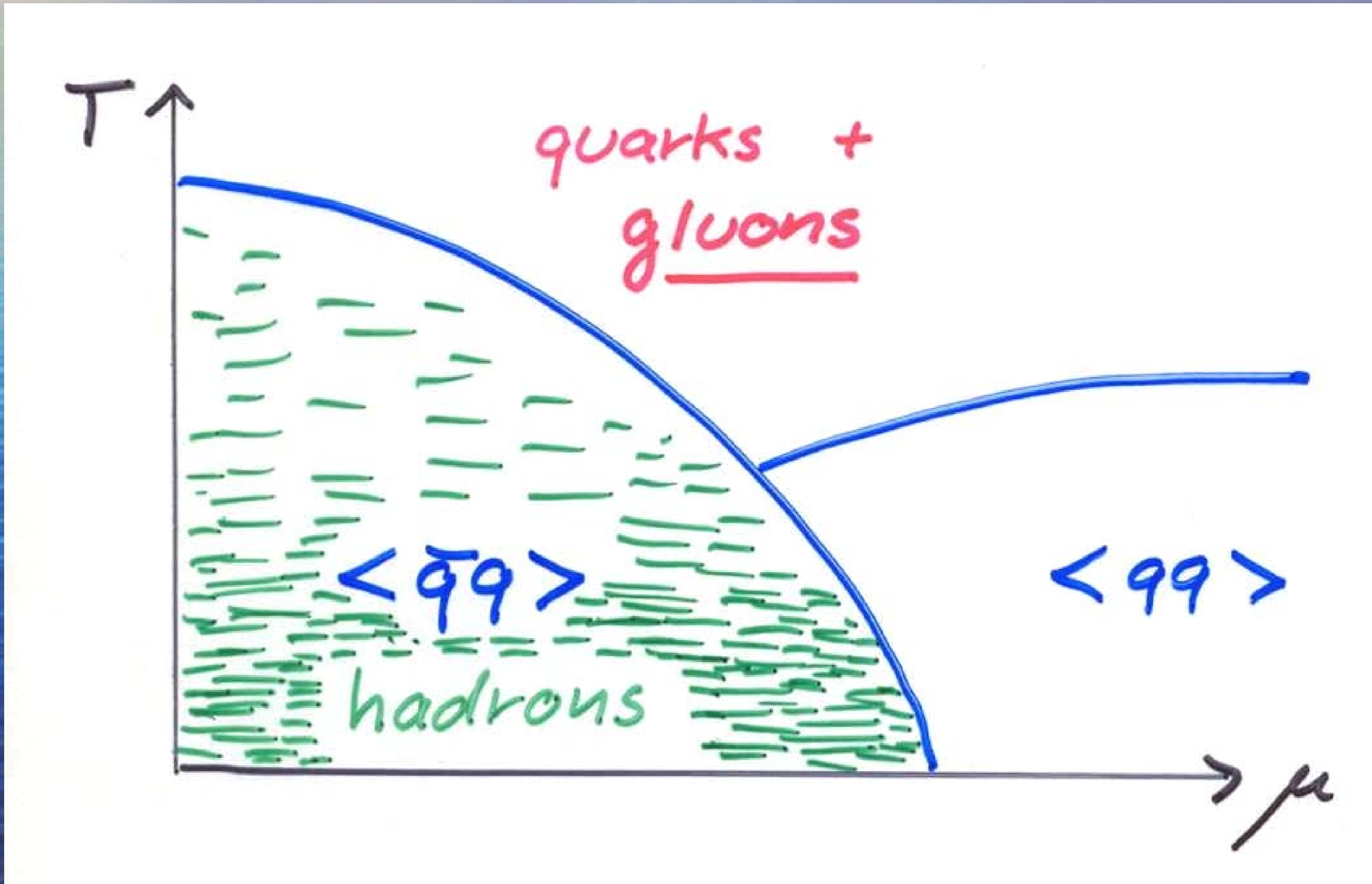
Critical lines and points

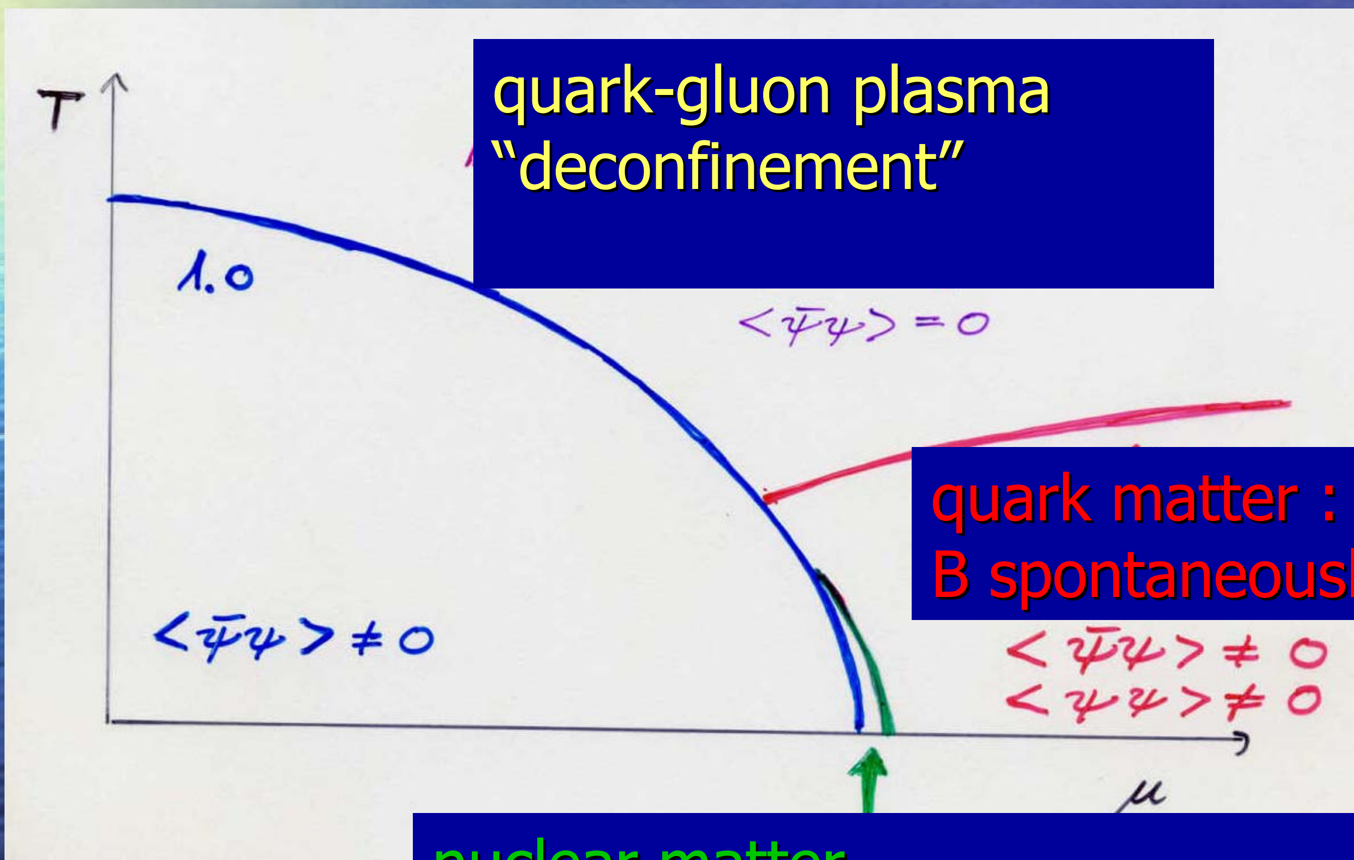
in the

QCD phase diagram

Understanding the phase diagram



Phase diagram for $m_s > m_{u,d}$



quark-gluon plasma
"deconfinement"

quark matter : superfluid
B spontaneously broken

nuclear matter
B,isospin (I_3) spontaneously broken, S conserved

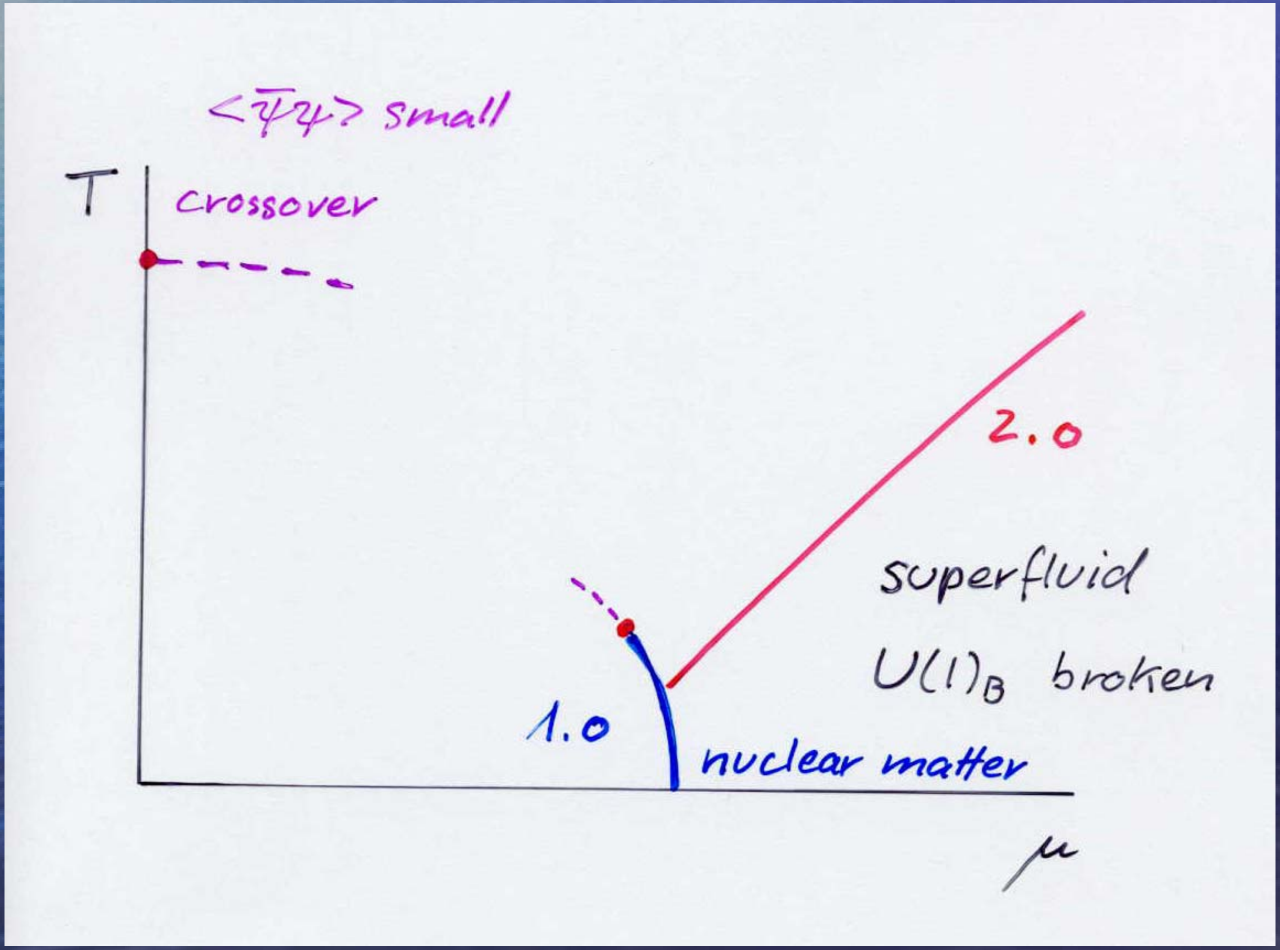
Order parameters

- Nuclear matter and quark matter are separated from other phases by true critical lines
- Different realizations of global symmetries
- Quark matter: SSB of baryon number B
- Nuclear matter: SSB of combination of B and isospin I_3

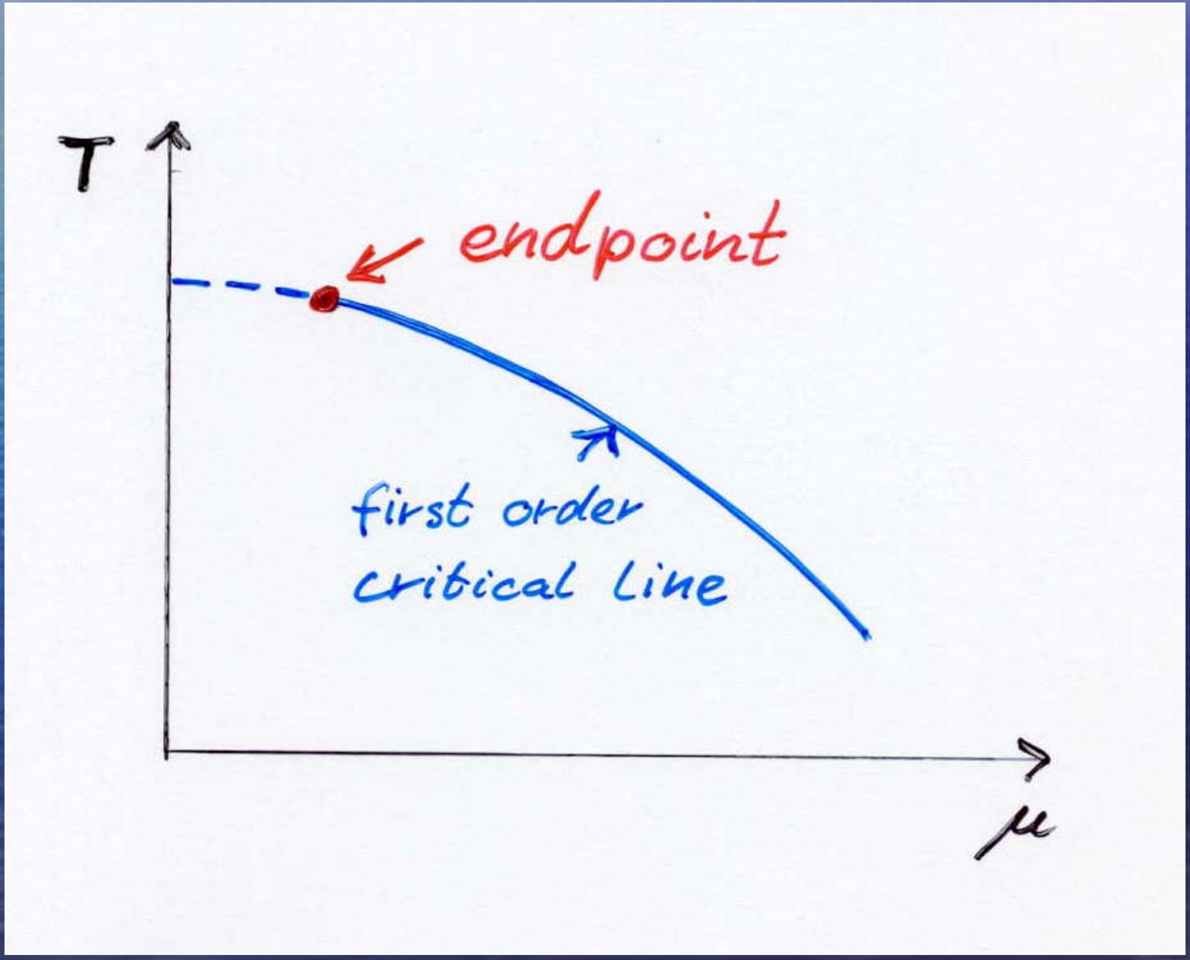
  neutron-neutron condensate

"minimal" phase diagram

for equal nonzero quark masses



Endpoint of critical line ?



How to find out ?

Methods

- Lattice : You have to wait until chiral limit is properly implemented !
- Models : Quark meson models cannot work
  Higgs picture of QCD ?
- Experiment : Has T_c been measured ?
  Indications for
  first order transition !

Lattice

Lattice results

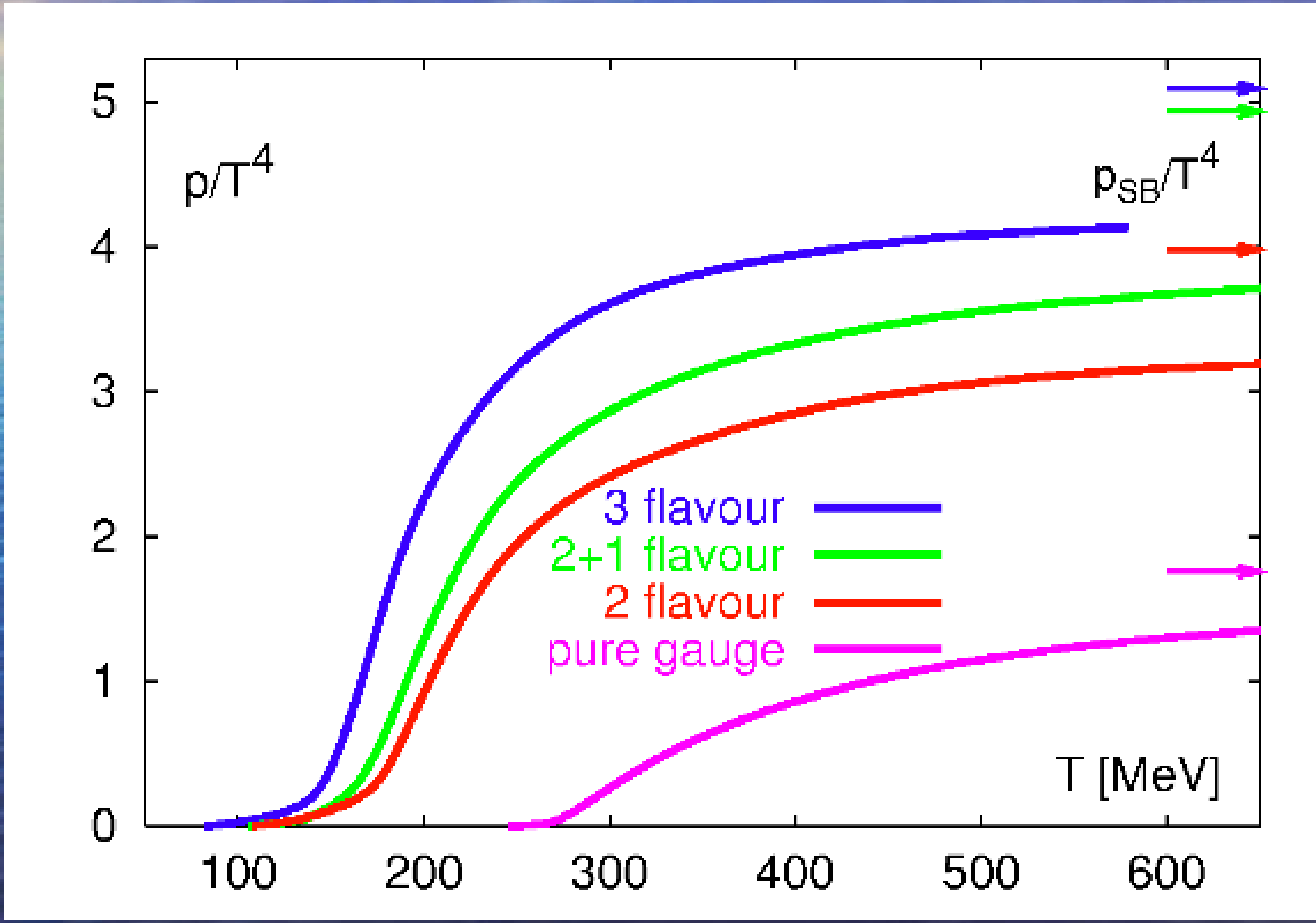
e.g. Karsch,Laermann,Peikert

Critical temperature in chiral limit :

$N_f = 3$: $T_c = (154 \pm 8)$ MeV
$N_f = 2$: $T_c = (173 \pm 8)$ MeV

Chiral symmetry restoration and deconfinement at same T_c

pressure



p/T4
pSB/T4
3 flavour
2+1 flavour
2 flavour
pure gauge
T [MeV]
0
1
2
3
4
5
100
200
300
400
500
600

realistic QCD

- precise lattice results not yet available
  for first order transition vs. crossover
- also uncertainties in determination of critical temperature (chiral limit …)
- extension to nonvanishing baryon number only for QCD with relatively heavy quarks

Models

Analytical description of phase transition

- Needs model that can account simultaneously for the correct degrees of freedom below and above the transition temperature.
- Partial aspects can be described by more limited models, e.g. chiral properties at small momenta.

Chiral quark meson model

- Limitation to chiral behavior
- Small up and down quark mass

  - large strange quark mass
- Particularly useful for critical behavior of second order phase transition or near endpoints of critical lines

(see N. Tetradis for possible QCD-endpoint)

Quark descriptions (NJL-model) fail to describe the high temperature and high density phase transitions correctly

High T : chiral aspects could be ok , but glue …
(pion gas to quark gas)

High density transition : different Fermi surface for quarks and baryons (T=0)
– in mean field theory factor 27 for density at given chemical potential –
Confinement is important : baryon enhancement

Berges,Jungnickel,…

Chiral perturbation theory even less complete

Universe cools below 170 MeV…

Both gluons and quarks disappear from thermal equilibrium : mass generation

- Chiral symmetry breaking
  - → mass for fermions
- Gluons ?

Analogous situation in electroweak phase transition understood by Higgs mechanism

Higgs description of QCD vacuum ?

Higgs picture of QCD

"spontaneous breaking of color "
in the QCD – vacuum

octet condensate

for $N_f = 3$ (u,d,s)

C.Wetterich, Phys.Rev.D64,036003(2001),hep-ph/0008150

Higgs phase and confinement

can be equivalent –

then simply two different descriptions (pictures) of the same physical situation

Is this realized for QCD ?

Necessary condition : spectrum of excitations with the same quantum numbers in both pictures

- known for QCD : mesons + baryons -

Quark –antiquark condensate

quarks : $\psi_{L,R\,a\,i}$

a – flavor, i – color

condensate in vacuum :

$$\langle \bar{\psi}_{L\,jb}\, \psi_{R\,ai} \rangle =$$

$$\frac{1}{\sqrt{6}}\, \bar{\xi}_0 \left(\delta_{ia}\, \delta_{jb} - \frac{1}{3}\, \delta_{ij}\, \delta_{ab} \right)$$

color octet

$$+ \frac{1}{\sqrt{3}}\, \bar{\sigma}_0\, \delta_{ij}\, \delta_{ab}$$

color singlet

Octet condensate

$< \text{octet} > \neq 0$:

- "Spontaneous breaking of color"
- Higgs mechanism
- Massive Gluons – all masses equal
- Eight octets have vev
- Infrared regulator for QCD

Flavor symmetry

for equal quark masses :

octet preserves global SU(3)-symmetry
"diagonal in color and flavor"
"color-flavor-locking"

(cf. Alford,Rajagopal,Wilczek ; Schaefer,Wilczek)

All particles fall into representations of the "eightfold way"

quarks : 8 + 1 , gluons : 8

Quarks and gluons carry the observed quantum numbers of isospin and strangeness of the baryon and vector meson octets !

They are integer charged!

Low energy effective action

$$\mathcal{L} = Z_\psi \{ i \bar\psi_i \gamma^\mu \partial_\mu \psi_i + g \bar\psi_i \gamma^\mu A_{ij,\mu} \psi_j \}$$

$$+ \frac{1}{2} G^{\mu\nu}_{ij} G_{ji,\mu\nu}$$ „QCD"

$$+ \mathrm{Tr}\{ (D^\mu \gamma_{ij})^\dagger (D_\mu \gamma_{ij}) \} + U(\gamma)$$

$$+ Z_\psi \bar\psi_i \left[(h \varphi \delta_{ij} + \tilde{h} \chi_{ij}) \frac{1+\gamma_5}{2} \right.$$

$$\left. - (h \varphi^\dagger \delta_{ij} + \tilde{h} \chi^\dagger_{ij}) \frac{1-\gamma_5}{2} \right] \psi_j$$

$$\gamma = \varphi + \chi$$

$$A_{ij,\mu} = \frac{1}{2} A^z_\mu (\lambda_z)_{ij}$$

…accounts for masses and couplings of light pseudoscalars, vector-mesons and baryons !

Phenomenological parameters

- 5 undetermined parameters

χ_0, σ_0, g, h, $\tilde{h}$

fixed by 5 observable quantities

(for $m_q = 0$, averages over $SU(3)$ multiplets)

$\bar{M}_\rho = 850$ MeV

$\bar{M}_N = 1150$ MeV

$M_1 = 1400$ MeV

$\bar{f} = 110$ MeV $\quad (\bar{f} = \frac{2}{3} f_K + \frac{1}{3} f_\pi)$

$\Gamma(\rho \to \mu^+\mu^-)$, $\Gamma(\rho \to e^+e^-) = 7$ keV

- predictions

* $\Gamma(\rho \to 2\pi) \approx 150$ MeV
* β-decay of neutrons: $g_A = 1$ (Exp: $g_A = 1.26$)
* vector dominance in electromagnetic interactions of pions, $g_{\gamma\pi\pi}/e = 0.04$

Chiral perturbation theory

+ all predictions of chiral perturbation theory
+ determination of parameters

		„Exp"
L_1	0.87	0.7 ± 0.3
L_2	1.74	1.7 ± 0.7
L_3	-5.2	$-(4.4 \pm 2.5)$

Chiral phase transition at high temperature

High temperature phase transition in QCD :

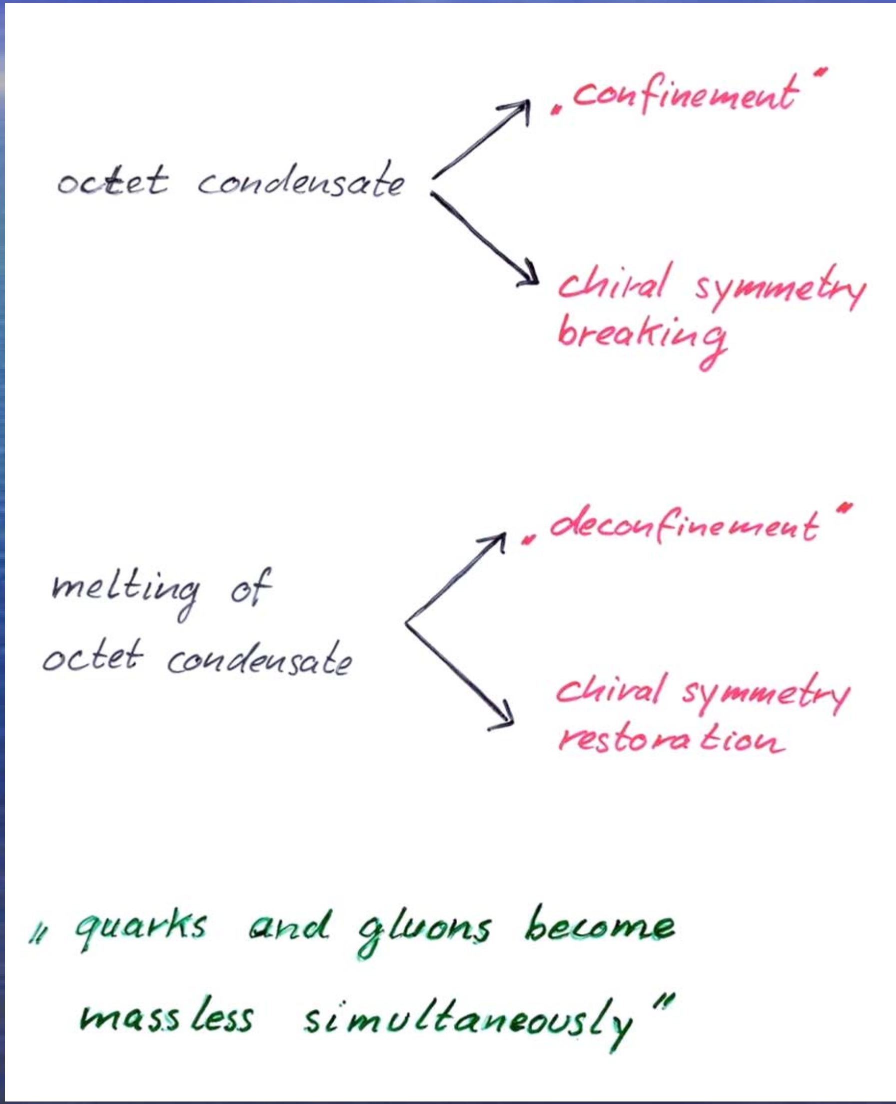
Melting of octet condensate

Lattice simulations :

Deconfinement temperature = critical temperature for restoration of chiral symmetry

Why ?

Simple explanation :



Higgs picture of the QCD-phase transition

A simple mean field calculation gives roughly reasonable description that should be improved.

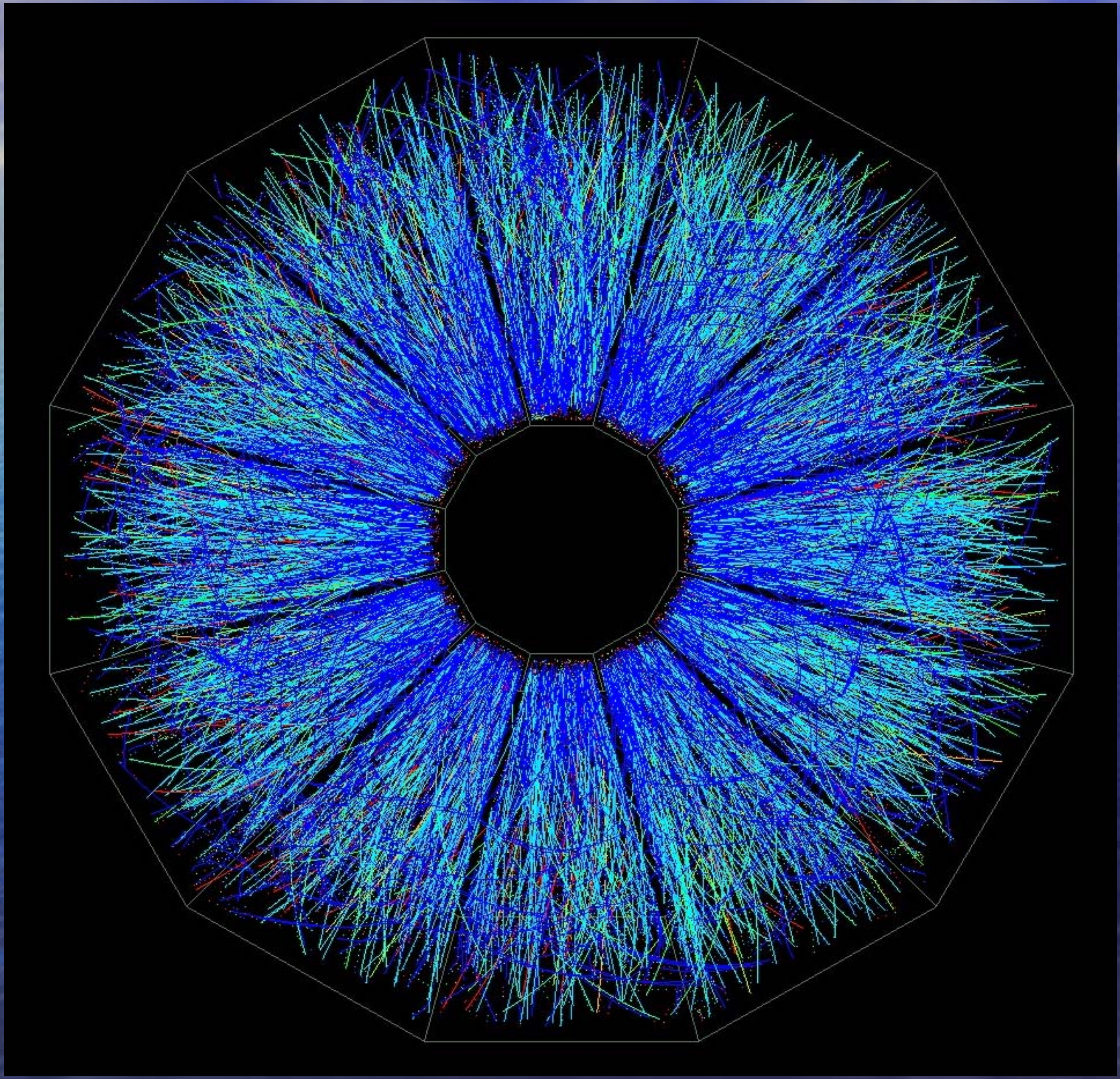
T_c =170 MeV

First order transition

Experiment

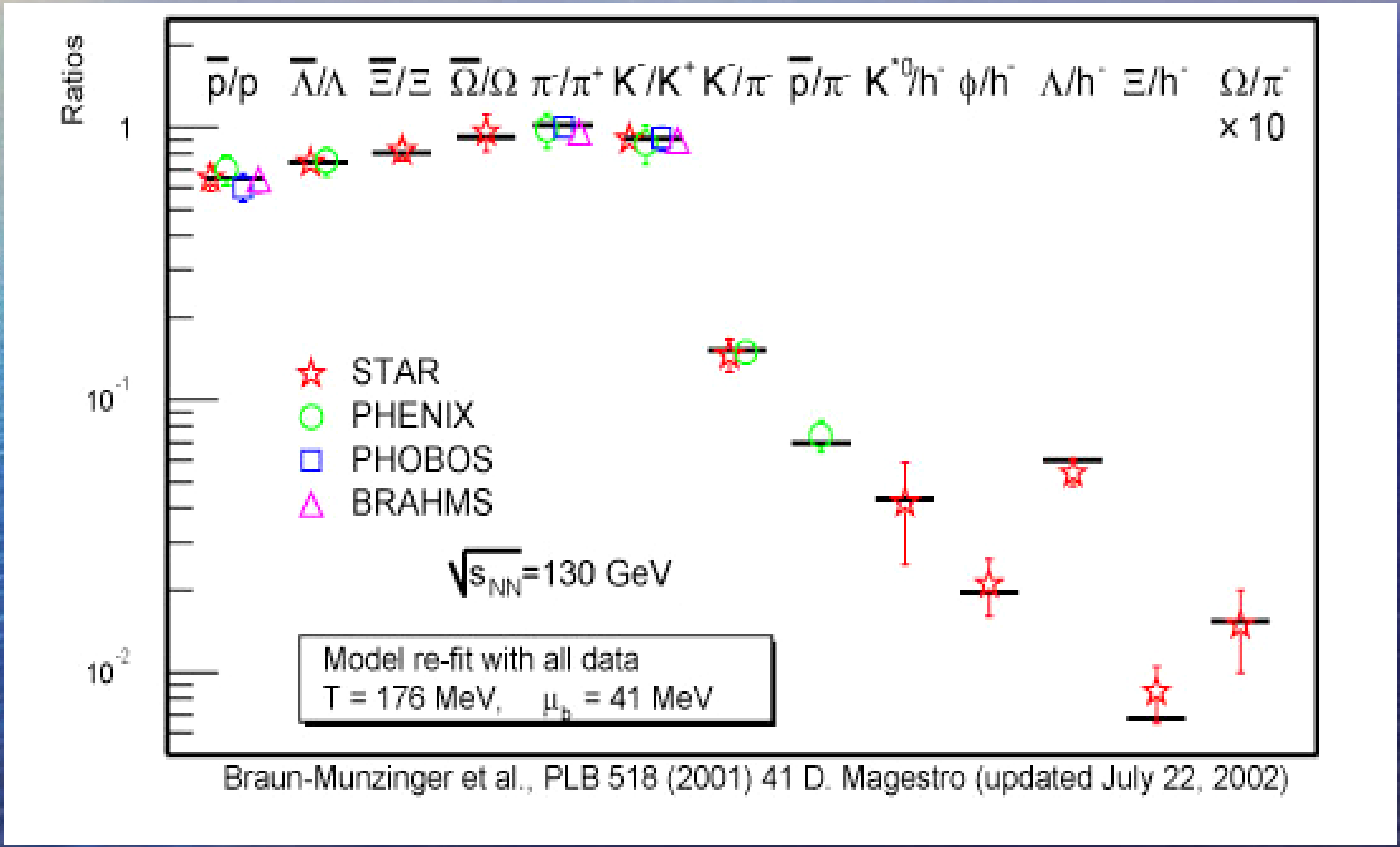
Has the critical temperature of the QCD phase transition been measured ?

Heavy ion collision



Chemical freeze-out temperature

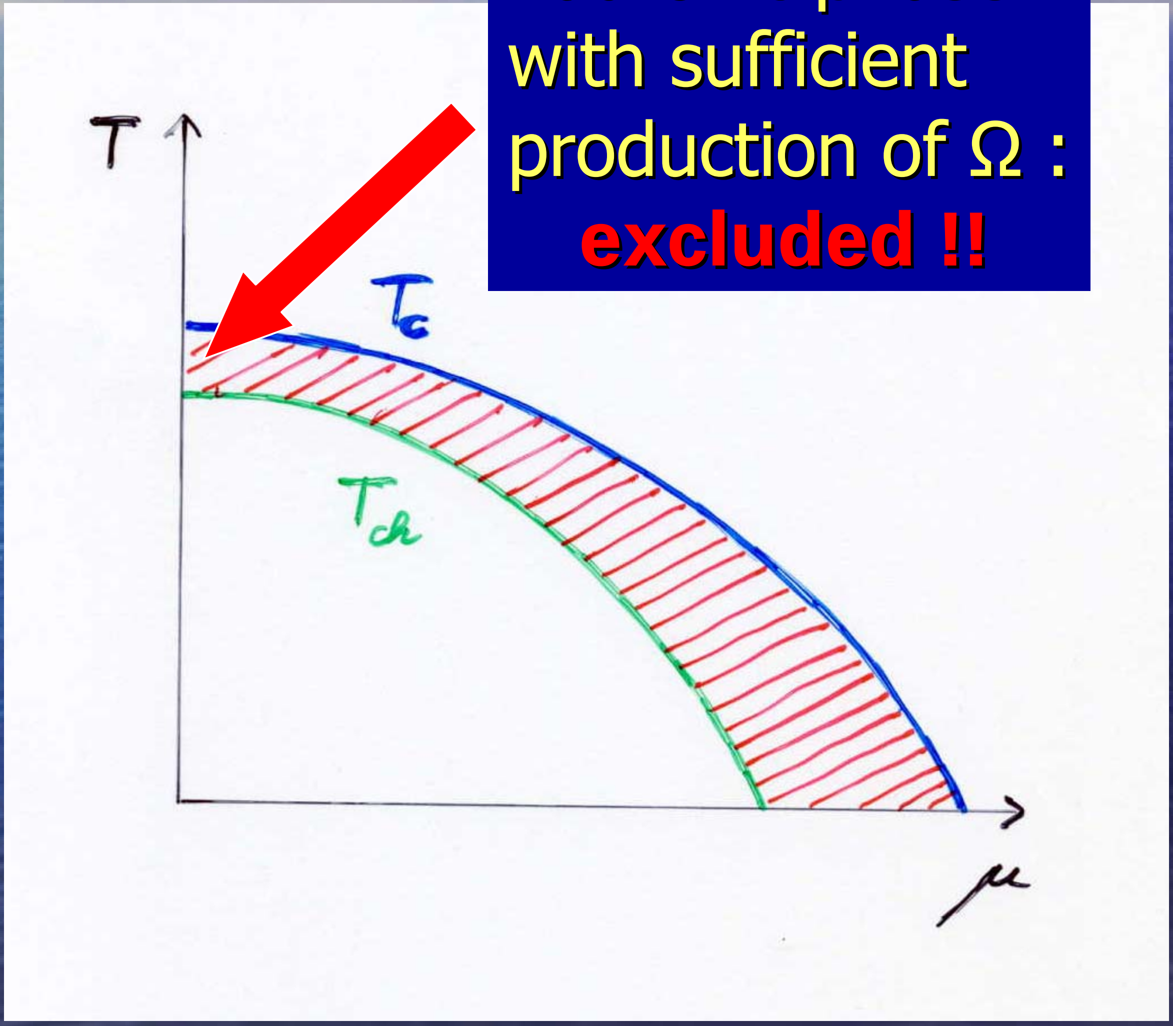
T_{ch} =176 MeV



hadron abundancies

Exclusion argument

hadronic phase with sufficient production of Ω :
excluded !!



Exclusion argument

Assume T is a meaningful concept - complex issue, to be discussed later

$T_{ch} < T_c$: hadrochemical equilibrium

Exclude T_{ch} much smaller than T_c :
say $T_{ch} > 0.95\ T_c$

$$0.95 < T_{ch} / T_c < 1$$

Has T_c been measured ?

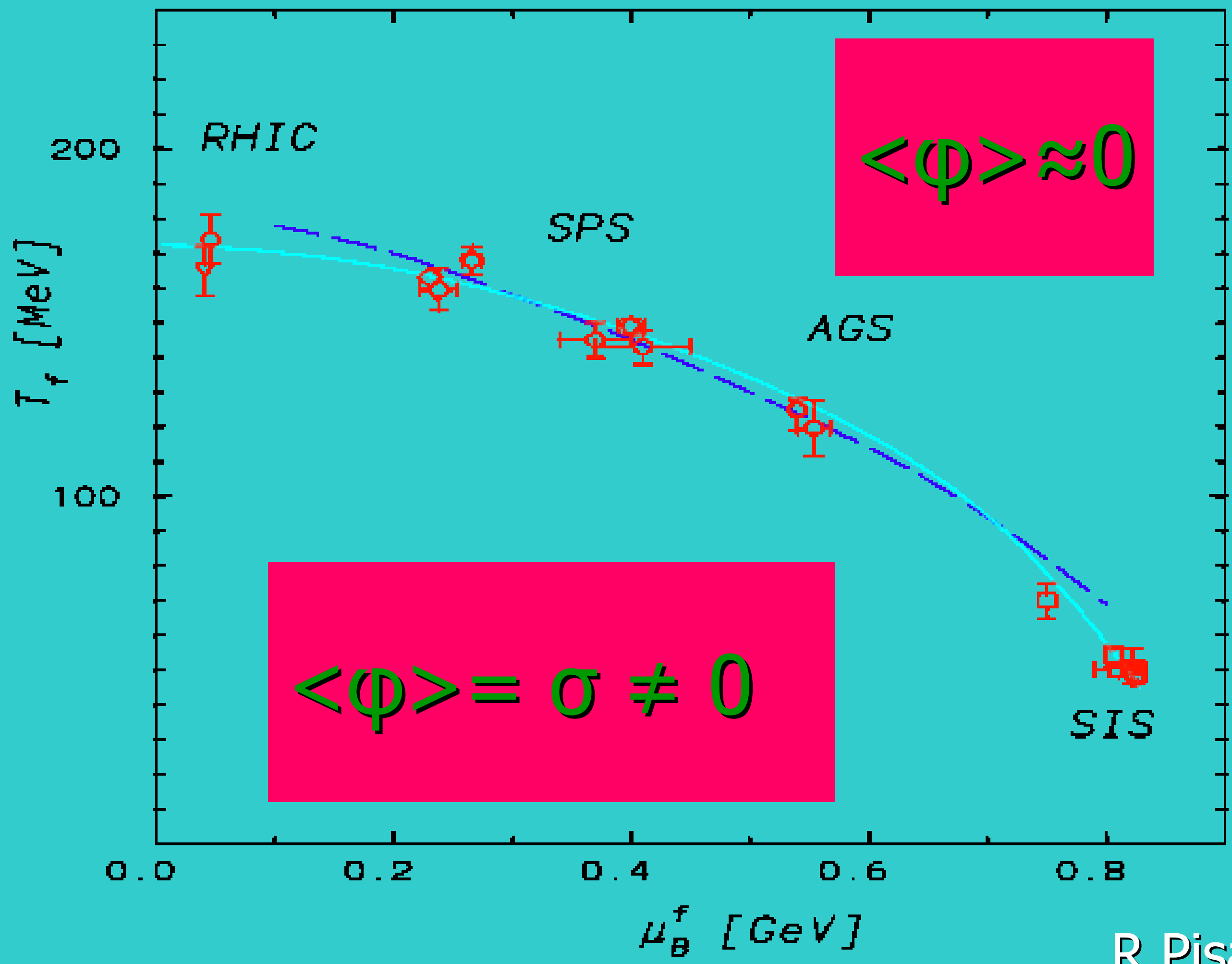
- Observation : statistical distribution of hadron species with "chemical freeze out temperature " T_{ch}=176 MeV
- T_{ch} cannot be much smaller than T_c : hadronic rates for $T< T_c$ are too small to produce multistrange hadrons (Ω,..)
- Only near T_c multiparticle scattering becomes important (collective excitations …) – proportional to high power of density

→ $T_{ch} \approx T_c$

P.Braun-Munzinger,J.Stachel,CW

$$T_{ch} \approx T_c$$

Phase diagram

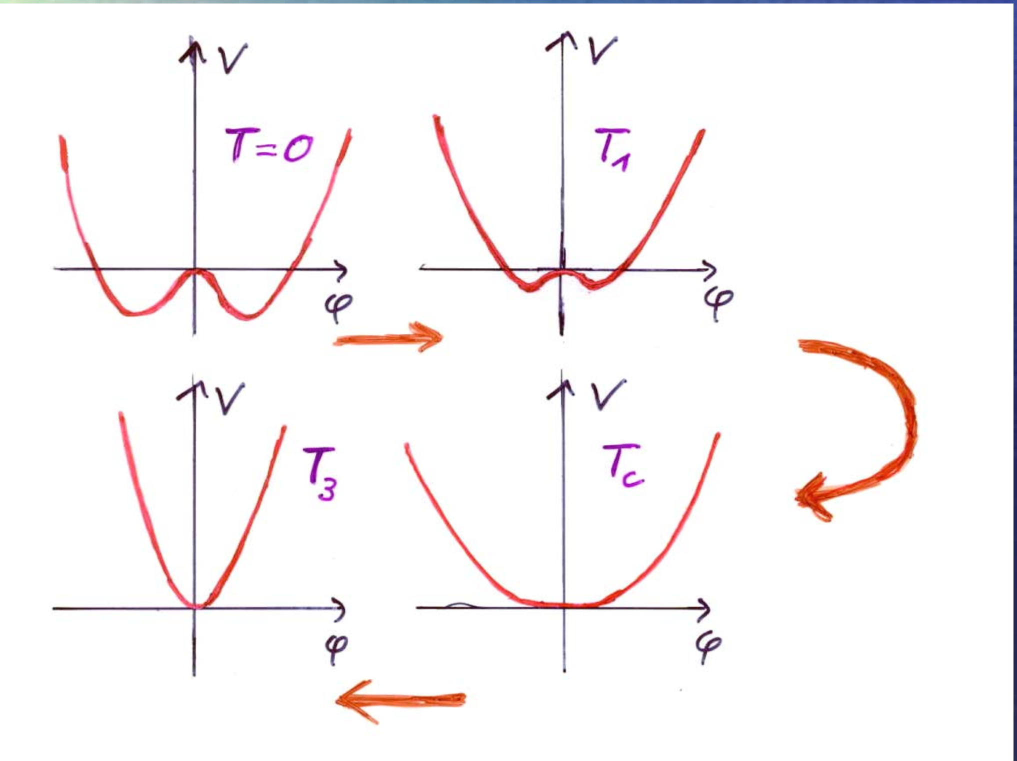


R.Pisarski

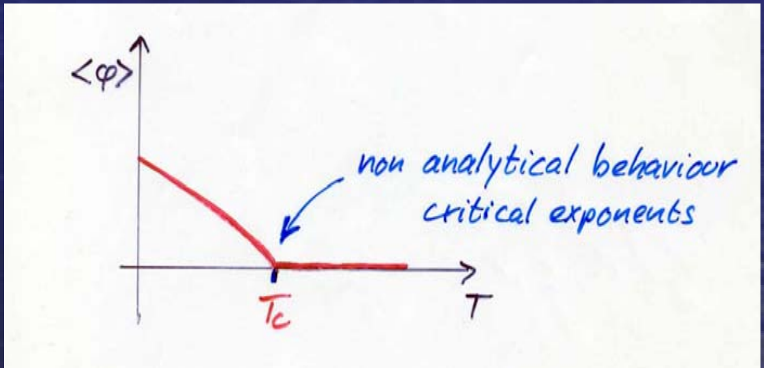
Temperature dependence of chiral order parameter

Does experiment indicate a first order phase transition for $\mu = 0$?

Second order phase transition



$$V = -\mu_0^2\varphi^\dagger\varphi + cT^2\varphi^\dagger\varphi + \frac{\lambda}{2}(\varphi^\dagger\varphi)$$



Second order phase transition

for T only somewhat below T_c :

the order parameter σ is expected to

be close to zero and

deviate substantially from its vacuum value

This seems to be disfavored by observation of chemical freeze out !

Temperature dependent masses

- Chiral order parameter σ depends on T
- Particle masses depend on σ
- Chemical freeze out measures m/T for many species
- Mass ratios at T just below T_c are close to vacuum ratios

Ratios of particle masses and chemical freeze out

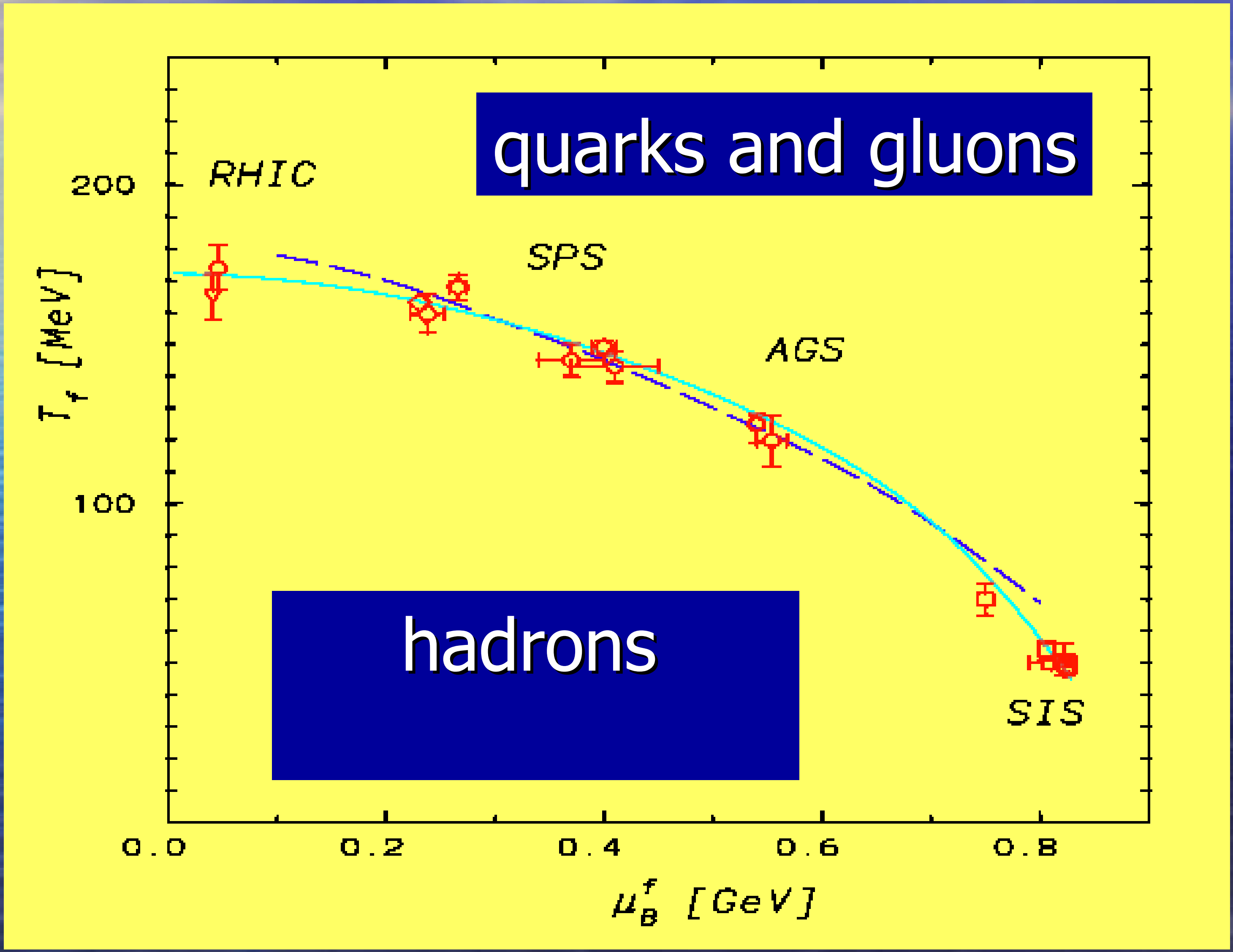
at chemical freeze out :

- ratios of hadron masses seem to be close to vacuum values
- nucleon and meson masses have different characteristic dependence on σ
- $m_{nucleon} \sim \sigma$, $m_{\pi} \sim \sigma^{-1/2}$
- $\Delta\sigma/\sigma < 0.1$ (conservative)

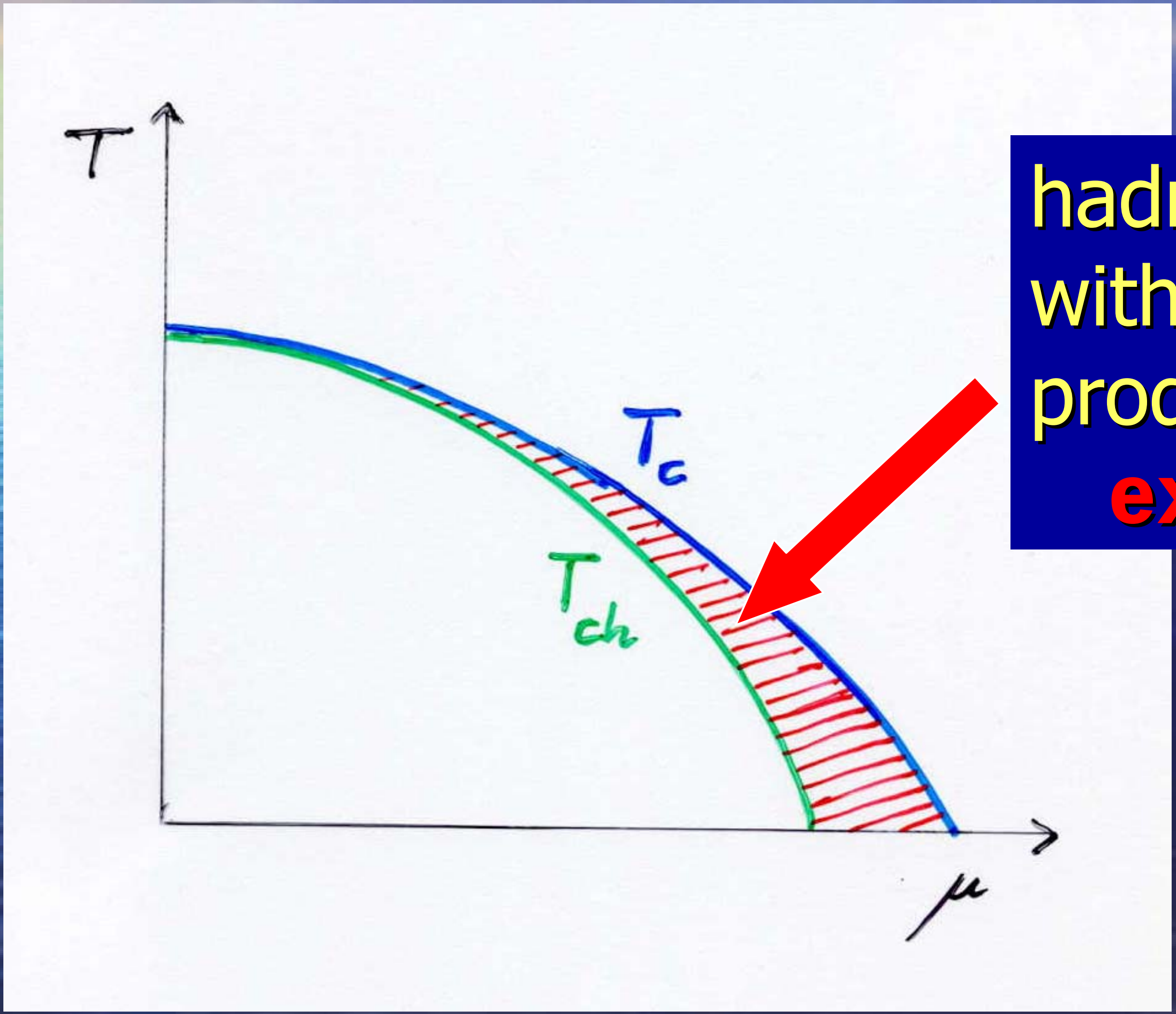
first order phase transition seems to be favored by chemical freeze out

...or extremely rapid crossover

How far has first order line been measured?



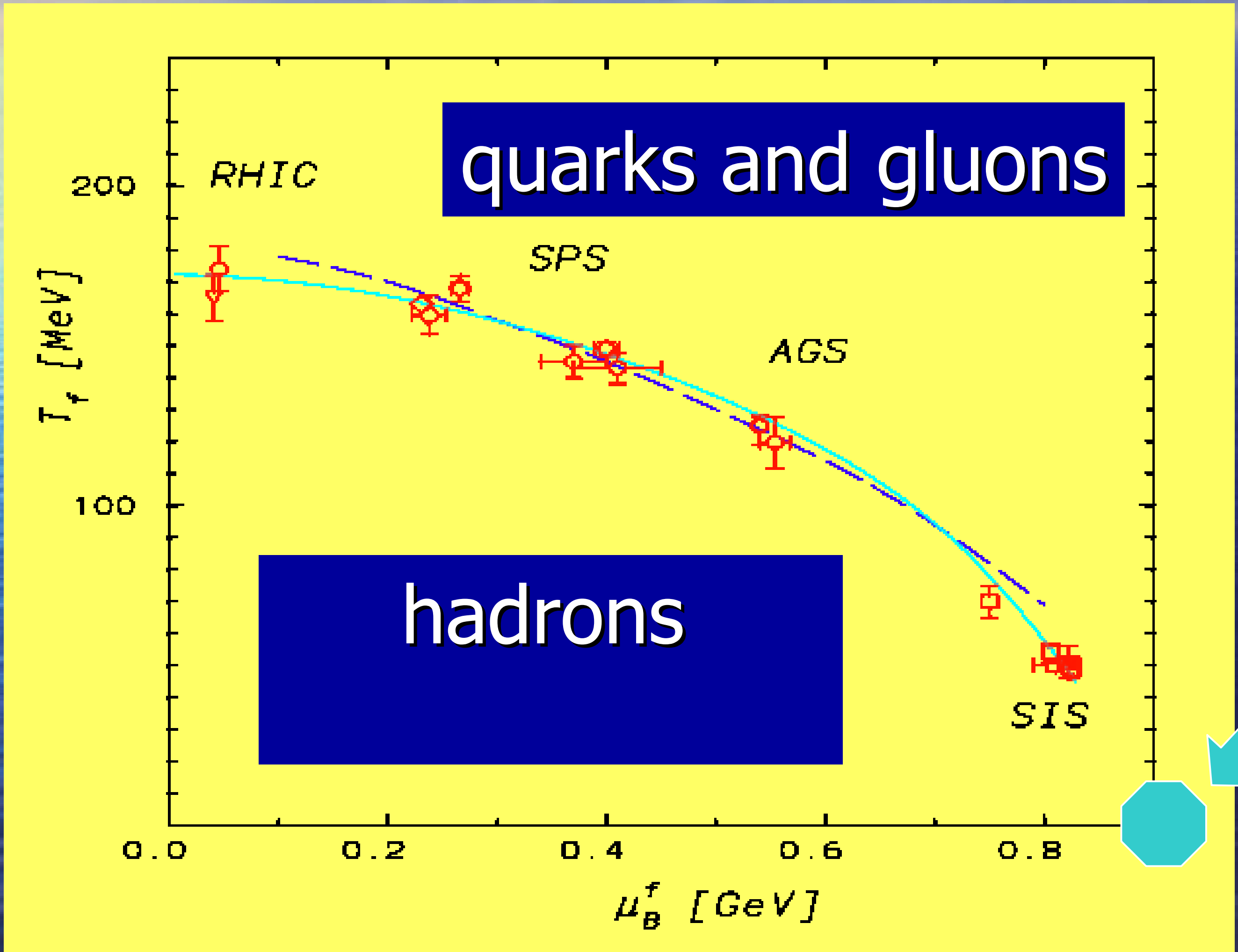
Exclusion argument for large density



hadronic phase with sufficient production of Ω :

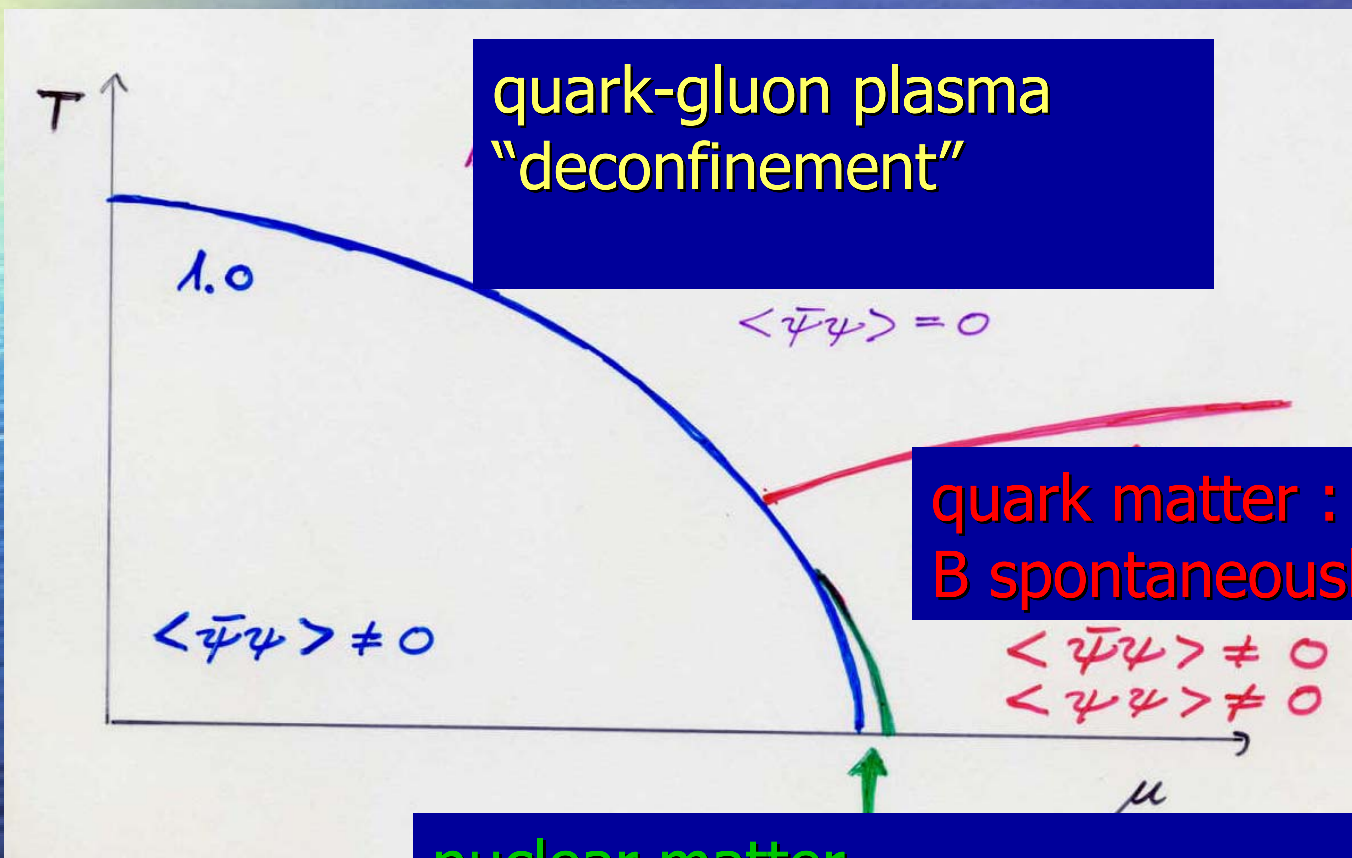
excluded !!

First order phase transition line



μ=923MeV transition to nuclear matter

Phase diagram for $m_s > m_{u,d}$



quark-gluon plasma
“deconfinement”

quark matter : superfluid
B spontaneously broken

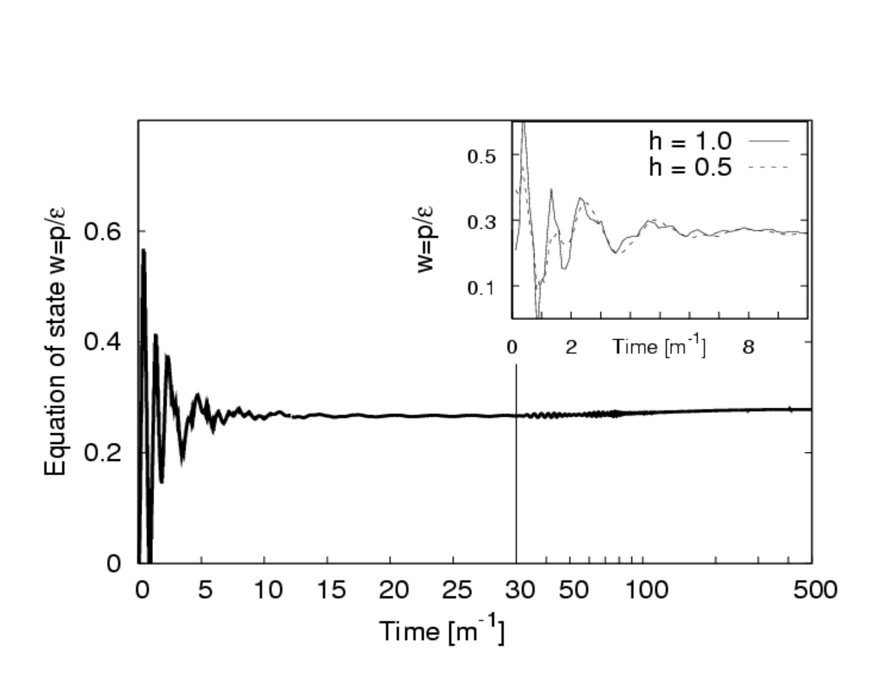
nuclear matter
B,isospin (I_3) spontaneously broken, S conserved

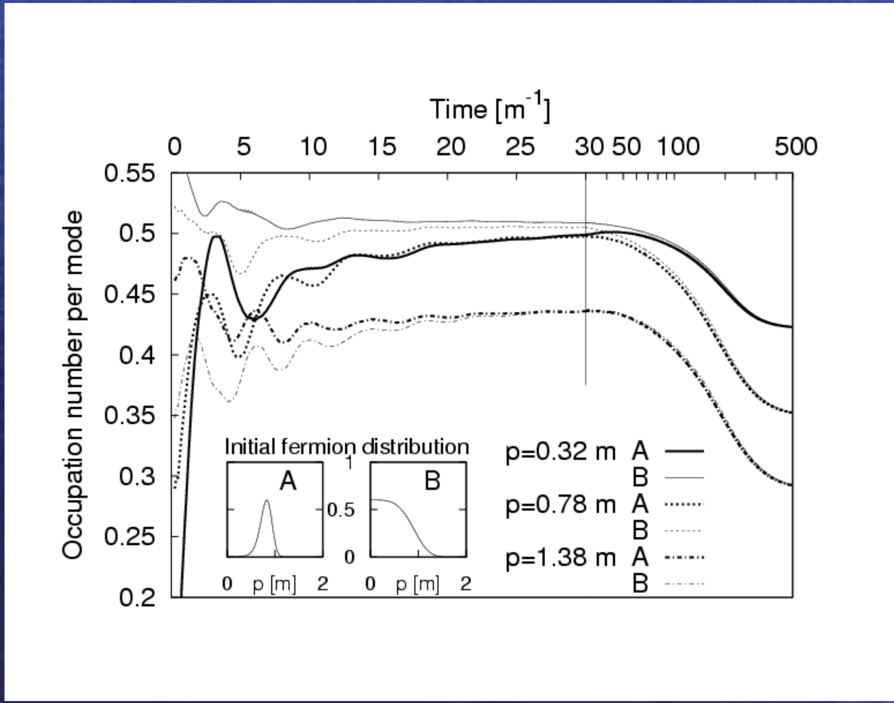
Is temperature defined ?

Does comparison with equilibrium critical temperature make sense ?

Prethermalization

J.Berges,Sz.Borsanyi,CW





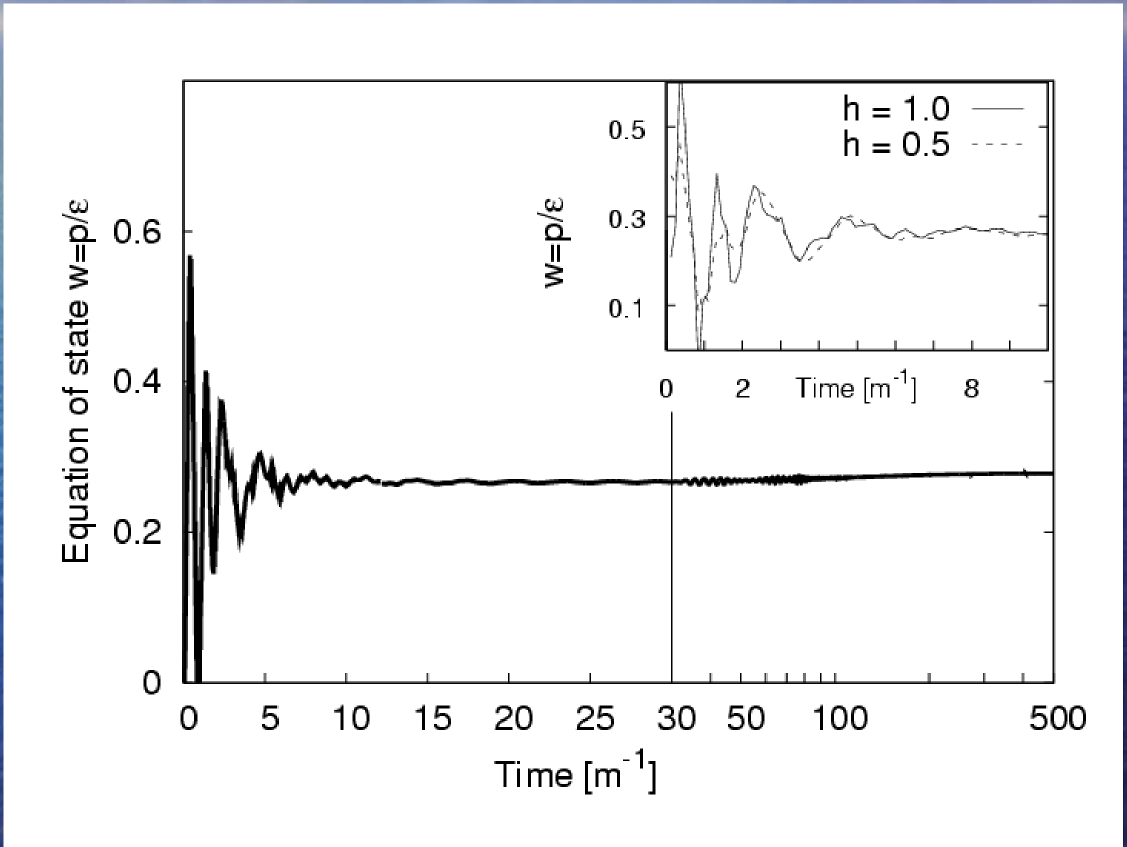
Vastly different time scales

for “thermalization” of different quantities

here : scalar with mass m coupled to fermions (linear quark-meson-model)

method : two particle irreducible non-equilibrium effective action (J.Berges et al)

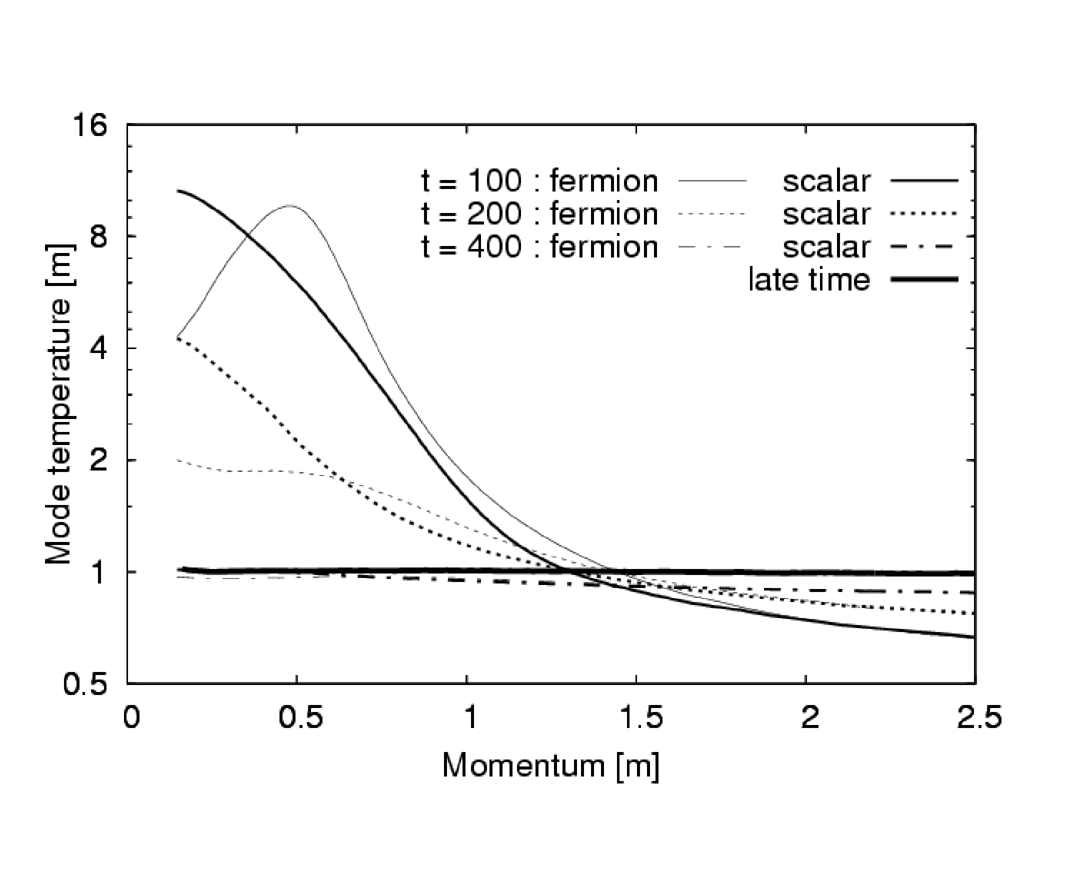
Prethermalization equation of state p/ε



similar for kinetic temperature

different “temperatures”

Mode temperature



$$n_p(t) \stackrel{!}{=} \frac{1}{\exp\left[\omega_p(t)/T_p(t)\right] \pm 1}$$

$\omega_p^{(f,s)}(t)$ determined by peak of spectral function

n_p :occupation number for momentum p

late time:

Bose-Einstein or Fermi-Dirac distribution

Global kinetic temperature $T_{\rm kin}$

Practical definition:

- association of temperature with average kinetic energy per d.o.f.

$$T_{\rm kin}(t) = E_{\rm kin}(t)/c_{\rm eq}$$

- $c_{\rm eq} = E_{\rm kin,eq}/T_{\rm eq}$ is given solely in terms of equilibrium quantities
  (E.g. relativistic plasma: $E_{\rm kin}/N = \epsilon/n = \alpha T$)

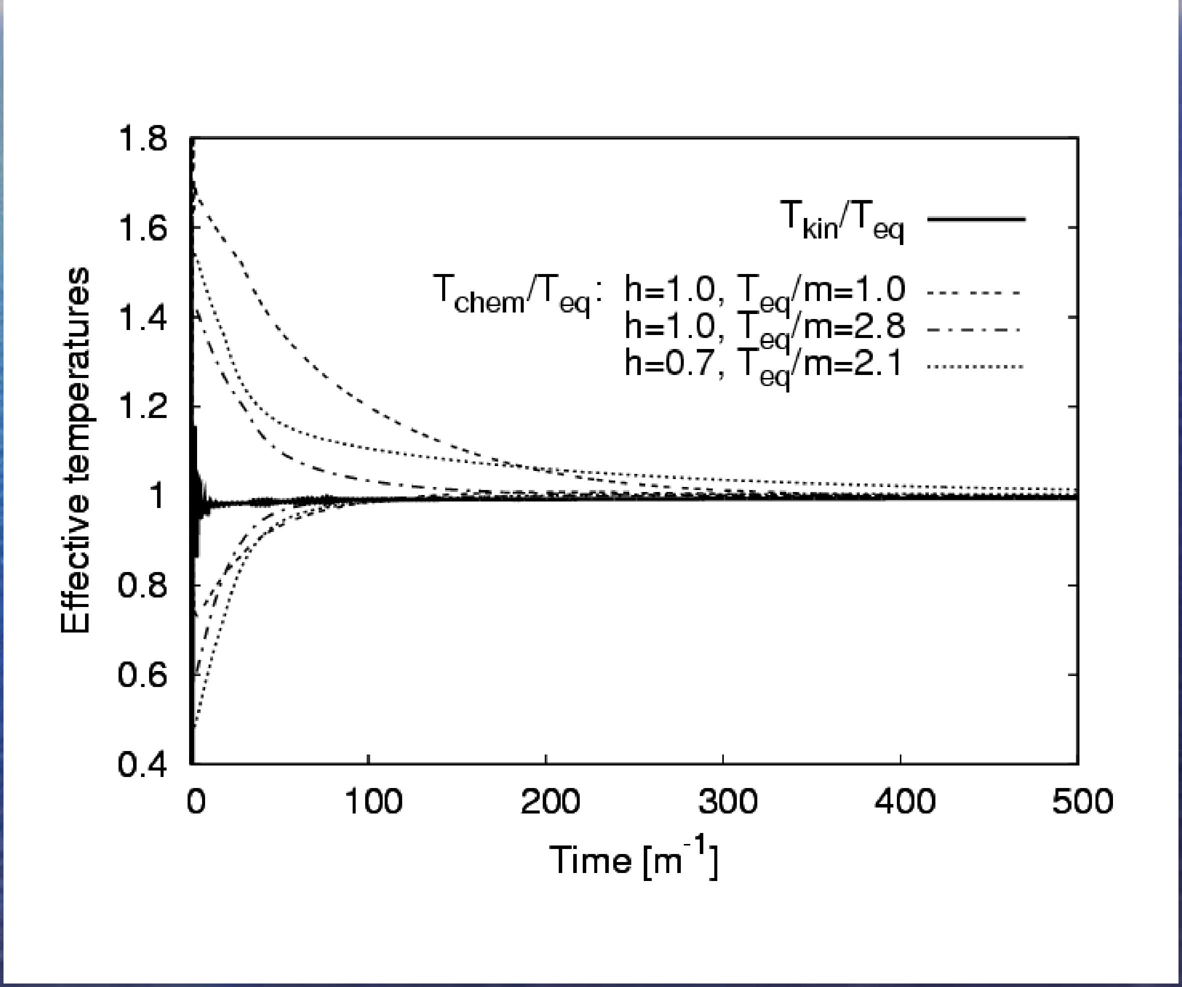
Kinetic equilibration: $T_{\rm kin}(t) = T_{\rm eq}$

Consider also *chemical temperatures* $T_{\rm ch}^{(f,s)}$ from integrated number density of each species, $n^{(f,s)}(t) = g^{(f,s)} \int {\rm d}^3p/(2\pi)^3\, n_p^{(f,s)}(t)$:

$$n(t) \stackrel{!}{=} \frac{g}{2\pi^2} \int_0^\infty {\rm d}p p^2 \left[\exp\left(\omega_p(t)/T_{\rm ch}(t)\right) \pm 1\right]^{-1}$$

Chemical equilibration: $T_{\rm ch}^{(f)}(t) = T_{\rm ch}^{(s)}(t)$

Kinetic equilibration before chemical equilibration



Once a temperature becomes stationary it takes the value of the equilibrium temperature.

Once chemical equilibration has been reached the chemical temperature equals the kinetic temperature and can be associated with the overall equilibrium temperature.

Comparison of chemical freeze out temperature with critical temperature of phase transition makes sense

Short and long distance degrees of freedom are different !

Short distances : quarks and gluons
Long distances : baryons and mesons

How to make the transition?

How to come from quarks and gluons to baryons and mesons ?

Find effective description where relevant degrees of freedom depend on momentum scale or resolution in space.

Microscope with variable resolution:

- High resolution , small piece of volume: quarks and gluons
- Low resolution, large volume : hadrons

Functional Renormalization Group

from small to large scales

Exact renormalization group equation

Exact flow equation

for scale dependence of average action

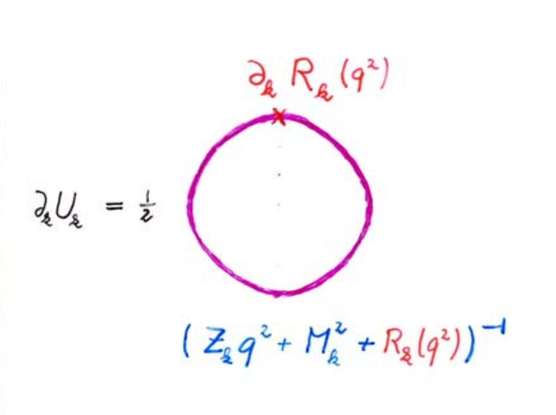
$$\partial_k \Gamma_k[\varphi] = \frac{1}{2} \mathrm{Tr} \left\{ \left(\Gamma_k^{(2)}[\varphi] + R_k \right)^{-1} \partial_k R_k \right\}$$

'92

$$\left(\Gamma_k^{(2)} \right)_{ab} (q, q') = \frac{\delta^2 \Gamma_k}{\delta \varphi_a(-q) \delta \varphi_b(q')}$$

$\mathrm{Tr} : \sum_a \int \frac{d^d q}{(2\pi)^d}$

(fermions : STr)



Infrared cutoff

$$R_k \quad : \quad \text{IR-cutoff}$$

$$\text{e.g} \qquad R_k = \frac{Z_k q^2}{e^{q^2/k^2} - 1}$$

$$\text{or} \qquad R_k = Z_k(k^2 - q^2)\Theta(k^2 - q^2) \qquad \text{(Litim)}$$

$$\lim_{k\to 0} R_k = 0$$

$$\lim_{k\to\infty} R_k \to \infty$$

Nambu Jona-Lasinio model

$$S = \int d^4x \left\{ i \bar{\psi}^i_a \gamma^\mu \partial_\mu \psi^i_a + 2\lambda_\sigma (\bar{\psi}^i_{Lb} \psi^i_{Ra})(\bar{\psi}^j_{Ra} \psi^j_{Lb}) \right\}$$

$$\psi_{L,R} = \frac{1 \pm \gamma^5}{2} \psi$$

$i, j = 1 \ldots N_c$ color ($N_c = 3$)

$a, b = 1 \ldots N_F$ flavor ($N_F = 3, 2$)

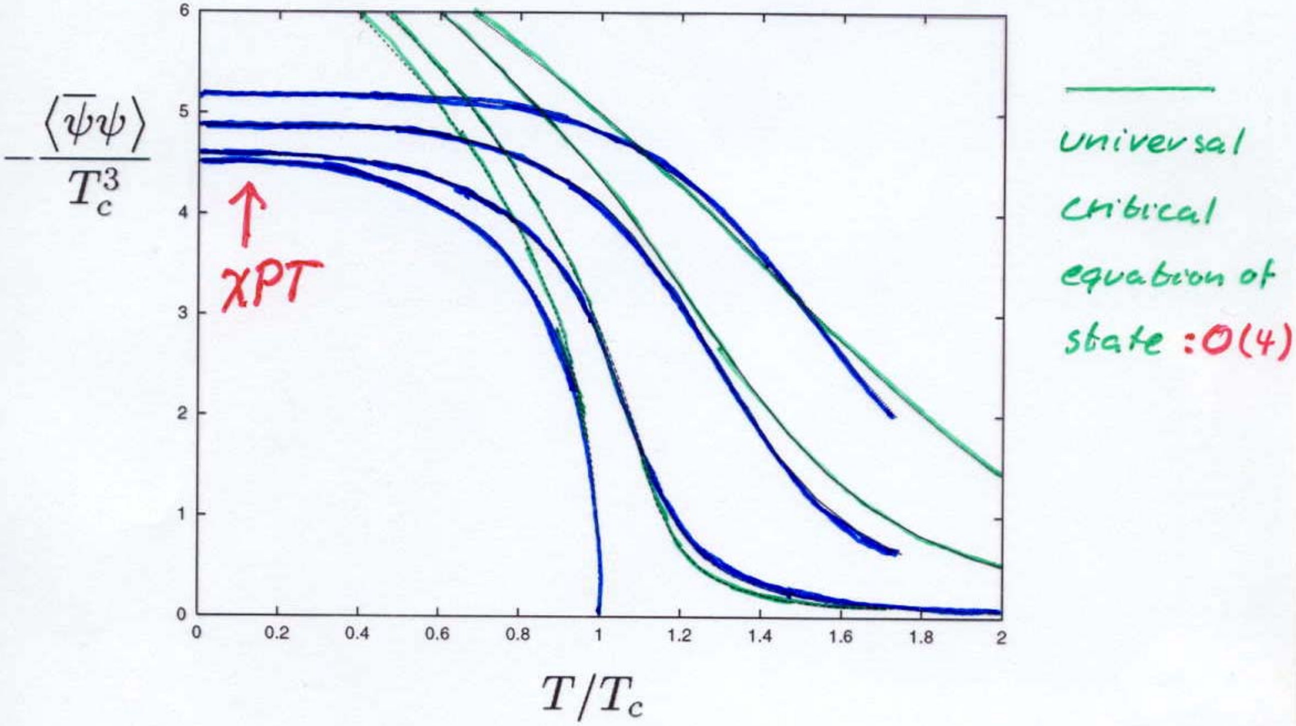
chiral flavor symmetry :

$$SU_L(N_F) \times SU_R(N_F)$$

…and more general quark meson models

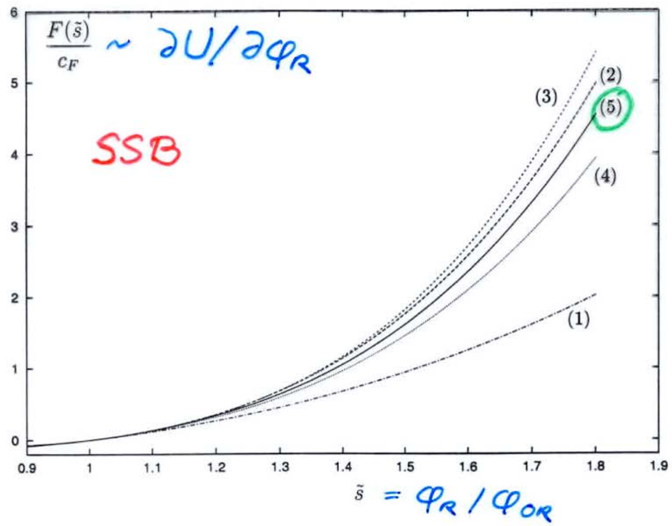
Chiral condensate

2nd order PT (expected for $O(4)$ Heisenberg model)



$\Longrightarrow$ Explicit link between χPT domain of validity (4d) and critical (universal) domain near T_c (3d)

Scaling form of equation of state



critical equation of state

(2) ERGE, lowest order derivative exp.; Berges, Tetradis, ...

(5) ERGE, first order derivative exp.; Seide, ...

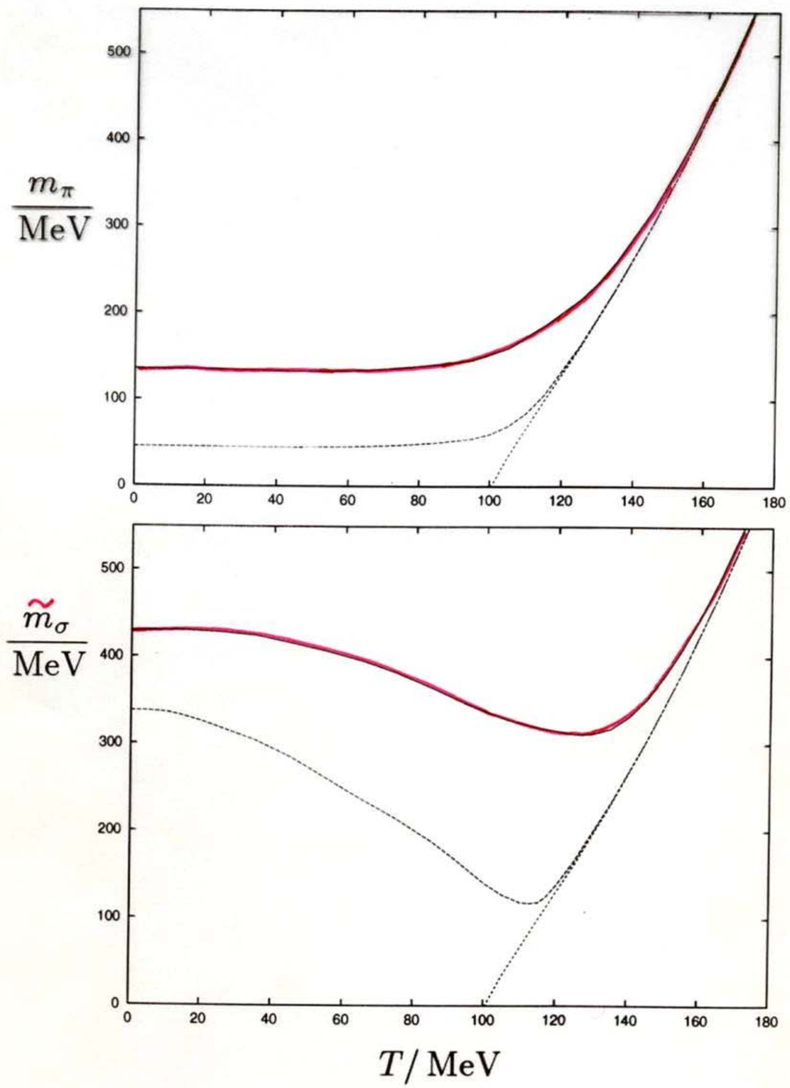
(1) mean field

(4) high-T-series, loop expansion, ε-expansion

(3) Monte Carlo

Berges, Tetradis,…

temperature dependent masses



pion mass

sigma mass

? $m_\sigma < 2m_\pi$ for $T \gtrsim 100\,\mathrm{MeV}$?

No long pion correlation length in thermal equilibrium!

conclusion

- Experimental determination of critical temperature may be more precise than lattice results
- Rather simple phase structure is suggested
- Analytical understanding is only at beginning

end

Cosmological phase transition…

…when the universe cools below 175 MeV

10^{-5} seconds after the big bang

QCD at high density

Nuclear matter

Heavy nuclei

Neutron stars

Quark stars …

QCD at high temperature

- Quark – gluon plasma
- Chiral symmetry restored
- Deconfinement (no linear heavy quark potential at large distances)
- Lattice simulations : both effects happen at the same temperature

“Solution” of QCD

Effective action (for suitable fields) contains all the relevant information of the solution of QCD

Gauge singlet fields, low momenta:

Order parameters, meson-(baryon-) propagators

Gluon and quark fields, high momenta:

Perturbative QCD

Aim: Computation of effective action

QCD – phase transition

Quark –gluon plasma	Hadron gas
• Gluons : 8 x 2 = 16	• Light mesons : 8
• Quarks : 9 x 7/2 =12.5	• (pions : 3)
• Dof : 28.5	• Dof : 8
Chiral symmetry	Chiral sym. broken

Large difference in number of degrees of freedom !

Strong increase of density and energy density at T_c !

Spontaneous breaking of color

- Condensate of colored scalar field
- Equivalence of Higgs and confinement description in real (N_f=3) QCD vacuum
- Gauge symmetries not spontaneously broken in formal sense (only for fixed gauge)

  Similar situation as in electroweak theory
- No "fundamental" scalars
- Symmetry breaking by quark-antiquark-condensate

A simple mean field calculation

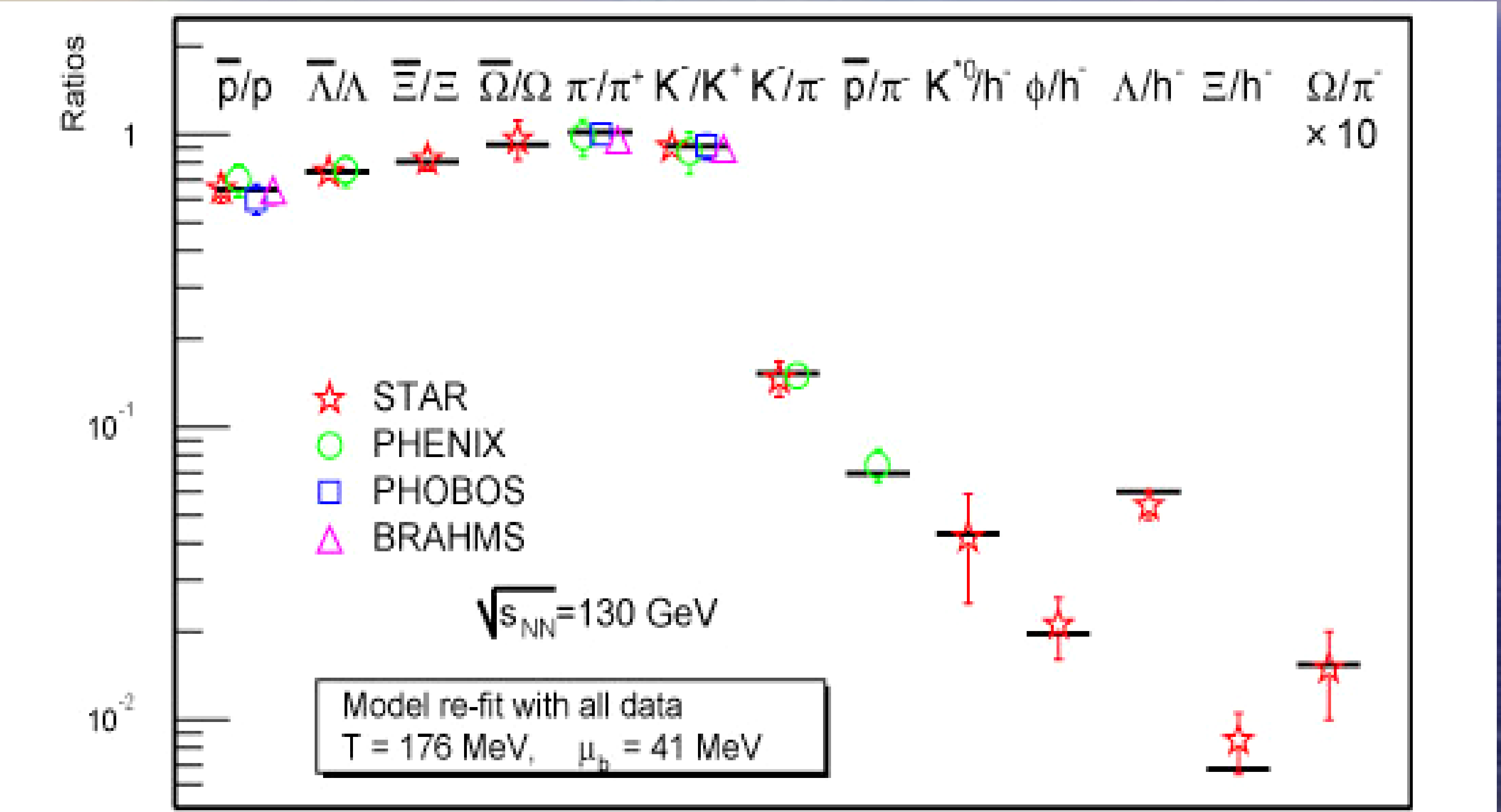
	vanishing quark masses	equal $m_u = m_d = m_s \neq 0$, $2M_K^2 + M_\pi^2 = (390\,\text{MeV})^2$	
$\bar{M}_\rho$	700 MeV	770 MeV	
f	68 MeV	116 MeV	
T_c	154 MeV	170 MeV	
$\bar{M}_\rho(T_c)$	290 MeV	290 MeV	screening masses
$\bar{M}_g(T_c)$	580 MeV	600 MeV	screening masses

equation of state : pion gas → QGP

$$\frac{\varepsilon - 3p}{\varepsilon + p} \approx \tau(T_c)\,\frac{T_c^4}{T^4} \qquad (T \gg T_c)$$

$\tau(T_c)$	0.37	0.53

Hadron abundancies



Braun-Munzinger et al., PLB 518 (2001) 41 D. Magestro (updated July 22, 2002)

Bound for critical temperature

$$0.95\, T_c < T_{ch} < T_c$$

- not : " I have a model where $T_c \approx T_{ch}$ "
- not : " I use T_c as a free parameter and find that in a model simulation it is close to the lattice value (or T_{ch}) "

$T_{ch} \approx 176$ MeV (?)

Estimate of critical temperature

For $T_{ch} \approx 176$ MeV :

$0.95 < T_{ch} / T_c$

- 176 MeV $< T_c <$ 185 MeV

$0.75 < T_{ch} / T_c$

- 176 MeV $< T_c <$ 235 MeV

Quantitative issue matters!

Key argument

- Two particle scattering rates not sufficient to produce Ω
- “multiparticle scattering for Ω-production “ : dominant only in immediate vicinity of T_c

needed :

lower bound on

T_{ch} / T_c

Exclude the hypothesis of a hadronic phase where multistrange particles are produced at T substantially smaller than T_c

Mechanisms for production of multistrange hadrons

Many proposals

- Hadronization
- Quark-hadron equilibrium
- Decay of collective excitation (σ – field)
- Multi-hadron-scattering

Different pictures !

Hadronic picture of Ω - production

Should exist, at least semi-quantitatively, if $T_{ch} < T_c$

(for $T_{ch} = T_c$: $T_{ch} > 0.95\, T_c$ is fulfilled anyhow)

e.g. collective excitations ≈ multi-hadron-scattering

(not necessarily the best and simplest picture)

multihadron -> Ω + X should have sufficient rate

Check of consistency for many models

Necessary if $T_{ch} \neq T_c$ and temperature is defined

Way to give quantitative bound on T_{ch} / T_c

Rates for multiparticle scattering

2 pions + 3 kaons -> Ω + antiproton

$$r(n_{in}, n_{out}) = \bar{n}(\mathrm{T})^{n_{in}} |\mathcal{M}|^2 \phi$$

$$\phi = \prod_{k=1}^{n_{out}} \left(\int \frac{d^3 p_k}{(2\pi)^3 (2E_k)} \right) (2\pi)^4 \delta^4 \left(\sum_k p_k^\mu \right)$$

$$r_\Omega = n_\pi^5 (n_K / n_\pi)^3 |\mathcal{M}|^2 \phi.$$

Very rapid density increase

…in vicinity of critical temperature

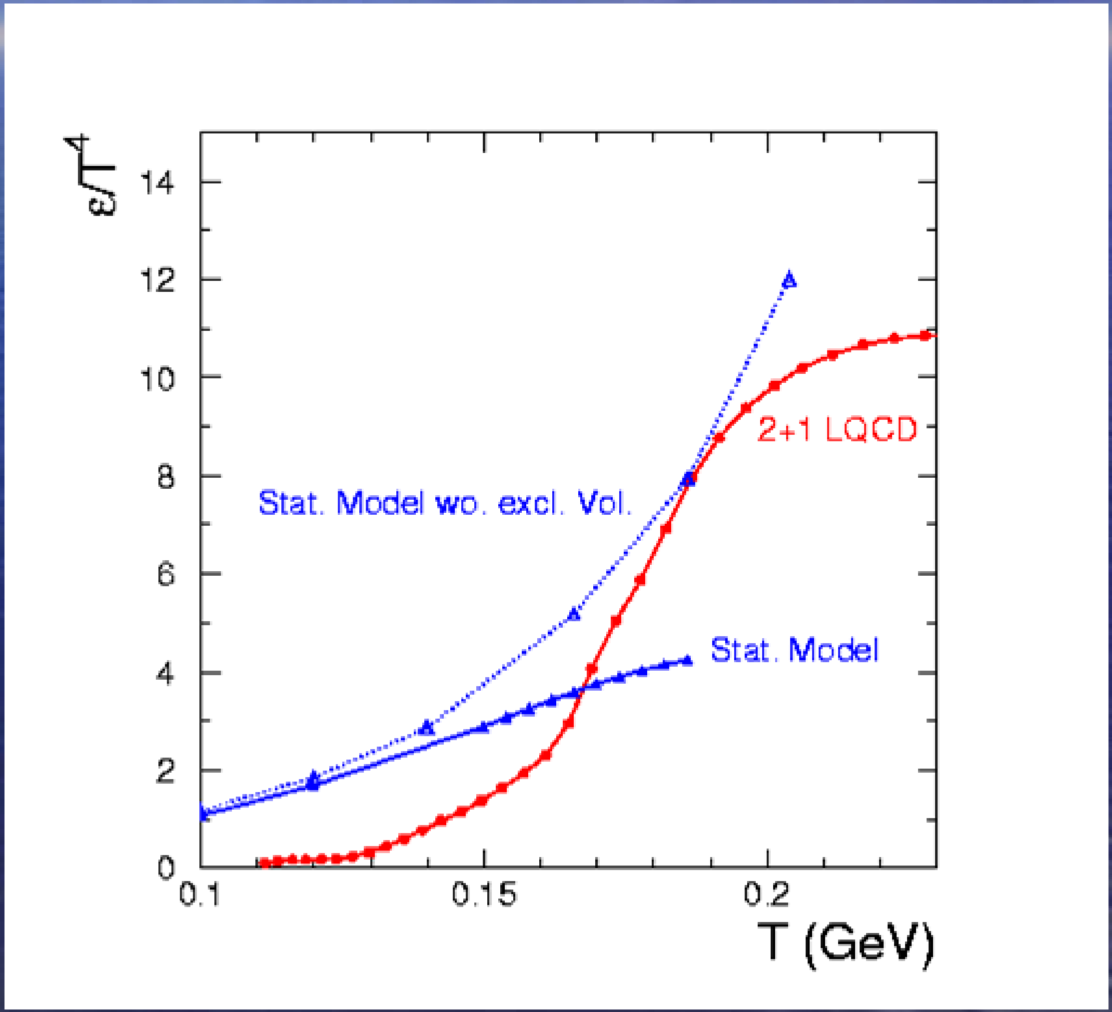
Extremely rapid increase of rate of multiparticle scattering processes

(proportional to very high power of density)

Energy density

Lattice simulations
Karsch et al

even more dramatic
for first order
transition



Phase space

- increases very rapidly with energy and therefore with temperature
- effective dependence of time needed to produce Ω

$$\tau_{\Omega} \sim T^{-60} \;!$$

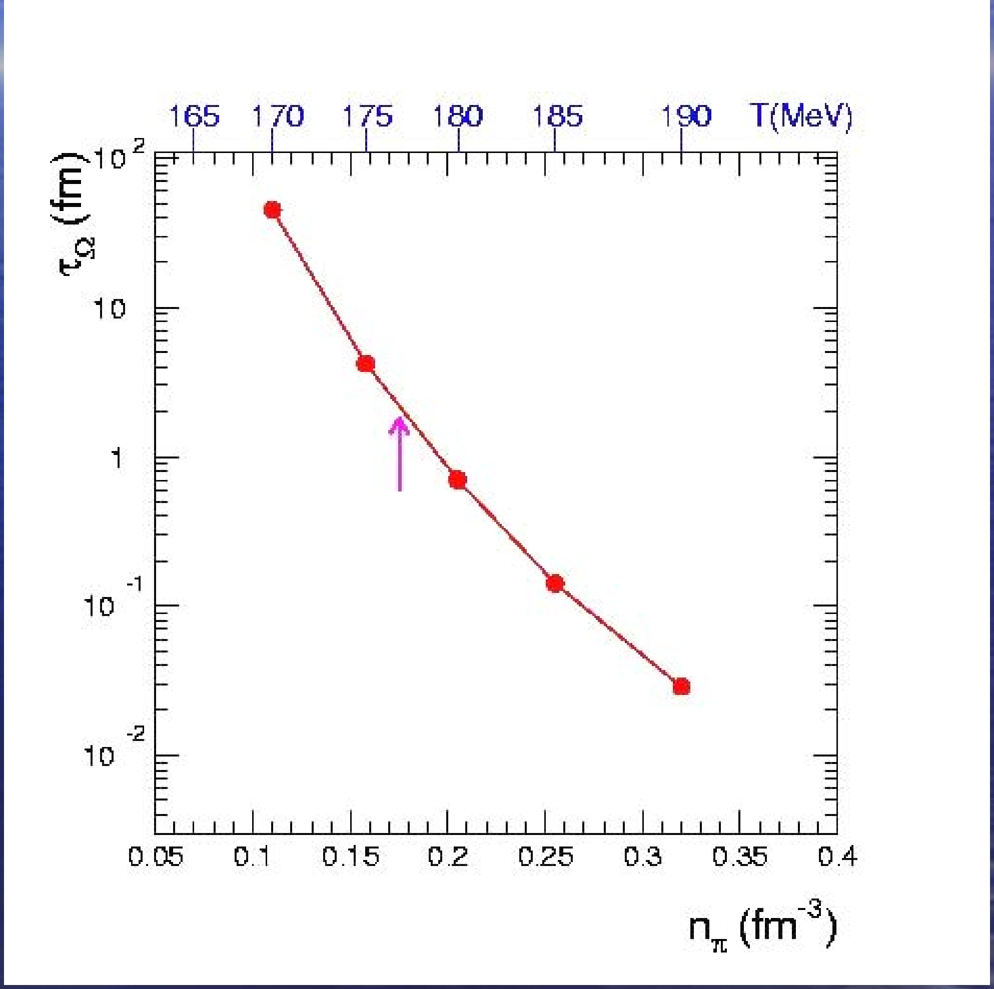
This will even be more dramatic if transition is closer to first order phase transition

Production time for Ω

multi-meson scattering

π+π+π+K+K -> Ω+p

strong dependence on "pion" density



P.Braun-Munzinger,J.Stachel,CW

extremely rapid change

lowering T by 5 MeV below critical temperature :

rate of Ω – production decreases by factor 10

This restricts chemical freeze out to close vicinity of critical temperature

$$0.95 < T_{ch} / T_c < 1$$

enough time for Ω - production

at T=176 MeV :

$\tau_{\Omega} \sim 2.3$ fm

consistency !

Relevant time scale in hadronic phase

rates needed for equilibration of Ω and kaons:

$$\bar{r}_j = \frac{\dot{N}_j}{V} = \dot{n}_j + n_j \dot{V}/V.$$

$$\left|\frac{\bar{r}_\Omega}{n_\Omega} - \frac{\bar{r}_K}{n_K}\right| = \frac{\ln F_{\Omega K}}{\tau_T}\frac{\mathrm{T_{ch}}}{\Delta \mathrm{T}} = (1.10\text{–}0.55)/\mathrm{fm}$$

ΔT = 5 MeV,
$F_{\Omega K}$ = 1.13 ,
τ_T =8 fm

two –particle – scattering :

$$\left|\frac{\bar{r}_\Omega}{n_\Omega} - \frac{\bar{r}_K}{n_K}\right| = (0.02\text{-}0.2)/\mathrm{fm}$$

A possible source of error : temperature-dependent particle masses

Chiral order parameter σ depends on T

chemical freeze out measures T/m !

$$M_j(\mathrm{T}) = h_j(\mathrm{T},\mu)\sigma(\mathrm{T},\mu)$$

$$\frac{\sigma(\mathrm{T}_{\mathrm{ch}},\mu)}{\mathrm{T}_{\mathrm{ch}}} = \frac{\sigma(0,0)}{\mathrm{T}_{\mathrm{obs}}}.$$

$$\mathrm{T}_c = 176^{+5}_{-18}\ \mathrm{MeV}.$$

uncertainty in m(T)

→

uncertainty in critical temperature

systematic uncertainty :

$$\Delta\sigma/\sigma = \Delta T_c/T_c$$

$\Delta\sigma$ is negative

$$M_j(\mathrm{T}) = h_j(\mathrm{T},\mu)\sigma(\mathrm{T},\mu)$$

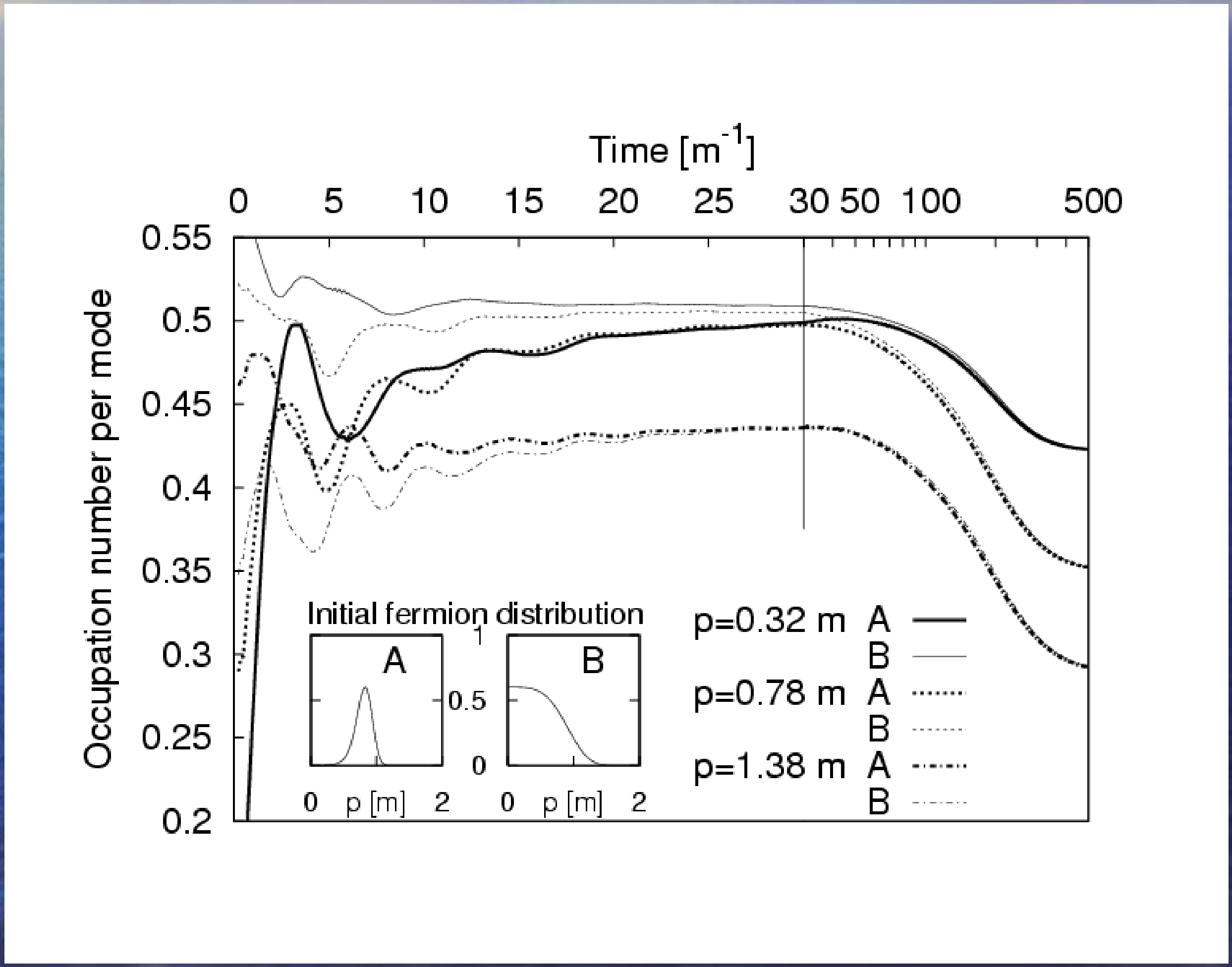
$$\frac{\sigma(\mathrm{T_{ch}},\mu)}{\mathrm{T_{ch}}} = \frac{\sigma(0,0)}{\mathrm{T_{obs}}}.$$

$$\mathrm{T}_c = 176^{+5}_{-18}\ \mathrm{MeV}.$$

conclusion

- experimental determination of critical temperature may be more precise than lattice results
- error estimate becomes crucial

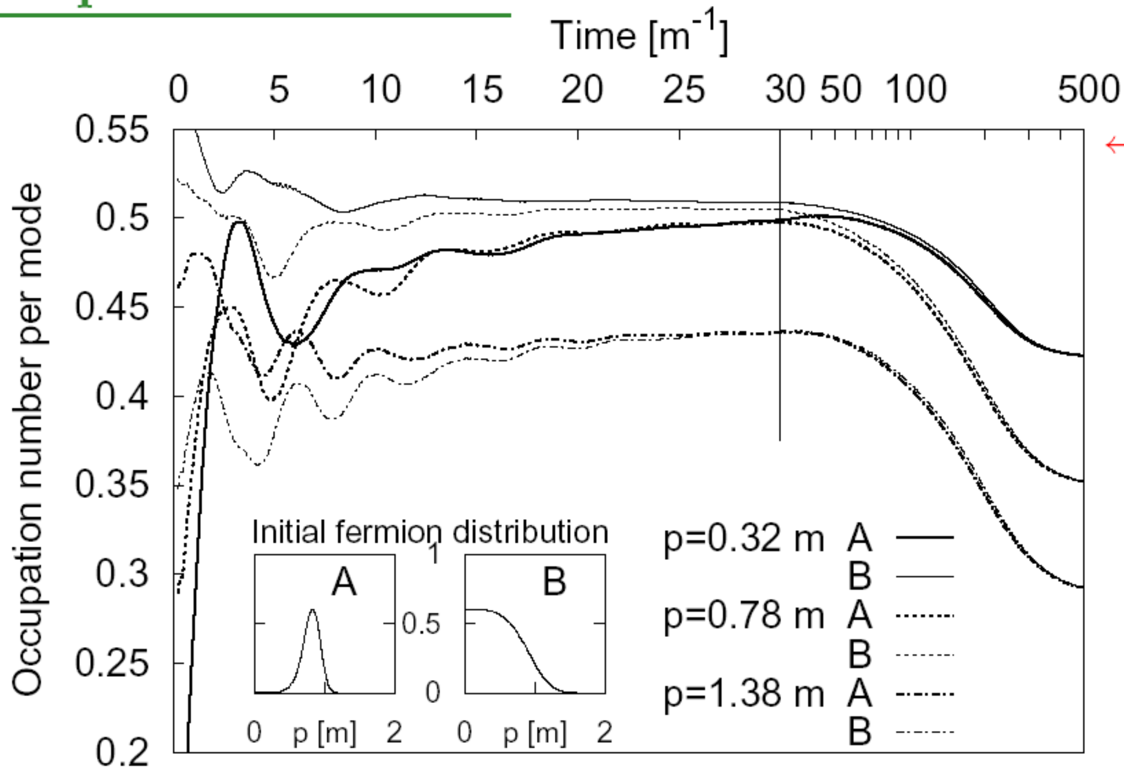
Thermal equilibration : occupation numbers



Effective loss of details of initial conditions (thermalization)

- Two different initial conditions (A), (B) with *same* energy density

Fermion occupation number:



← log scale!

Characteristic damping time: $t_{\mathrm{damp}}(p/m \simeq 1) \simeq 25\, m^{-1}$,

thermalization time: $t_{\mathrm{eq}} \simeq 95\, m^{-1}$

in units of scalar *thermal* mass m $\left(n(p) \sim \mathrm{tr}\, \frac{p^i \gamma^i}{p} \langle [\psi, \bar{\psi}] \rangle_p \; ; \; 0 \leq n(p) \leq 1\right)$

$\rightsquigarrow$ t_{pt} is of the order of the characteristic inverse mass scale m^{-1}

- consequence of rapid loss of phase information ("dephasing")
- unrelated to the scattering-driven process of thermalization

$\rightsquigarrow$ $t_{\mathrm{pt}} \quad \ll \quad t_{\mathrm{damp}} \quad \ll \quad t_{\mathrm{eq}}$

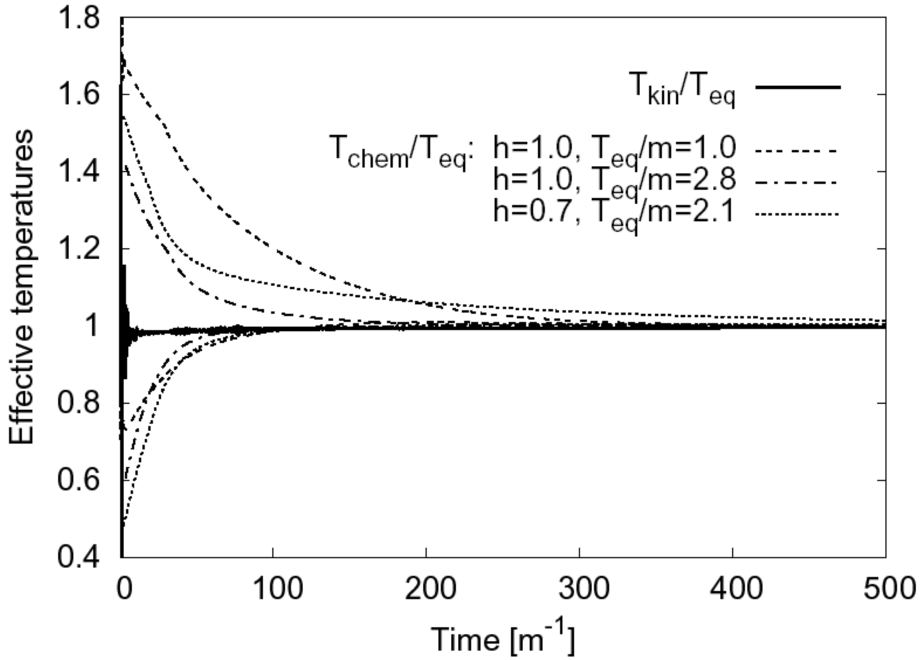
Prethermalization of the equation of state occurs on time scales dramatically shorter than the thermal equilibration time!

Given an EOS, the crucial question arises:

- *Does a suitable global kinetic temperature T_{kin} also exist at t_{pt}?*

$\rightsquigarrow$ "quasi-thermal" description in a far-from-equilibrium situation!

Kinetic vs chemical equilibration:



upper curves: scalars

lower curves: fermions

$\rightsquigarrow$ *$T_{\rm kin}(t)$ prethermalizes on a very short time scale $\sim m^{-1}$ in contrast to chemical equilibration*

$\rightsquigarrow$ late-time chemical equilibration for $t_{\rm ch} \simeq t_{\rm eq}$
($t_{\rm ch}$ depends on details of particle number changing interactions;
deviation from thermal result can become relatively small for $t \ll t_{\rm eq}$)

Prethermalization: *far-from-equilibrium phenomenon* which describes

- very rapid establishment of an approximately constant ratio of pressure over energy density (equation of state)
- as well as a kinetic temperature based on average kinetic energy

$\rightsquigarrow$ *Crucial for the use of efficient hydrodynamic descriptions!*

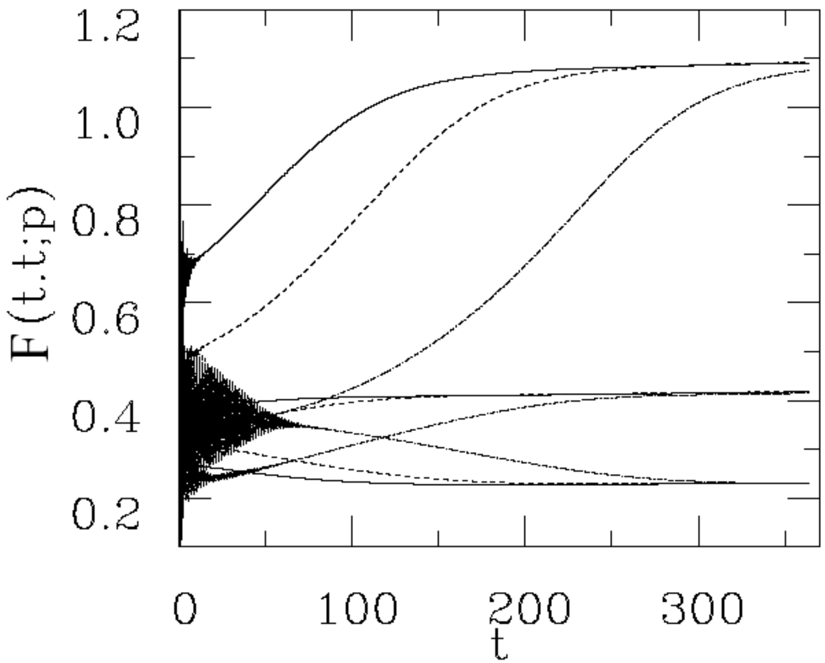
($p = p(e)$ important ingredient to close system of equations $\partial_\mu T^{\mu\nu} = 0$)

More generally:

Extremely different time scales for loss of initial conditions for

- certain "bulk quantities" which average over all momentum modes
- "mode quantities" characterizing the evolution of individual modes

Compare with 2PI:



Propagator for (very) different initial conditions with *same* $\langle E \rangle$

(momenta $p = 0, 3, 5$; all in initial mass units, $\phi = 0$)

here: scalar ϕ^4 to *3-loop*

J.B., Cox, PLB**517** (2001) 369

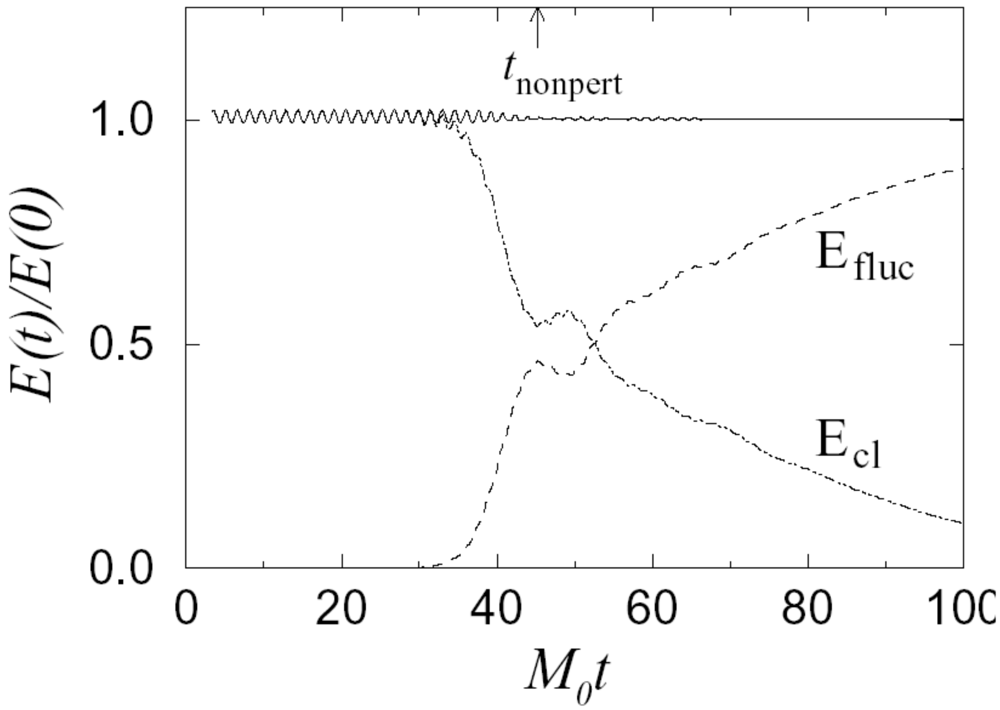
'Damping'	'Drifting'	'Thermalization'
exponential suppression of correlations with initial time	smooth, parametrically slow change of modes	approach to quantum thermal equilibrium
rate: $\gamma^{(\text{damp})}$	weak dependence of $F(t,t')$ on $t+t'$	rate: $\gamma^{(\text{therm})} \neq \gamma^{(\text{damp})}$
$F(t,0) = \langle \phi(t)\, \phi(0) \rangle \longrightarrow 0^+$		$F(t,t') \longrightarrow F^{(\text{eq})}(t-t')$

Initial conditions

Effective loss of initial conditions

early **intermediate** **late** **time**

Coherent field regime $\xrightarrow{\text{`particle production'}}$ fluctuation dominated regime:



$t_{\text{nonpert}}: \; E_{\text{fluc}} \simeq E_{\text{cl}}$

here: $N = 4, \; \lambda = 10^{-6}$

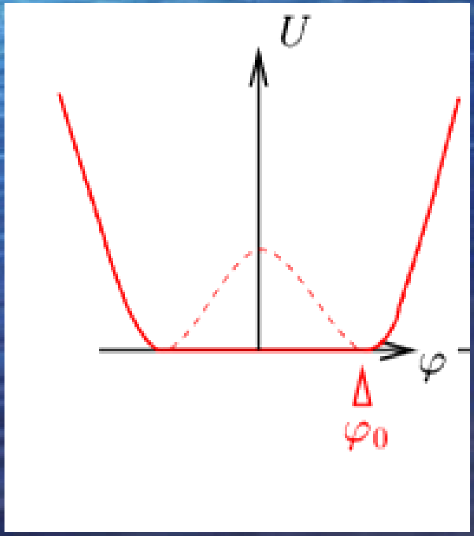
$\rightsquigarrow$ dramatic phenomenon for 'arbitrarily' weakly coupled theory!

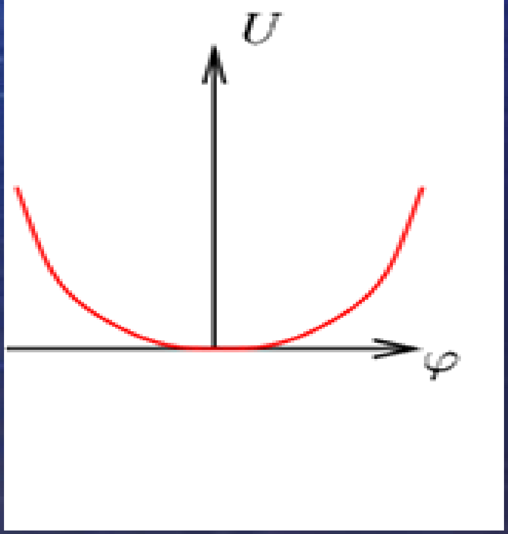
Chiral symmetry restoration at high temperature

Low T
SSB
$<\varphi>=\varphi_0 \neq 0$

High T
SYM
$<\varphi>=0$

at high T :
less order
more symmetry



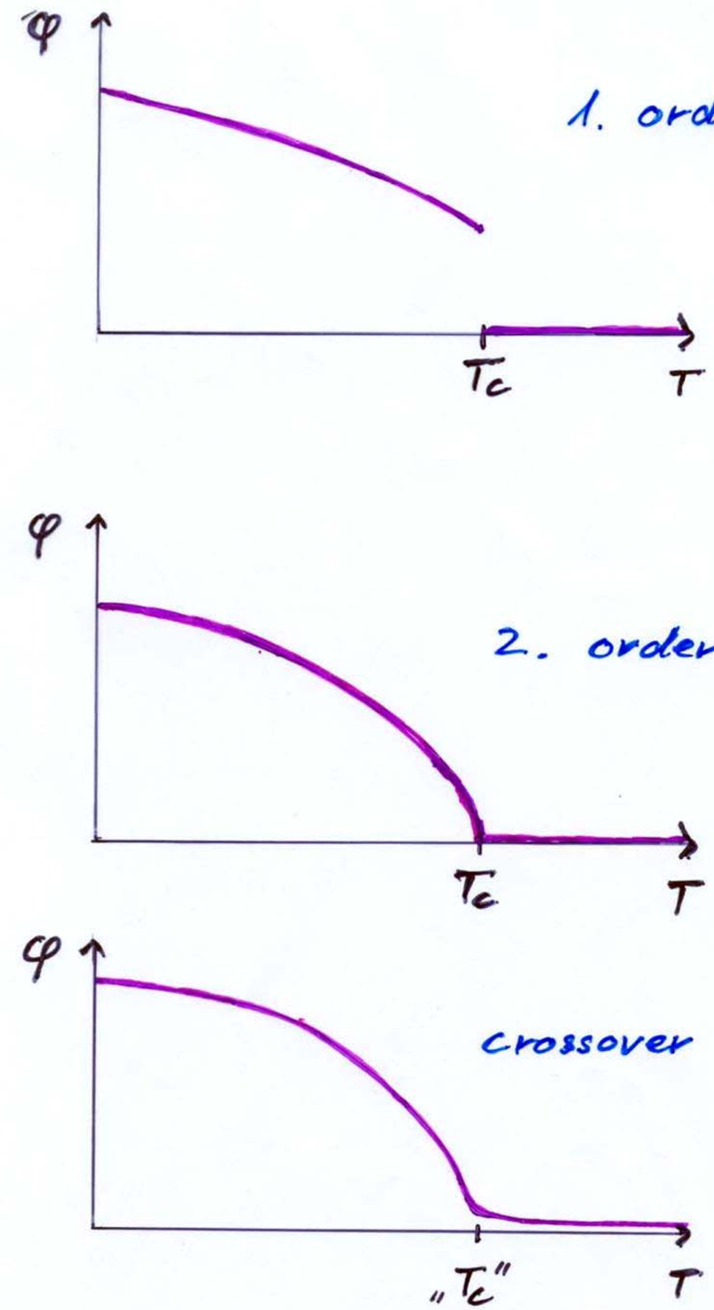


examples:

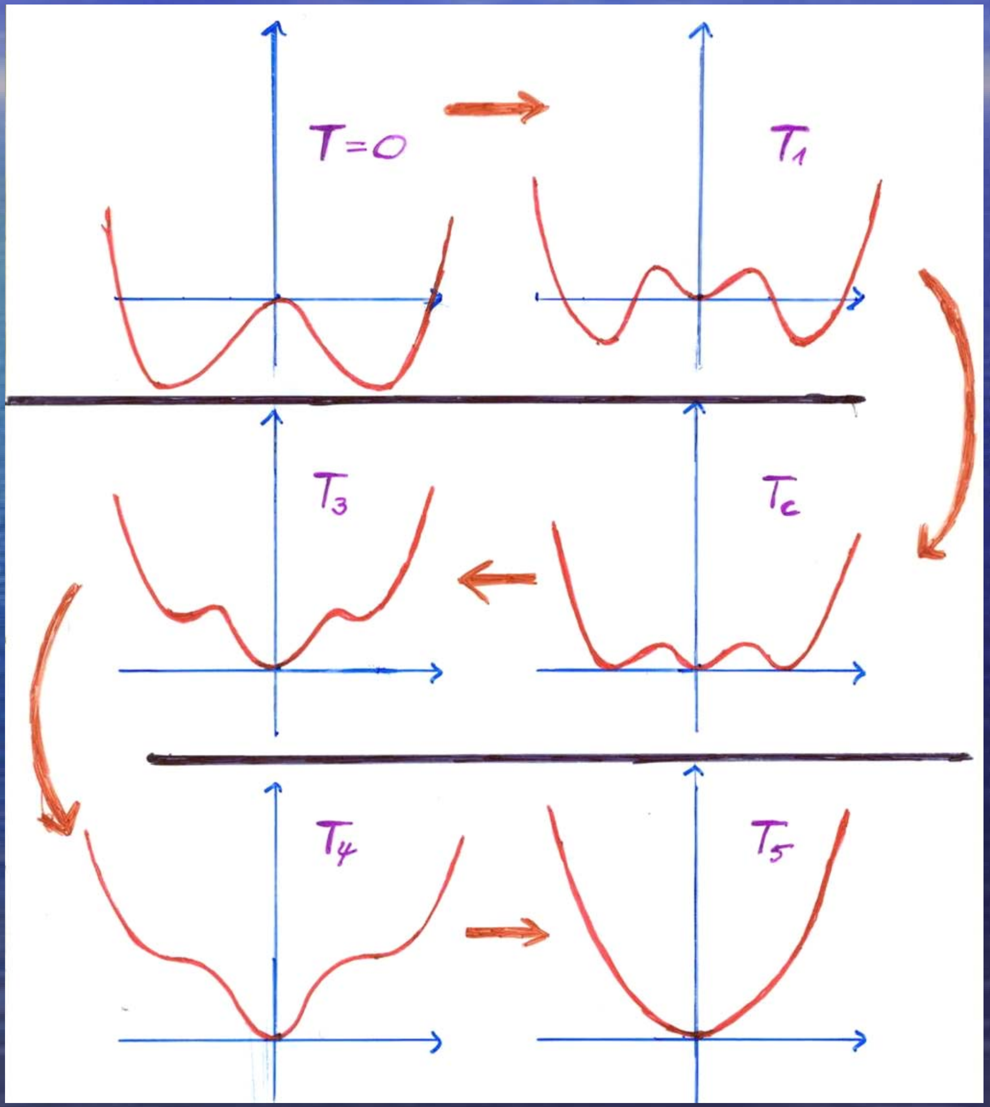
magnets, crystals

Order of the phase transition is crucial ingredient for experiments (heavy ion collisions)
and cosmological phase transition

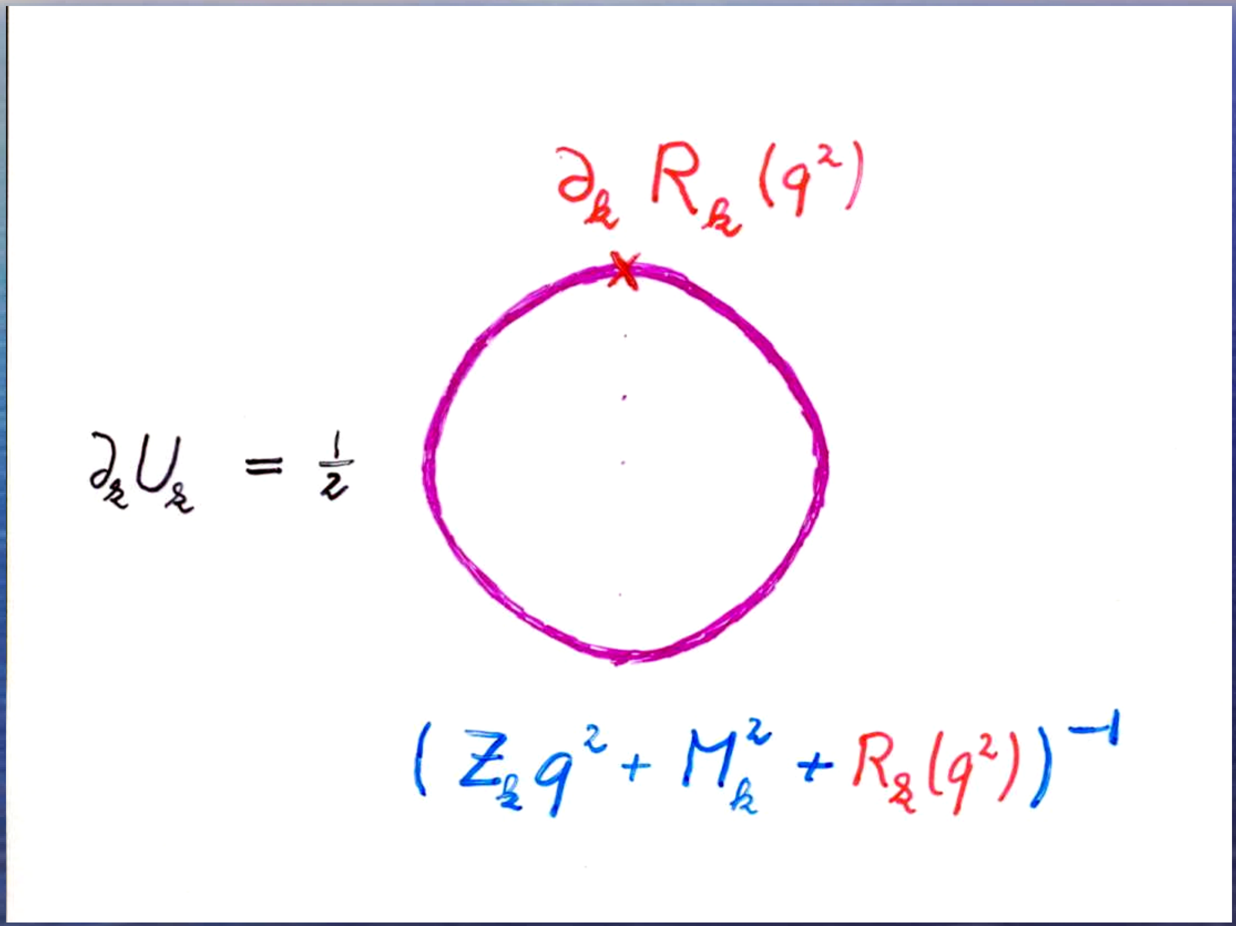
Order of the phase transition



First order phase transition



Simple one loop structure – nevertheless (almost) exact



Flow equation for average potential

$$\partial_k U_k(\varphi) = \frac{1}{2} \sum_i \int \frac{d^d q}{(2\pi)^d} \frac{\partial_k R_k(q^2)}{Z_k q^2 + R_k(q^2) + \bar{M}^2_{k,i}(\varphi)}$$

$\bar{M}^2_{k,ab} = \frac{\partial^2 U_k}{\partial\varphi_a \partial\varphi_b}$: Mass matrix

$\bar{M}^2_{k,i}$: Eigenvalues of mass matrix

Critical temperature , N_f = 2

$\frac{m_\pi}{\mathrm{MeV}}$	0	45	135	230
$\frac{T_{pc}}{\mathrm{MeV}}$	100.7	$\simeq 110$	$\simeq 130$	$\simeq 150$

for $f_\pi = 93$ MeV

Lattice simulation

J.Berges,D.Jungnickel,…