \) (?)
Estimate of critical temperature

For $T_{ch} \approx 176$ MeV:

$0.95 < \frac{T_{ch}}{T_c}$

- $176$ MeV $< T_c < 185$ MeV

$0.75 < \frac{T_{ch}}{T_c}$

- $176$ MeV $< T_c < 235$ MeV

Quantitative issue matters!
Key argument

- Two particle scattering rates not sufficient to produce $\Omega$
- “multiparticle scattering for $\Omega$-production “ : dominant only in immediate vicinity of $T_c$
needed:

**lower bound on** \( T_{ch} / T_c \)
Exclude the hypothesis of a hadronic phase where multistrange particles are produced at $T$ substantially smaller than $T_c$. 
Mechanisms for production of multistrange hadrons

Many proposals

- Hadronization
- Quark-hadron equilibrium
- Decay of collective excitation ($\sigma$ – field)
- Multi-hadron-scattering

Different pictures!
Hadronic picture of \( \Omega \) - production

Should exist, at least semi-quantitatively, if \( T_{ch} < T_c \)

\(( \text{for } T_{ch} = T_c : T_{ch} > 0.95 T_c \text{ is fulfilled anyhow} )\)

e.g. collective excitations \( \approx \) multi-hadron-scattering

(not necessarily the best and simplest picture)

\[ \text{multihadron} \rightarrow \Omega + X \text{ should have sufficient rate} \]

Check of consistency for many models

Necessary if \( T_{ch} \neq T_c \) and temperature is defined

Way to give \textbf{quantitative} bound on \( T_{ch} / T_c \)
Rates for multiparticle scattering

2 pions + 3 kaons $\rightarrow$ $\Omega$ + antiproton

$$r(n_{in}, n_{out}) = \bar{n}(T)^{n_{in}} |\mathcal{M}|^2 \phi$$

$$\phi = \prod_{k=1}^{n_{out}} \left( \int \frac{d^3 p_k}{(2\pi)^3(2E_k)} \right) (2\pi)^4 \delta^4 \left( \sum_k p_k^\mu \right)$$

$$r_{\Omega} = n_\pi^5 (n_K/n_\pi)^3 |\mathcal{M}|^2 \phi.$$
Very rapid density increase

...in vicinity of critical temperature

Extremely rapid increase of rate of multiparticle scattering processes

( proportional to very high power of density )
Energy density

Lattice simulations Karsch et al

even more dramatic for first order transition
Phase space

- increases very rapidly with energy and therefore with temperature
- effective dependence of time needed to produce $\Omega$

$$T_\Omega \sim T^{-60}$$

This will even be more dramatic if transition is closer to first order phase transition
Production time for $\Omega$

multi-meson scattering

$\pi^+\pi^+\pi^+K^+K^- \rightarrow \Omega+p$

strong dependence on “pion” density

P. Braun-Munzinger, J. Stachel, CW
extremely rapid change

lowering T by 5 MeV below critical temperature:

rate of $\Omega$ – production decreases by factor 10

This restricts chemical freeze out to close vicinity of critical temperature:

$$0.95 < \frac{T_{ch}}{T_c} < 1$$
enough time for $\Omega$ - production

at $T=176$ MeV:

$\tau_\Omega \sim 2.3$ fm

consistency!
Relevant time scale in hadronic phase

rates needed for equilibration of $\Omega$ and kaons:

$$
\Delta T = 5 \text{ MeV}, \\
F_{\Omega K} = 1.13, \\
T_T = 8 \text{ fm}
$$

$$
\frac{\tilde{r}_j}{V} = \frac{\dot{N}_j}{V} = \dot{n}_j + n_j \dot{V}/V.
$$

$$
|\frac{\tilde{r}_\Omega}{n_\Omega} - \frac{\tilde{r}_K}{n_K}| = \frac{\ln F_{\Omega K} T_{ch}}{\tau_T} \frac{T_{ch}}{\Delta T} = (1.10 - 0.55)/\text{fm}
$$

two–particle–scattering:

$$
|\frac{\tilde{r}_\Omega}{n_\Omega} - \frac{\tilde{r}_K}{n_K}| = (0.02 - 0.2)/\text{fm}
$$
A possible source of error: temperature-dependent particle masses

Chiral order parameter $\sigma$ depends on $T$

Chemical freeze-out measures $T/m$!

\[
M_j(T) = h_j(T, \mu)\sigma(T, \mu)
\]

\[
\frac{\sigma(T_{ch}, \mu)}{T_{ch}} = \frac{\sigma(0, 0)}{T_{obs}}.
\]

\[
T_c = 176^{+5}_{-18} \text{ MeV}.
\]
uncertainty in $m(T)$

uncertainty in critical temperature
systematic uncertainty:

$$\Delta \sigma/\sigma = \Delta T_c / T_c$$

$\Delta \sigma$ is negative

$$M_j(T) = h_j(T, \mu) \sigma(T, \mu)$$

$$\frac{\sigma(T_{ch}, \mu)}{T_{ch}} = \frac{\sigma(0, 0)}{T_{obs}}.$$
• experimental determination of critical temperature may be more precise than lattice results
• error estimate becomes crucial
Thermal equilibration: occupation numbers
Effective loss of details of initial conditions (thermalization)

- Two different initial conditions (A, B) with same energy density

**Fermion occupation number:**

Characteristic damping time: \( t_{\text{damp}}(p/m \simeq 1) \simeq 25 \, m^{-1} \), thermalization time: \( t_{\text{eq}} \simeq 95 \, m^{-1} \)

in units of scalar thermal mass \( m \) \( \langle n(p) \rangle \sim \text{tr} \frac{p^i \gamma^i}{p} \langle [\psi, \bar{\psi}] \rangle_p \; ; \; 0 \leq n(p) \leq 1 \)
$\sim t_{pt}$ is of the order of the characteristic inverse mass scale $m^{-1}$

- consequence of rapid loss of phase information (“dephasing”)
- unrelated to the scattering-driven process of thermalization

$\sim t_{pt} \ll t_{damp} \ll t_{eq}$

**Prethermalization of the equation of state occurs on time scales dramatically shorter than the thermal equilibration time!**

Given an EOS, the crucial question arises:

- *Does a suitable global kinetic temperature $T_{kin}$ also exist at $t_{pt}$?*

$\sim$ “quasi-thermal” description in a far-from-equilibrium situation!
Kinetic vs chemical equilibration:

\[ T_{\text{kin}}/T_{\text{eq}} \]

\[ T_{\text{chem}}/T_{\text{eq}}: \begin{align*}
    h=1.0, & T_{\text{eq}}/m=1.0 \\
    h=1.0, & T_{\text{eq}}/m=2.8 \\
    h=0.7, & T_{\text{eq}}/m=2.1 \\
\end{align*} \]

- upper curves: scalars
- lower curves: fermions

\[ T_{\text{kin}}(t) \text{ prethermalizes on a very short time scale } \sim m^{-1} \]

in contrast to chemical equilibration

\[ \sim \text{ late-time chemical equilibration for } t_{\text{ch}} \sim t_{\text{eq}} \]

\( t_{\text{ch}} \) depends on details of particle number changing interactions; deviation from thermal result can become relatively small for \( t \ll t_{\text{eq}} \)
**Prethermalization**: far-from-equilibrium phenomenon which describes

- very rapid establishment of an approximately constant ratio of pressure over energy density (equation of state)
- as well as a kinetic temperature based on average kinetic energy

\[ p = p(e) \text{ important ingredient to close system of equations } \partial_\mu T^{\mu\nu} = 0 \]

More generally:

Extremely different time scales for loss of initial conditions for

- certain “bulk quantities” which average over all momentum modes
- “mode quantities” characterizing the evolution of individual modes
Compare with 2PI:

Propagator for (very) different initial conditions with same $\langle E \rangle$

(momenta $p = 0, 3, 5$; all in initial mass units, $\phi = 0$)

here: scalar $\phi^4$ to 3-loop

J.B., Cox, PLB517 (2001) 369

'Damping'

exponential suppression of correlations with initial time
rate: $\gamma^{(damp)}$

$F(t,0) = \langle \phi(t) \phi(0) \rangle \to 0^+$

early

'Drifting'

smooth, parametrically slow change of modes
weak dependence of $F(t,t')$
on $t + t'$

intermediate

'Thermalization'

approach to quantum thermal equilibrium
rate: $\gamma^{(therm)} \neq \gamma^{(damp)}$

$F(t,t') \to F^{(eq)}(t-t')$

late time

Effective loss of initial conditions
Coherent field regime $\rightarrow$ *particle production* $\rightarrow$ fluctuation dominated regime:

$E(t)/E(0)$

$E_{\text{fluc}}$, $E_{\text{cl}}$

$t_{\text{nonpert}}$: $E_{\text{fluc}} \simeq E_{\text{cl}}$

here: $N = 4$, $\lambda = 10^{-6}$

$\sim$ dramatic phenomenon for ‘arbitrarily’ weakly coupled theory!
Chiral symmetry restoration at high temperature

Low T
SSB
$\langle \phi \rangle = \phi_0 \neq 0$

High T
SYM
$\langle \phi \rangle = 0$

at high T:
less order
more symmetry

examples:
magnets, crystals
Order of the phase transition is a crucial ingredient for experiments (heavy ion collisions) and cosmological phase transition.
Order of the phase transition
First order phase transition
Simple one loop structure - nevertheless (almost) exact

\[ \partial_k U_k = \frac{1}{2} \left( \partial_k q^2 + M_k^2 + R_k(q^2) \right)^{-1} \]
Flow equation for average potential

\[ \partial_k U_k(\varphi) = \frac{1}{2} \sum_i \int \frac{d^d q}{(2\pi)^d} \frac{\partial_k R_k(q^2)}{Z_k q^2 + R_k(q^2) + \overline{M}^2_{k,i}(\varphi)} \]

\[ \overline{M}^2_{k,ab} = \frac{\partial^2 U_k}{\partial \varphi_a \partial \varphi_b} : \text{ Mass matrix} \]

\[ \overline{M}^2_{k,i} : \text{ Eigenvalues of mass matrix} \]
Critical temperature, $N_f = 2$

<table>
<thead>
<tr>
<th>$m_\pi$ (MeV)</th>
<th>0</th>
<th>45</th>
<th>135</th>
<th>230</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{pc}$ (MeV)</td>
<td>100.7</td>
<td>$\sim$ 110</td>
<td>$\sim$ 130</td>
<td>$\sim$ 150</td>
</tr>
</tbody>
</table>

For $f_\pi = 93$ MeV

Lattice simulation

J. Berges, D. Jungnickel, …