

**Chiral freedom
and the
scale of
weak interactions**

proposal for solution of gauge hierarchy problem

- model without fundamental scalar
- new anti-symmetric tensor fields
- local mass term forbidden by symmetry
- chiral couplings to quarks and leptons
- chiral couplings are asymptotically free
- weak scale by dimensional transmutation

antisymmetric tensor fields

- two irreducible representations of Lorentz – symmetry : $(3,1) + (1,3)$
- complex representations : $(3,1)^* = (1,3)$
- similar to left/right handed spinors

$$\beta_{mn}^{\pm} = \frac{1}{2}\beta_{mn} \pm \frac{i}{4}\epsilon_{mn}{}^{pq}\beta_{pq}$$

chiral couplings to quarks and leptons

$$\begin{aligned} -\mathcal{L}_{ch} = & \bar{u}_R \bar{F}_U \tilde{\beta}_+ q_L - \bar{q}_L \bar{F}_U^\dagger \tilde{\beta}_+ u_R \\ & + \bar{d}_R \bar{F}_D \bar{\beta}_- q_L - \bar{q}_L \bar{F}_D^\dagger \bar{\beta}_- d_R \\ & + \bar{e}_R \bar{F}_L \bar{\beta}_- l_L - \bar{l}_L \bar{F}_L^\dagger \bar{\beta}_- e_R \end{aligned}$$

$$\beta_\pm = \frac{1}{2} \beta_{mn}^\pm \sigma^{mn}$$

- most general interaction consistent with Lorentz and gauge symmetry : β are weak doublets with hypercharge
- consistent with chiral parity :
 d_R , e_R , β^- have odd chiral parity

no local mass term allowed for chiral tensors

- Lorentz symmetry forbids $(\beta^+)^* \beta^+$
- Gauge symmetry forbids $\beta^+ \beta^+$
- Chiral parity forbids $(\beta^-)^* \beta^+$

kinetic term

$$-\mathcal{L}_{\beta,kin}^{ch} = \frac{1}{4} \int d^4x \{ (\partial^\rho \beta^{\mu\nu})^* \partial_\rho \beta_{\mu\nu} - 4 (\partial_\mu \beta^{\mu\nu})^* \partial_\rho \beta^\rho{}_\nu \}$$

- does not mix β^+ and β^-
- unique possibility consistent with all symmetries, including chiral parity

quartic couplings

$$\begin{aligned} -\mathcal{L}_{\beta,4} = & \frac{\tau_+}{16} [(\beta_{\mu\rho}^+)^\dagger \beta^{+\rho\nu}] [(\beta^{+\mu\sigma})^\dagger \beta_{\sigma\nu}^+] + (+ \rightarrow -) \\ & + \frac{\tau_1}{16} [(\beta_{\mu\nu}^+)^\dagger \beta^{-\mu\nu}] [(\beta_{\rho\sigma}^-)^\dagger \beta^{+\rho\sigma}] \\ & + \frac{\tau_2}{16} [(\beta_{\mu\nu}^+)^\dagger \vec{\tau} \beta^{-\mu\nu}] [(\beta_{\rho\sigma}^-)^\dagger \vec{\tau} \beta^{+\rho\sigma}] \\ & + \frac{\tau_3}{64} [(\beta_{\mu\nu}^+)^\dagger \beta^{-\mu\nu}] [(\beta_{\rho\sigma}^+)^\dagger \beta^{-\rho\sigma}] + c.c. \\ & + \frac{\tau_4}{64} [(\beta_{\mu\nu}^+)^\dagger \beta^{-\rho\sigma}] [(\beta^{+\mu\nu})^\dagger \beta_{\rho\sigma}^-] + c.c. \\ & + \frac{\tau_5}{64} [(\beta_{\mu\nu}^+)^\dagger \beta^{-\rho\sigma}] [(\beta_{\rho\sigma}^+)^\dagger \beta^{-\mu\nu}] + c.c. \end{aligned}$$

add gauge interactions and
gauge invariant kinetic term for fermions ...

classical dilatation symmetry

- action has no parameter with dimension mass
- all couplings are dimensionless

flavor and CP violation

- chiral couplings can be made diagonal and real by suitable phases for fermions

→ Kobayashi – Maskawa Matrix

$$\begin{aligned} -\mathcal{L}_{ch} = & \bar{u}_R \bar{F}_U \tilde{\beta}_+ q_L - \bar{q}_L \bar{F}_U^\dagger \tilde{\beta}_+ u_R \\ & + \bar{d}_R \bar{F}_D \bar{\beta}_- q_L - \bar{q}_L \bar{F}_D^\dagger \bar{\beta}_- d_R \\ & + \bar{e}_R \bar{F}_L \bar{\beta}_- l_L - \bar{l}_L \bar{F}_L^\dagger \bar{\beta}_- e_R \end{aligned}$$

- same flavor violation and CP violation as in standard model
- additional CP violation through quartic couplings possible

asymptotic freedom

evolution equations for chiral couplings

$$k \frac{\partial}{\partial k} F_U = -\frac{9}{8\pi^2} F_U F_U^\dagger F_U - \frac{3}{8\pi^2} F_U F_D^\dagger F_D + \frac{1}{4\pi^2} F_U \text{tr}(F_U^\dagger F_U) - \frac{1}{2\pi^2} g_s^2 F_U$$

$$k \frac{\partial}{\partial k} F_D = -\frac{9}{8\pi^2} F_D F_D^\dagger F_D - \frac{3}{8\pi^2} F_D F_U^\dagger F_U + \frac{1}{4\pi^2} F_D \text{tr}(F_D^\dagger F_D + \frac{1}{3} F_L^\dagger F_L) - \frac{1}{2\pi^2} g_s^2 F_D$$

$$k \frac{\partial}{\partial k} F_L = -\frac{9}{8\pi^2} F_L F_L^\dagger F_L + \frac{1}{4\pi^2} F_L \text{tr}(F_D^\dagger F_D + \frac{1}{3} F_L^\dagger F_L)$$

$$F_U = Z_u^{-1/2} \bar{F}_U Z_q^{-1/2} Z_+^{-1/2}$$

evolution equations for top coupling

$$k \frac{\partial}{\partial k} F_U = -\frac{9}{8\pi^2} F_U F_U^\dagger F_U$$

fermion anomalous
dimension

$$+\frac{1}{4\pi^2} F_U \text{tr}(F_U^\dagger F_U)$$

tensor anomalous
dimension

no vertex correction

asymptotic freedom !

dimensional transmutation

$$f_t^2(k) = \frac{4\pi^2}{7 \ln(k/\Lambda_{ch}^{(t)})}$$

Chiral coupling for top grows large
at chiral scale Λ_{ch}

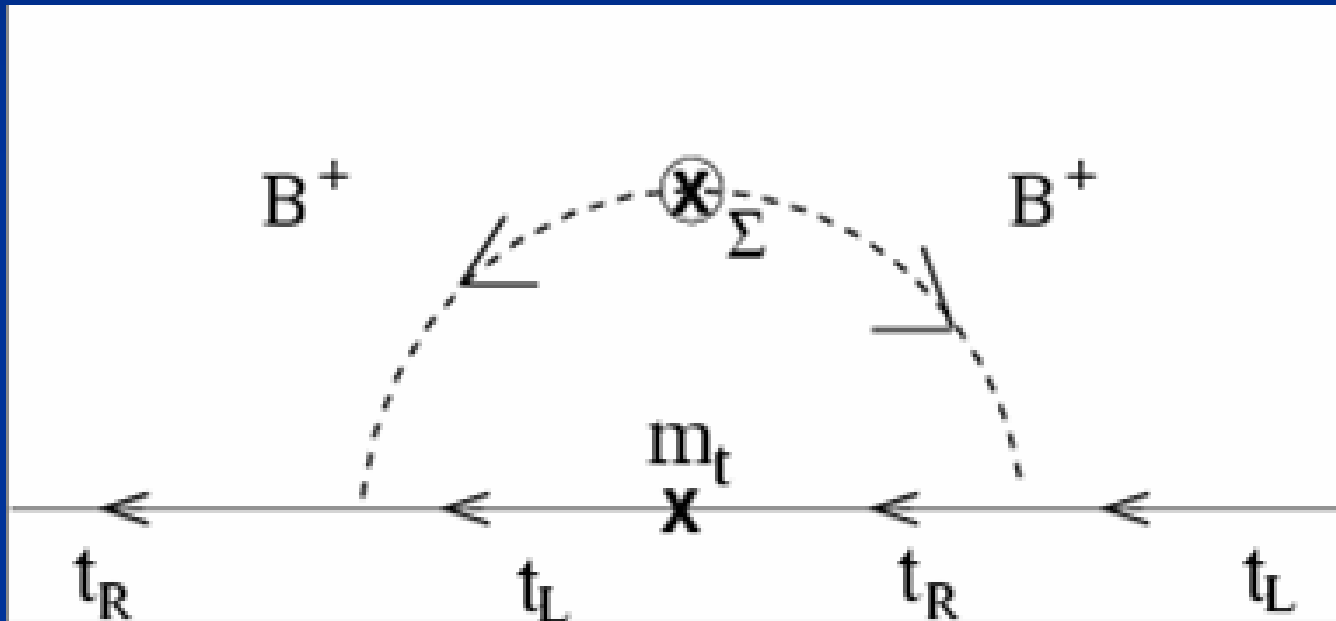
This sets physical scale : dimensional transmutation -
similar to Λ_{QCD} in strong QCD- gauge interaction

**spontaneous electroweak
symmetry breaking**

top – anti-top condensate

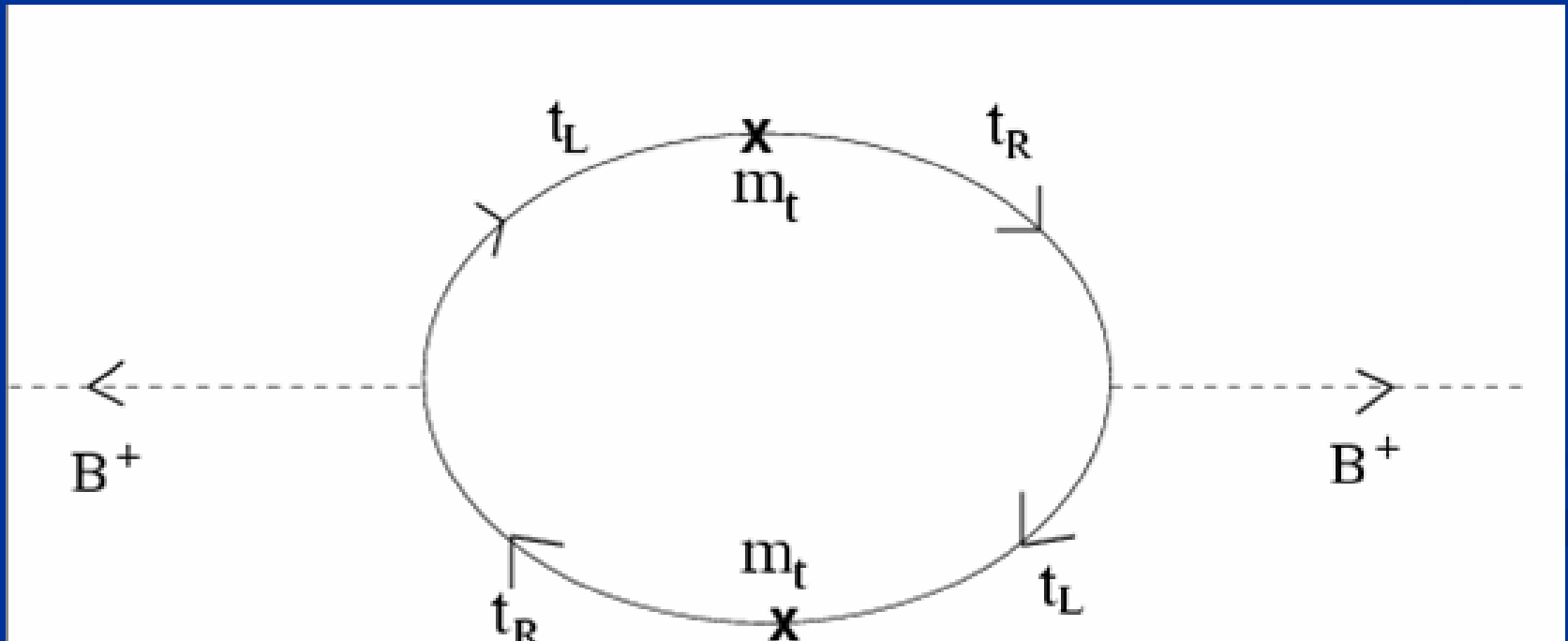
- large chiral coupling for top leads to large effective attractive interaction for top quark
- this triggers condensation of top – anti-top pairs
- electroweak symmetry breaking : effective Higgs mechanism provides mass for weak bosons
- effective Yukawa couplings of Higgs give mass to quarks and leptons

Schwinger - Dyson equation for top quark mass



solve gap equation for top quark propagator

SDE for B-B-propagator



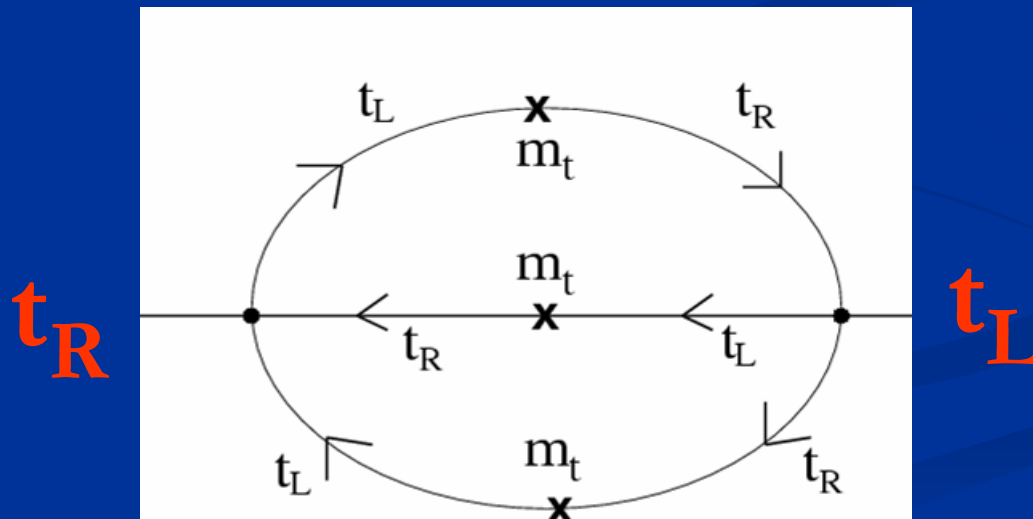
gap equation for top quark mass

$$\int_{M_{\beta}^2/m_t^2}^{\infty} \frac{dx}{x(x+1)} \frac{\ln \left(\frac{\Lambda_t^4}{m_t^4(1+x/4)^2} + 1 \right)}{\left(\ln \left(\frac{m_t^2}{\Lambda_{ch^2}^{(t)}} \right) + \ln x \right)^2} = \frac{49}{36}$$

has reasonable solutions for m_t :
somewhat above the chiral scale

two loop SDE for top-quark mass

contract B- exchange to pointlike four fermion interaction



effective interactions

- introduce composite field for top- antitop bound state
- plays role of Higgs field
- new effective interactions involving the composite scalar φ

effective scalar tensor interactions

$$\begin{aligned}
 -\mathcal{L}_{M\beta} = & \frac{1}{8} \text{tr} \{ \sigma_1 [\varphi_t^\dagger \varphi_b] [\bar{\beta}_- \beta_+] + \sigma_2 [\varphi_t^\dagger \beta_+] [\bar{\beta}_- \varphi_b] + \\
 & + \sigma_+ [\bar{\beta}_+ \varphi_t] [\bar{\beta}_+ \varphi_t] + \sigma_- [\bar{\beta}_- \varphi_b] [\bar{\beta}_- \varphi_b] \} \\
 & + \sigma_{v1} [\varphi_b^\dagger \varphi_t] [\bar{\beta}_- \beta_+] + \sigma_{v2} [\varphi_b^\dagger \beta_+] [\bar{\beta}_- \varphi_t] \\
 & + \sigma_{v+} [\bar{\beta}_+ \varphi_b] [\bar{\beta}_+ \varphi_b] + \sigma_{v-} [\bar{\beta}_- \varphi_t] [\bar{\beta}_- \varphi_t] \} + c.c.
 \end{aligned}$$

$$\frac{1}{8} \text{tr} \bar{\beta}_- \beta_+ = \frac{1}{4} \beta_{\mu\nu}^{-*} \beta^{+\mu\nu} = B_k^{-*} B_k^+ ,$$

$$\frac{1}{8} \text{tr} \beta_{\pm} \beta_{\pm} = \frac{1}{4} \beta_{\mu\nu}^{\pm} \beta^{\pm\mu\nu} = B_k^{\pm} B_k^{\pm}$$

chiral tensor – gauge boson - mixing

$$\begin{aligned} -\mathcal{L}_{F\beta} = & \nu_{y+}[\varphi_t^\dagger \beta_{\mu\nu}^+] Y^{\mu\nu} + \nu_{y+}^*[(\beta_{\mu\nu}^+)^\dagger \varphi_t] Y^{\mu\nu} \\ & + \nu_{w+}[\varphi_t^\dagger \vec{\tau} \beta_{\mu\nu}^+] \vec{W}^{\mu\nu} + \nu_{w+}^*[(\beta_{\mu\nu}^+)^\dagger \vec{\tau} \varphi_t] \vec{W}^{\mu\nu} \\ & + \nu_{y-}[\varphi_b^\dagger \beta_{\mu\nu}^-] Y^{\mu\nu} + \nu_{y-}^*[(\beta_{\mu\nu}^-)^\dagger \varphi_b] Y^{\mu\nu} \\ & + \nu_{w-}[\varphi_b^\dagger \vec{\tau} \beta_{\mu\nu}^-] \vec{W}^{\mu\nu} + \nu_{w-}^*[(\beta_{\mu\nu}^-)^\dagger \vec{\tau} \varphi_b] \vec{W}^{\mu\nu} \end{aligned}$$

and more ...

massive chiral tensor fields

chirons

- irreducible representation for anti-symmetric tensor fields has three components
- in presence of mass : little group $SO(3)$
- with respect to $SO(3)$: anti-symmetric tensor equivalent to vector
- massive chiral tensors = massive spin one particles : chirons

massive spin one particles

- new basis of vector fields:

$$S_{\mu}^{\pm} = \frac{\partial_{\nu}}{\sqrt{\partial^2}} \beta^{\pm\nu}{}_{\mu}, \quad \partial_{\mu} S^{\pm\mu} = 0$$

- standard action for massive vector fields

$$\begin{aligned} \Gamma_{\beta, kin}^{ch} &= - \int_q Z(q) q_{\mu} q_{\nu} (\beta^{\mu\rho}(q))^{\dagger} \beta^{\nu}{}_{\rho}(q) \\ &= \int_q (q^2 + m^2) S^{\mu\dagger}(q) S_{\mu}(q) \end{aligned}$$

$$Z(q) = 1 + m^2 / q^2$$

- classical stability !

classical stability

- massive spin one fields : stable
- free theory for chiral tensors:
borderline stability/instability,
actually unstable (secular solutions , no ghosts)
- mass term moves theory to stable region
- positive energy density for solutions of field equations

consistency of chiral tensors ?

B - basis

$$\beta_{jk}^+ = \epsilon_{jkl} B_l^+ , \beta_{0k}^+ = iB_k^+ \\ \beta_{jk}^- = \epsilon_{jkl} B_l^- , \beta_{0k}^- = -iB_k^-$$

- B –fields are unconstrained
- six complex doublets
- vectors under space – rotations
- irreducible under Lorentz -transformations

free propagator

$$-\mathcal{L}_{\beta,kin}^{ch} = \Omega^{-1} \int \frac{d^4q}{(2\pi)^4} \{ B_k^{+*}(q) P_{kl}(q) B_l^+(q) + B_k^{-*}(q) P_{kl}^*(q) B_l^-(q) \}$$

inverse propagator has unusual form :

$$P_{kl} = -(q_0^2 + q_j q_j) \delta_{kl} + 2q_k q_l - 2i \epsilon_{klj} q_0 q_j$$

$$P^\dagger = P, \quad PP^* = q^4, \quad P^{-1} = \frac{1}{q^4} P^*$$

propagator is invertible ! except for pole at $q^2 = 0$

energy density

$$\rho = -T_0^0 = Z_+ \{ \partial_0 B_k^{+*} \partial_0 B_k^+ + 2 \partial_k B_k^{+*} \partial_l B_l^+ - \partial_l B_k^{+*} \partial_l B_k^+ \} + (+ \rightarrow -)$$

for plane waves :

$$\rho = 2Z_+ \partial_k b_3^{+*} \partial_k b_3^+ + (+ \rightarrow -)$$

positive for longitudinal mode b_3

vanishes for transversal modes $b_{1,2}$ (borderline to stability)

unstable secular classical solutions in free theory

quantum theory : free Hamiltonian is not bounded

secular instability

$$b_1 = (B_1 + iB_2)/\sqrt{2}, \quad b_2 = (B_1 - iB_2)/\sqrt{2}$$

$$\pi_1 = \dot{b}_1 + iqb_1$$

$$b_1 = Q, \quad \pi_1 = P$$

$$h_1 = P^*P - iq(P^*Q - Q^*P)$$

$$\dot{Q} = P - iqQ, \quad \dot{P} = -iqP$$

$$\ddot{Q} + 2iq\dot{Q} - q^2Q = 0$$

$$Q = (Q_0 + P_0t)e^{-iqt}, \quad P = P_0e^{-iqt}$$

solutions grow linearly with time !

no consistent free theory !

mechanical analogue


$$d^2x / dt^2 = \epsilon x$$

- $\epsilon > 0$: exponentially growing mode
(tachyon or ghost)
- $\epsilon < 0$: stable mode
- $\epsilon = 0$: borderline (secular solution growing
linearly with time)

even tiny ϵ decides on stability !

interactions will decide on stability !

interacting chiral tensors can be consistently quantized

- Hamiltonian permits canonical quantization
- Interactions will decide on which side of the borderline between stability and instability the model lies.
- Vacuum not perturbative
- Non – perturbative generation of mass:

stable massive spin one particles !

Chirons

chiron mass

non – perturbative mass term

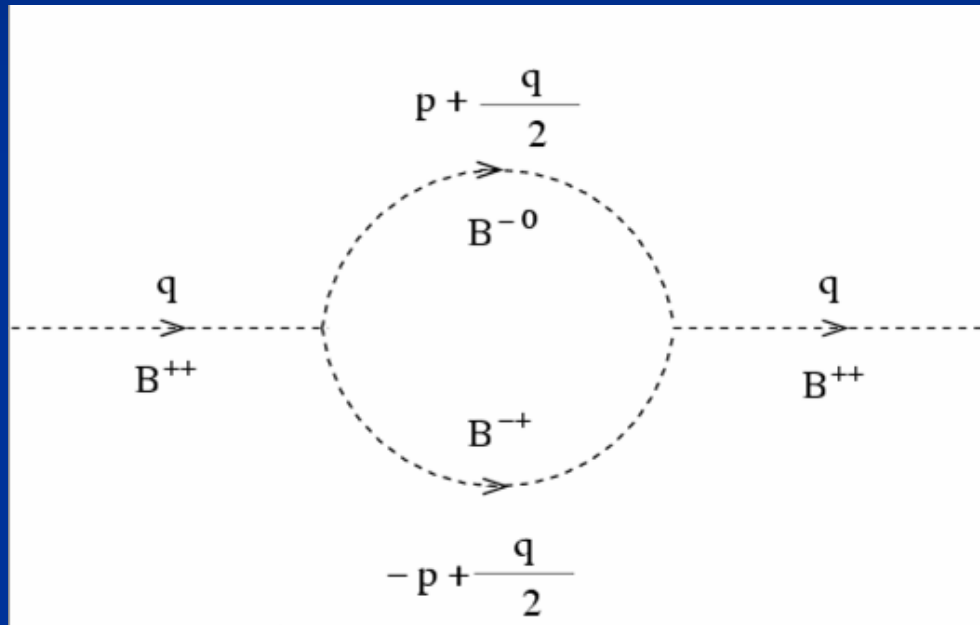
- m^2 : local in S - basis , non-local in B – basis
- cannot be generated in perturbation theory in absence of electroweak symmetry breaking
- plausible infrared regularization for divergence of inverse quantum propagator as chiral scale is approached
- in presence of electroweak symmetry breaking : generated by loops involving chiral couplings

effective cubic tensor interactions

$$-\mathcal{L}_{3\beta} = \gamma_t \epsilon_{klm} [\varphi_t^\dagger B_k^-] [(B_l^+)^\dagger B_m^-] \\ + \gamma_b \epsilon_{klm} [\varphi_b^\dagger B_k^+] [(B_l^-)^\dagger B_m^+] + c.c.$$

generated by electroweak symmetry breaking

propagator corrections from cubic couplings



$$iJ_{kl}(q) = \frac{1}{16\pi^2} \frac{P_{kl}(q)}{q^2}$$

non - local !

effective propagator for chiral tensors

$$\tilde{P}_{kl}(q) = P_{kl}(q) + i(|\gamma_t^* \varphi_t|^2 + |\gamma_b^* \varphi_b|^2) J_{kl}(q)$$

$$iJ_{kl}(q) = \frac{1}{16\pi^2} \frac{P_{kl}(q)}{q^2}$$

massive effective
inverse propagator :
pole for massive field

$$\tilde{P}_{kl}(q) = \frac{P_{kl}(q)}{q^2} (q^2 + m^2)$$

mass term :

$$m^2 = \frac{1}{16\pi^2} (|\gamma_t^* \varphi_t|^2 + |\gamma_b^* \varphi_b|^2)$$

phenomenology

new resonances at LHC ?

- production of massive chiralons at LHC ?
- signal : massive spin one resonances
- rather broad : decay into top quarks
- relatively small production cross section : small chiral couplings to lowest generation quarks , no direct coupling to gluons

effects at low energy

- mixing with gauge bosons is important
- also direct four fermion interactions with tensor structure

mixing between chiral tensor and photon

$$\Gamma_c^{(2)} = \begin{pmatrix} q^2 + m_R^2 & \beta\sqrt{-q^2} \\ \beta\sqrt{-q^2} & q^2 \end{pmatrix}.$$

$$\det = q^2(q^2 + m_R^2 + \beta^2)$$

photon remains massless but acquires new tensor interaction

$$-\mathcal{L}_{ch} \rightarrow \alpha_\gamma \bar{e}_L F_L^\dagger \sigma^{\mu\nu} e_R F_{\mu\nu} + h.c.$$

Pauli term contributes to g-2

$$-\mathcal{L}_{ch} \rightarrow \alpha_\gamma \bar{e}_L F_L^\dagger \sigma^{\mu\nu} e_R F_{\mu\nu} + h.c.$$

suppressed by

- inverse mass of chiral tensor
- small chiral coupling of muon and electron
- small mixing between chiral tensor and photon
- for $M_c \approx 300 \text{ GeV}$ and small chiral couplings :
 $\Delta(g-2) \approx 5 \cdot 10^{-9}$ for muon
larger chiral couplings : $M_c \approx \text{few TeV}$

anomalous magnetic moment of muon

$$\Delta(g-2) = -4 \cdot 10^{-7} c_{\beta} \sigma f_b^2 \left(\frac{m_t}{M_c} \right)^2, \quad \sigma = \frac{f_{\mu} m_b}{f_b m_{\mu}}$$

electroweak precision tests

chiron exchange and mixing:
compatible with LEP experiments
for $M_c > 300 \text{ GeV}$

rough
estimate :

$$\Delta \hat{S} \approx -0.05(m_t/M_c)^2$$

for $M_c = 1 \text{ TeV}$:

$$\Delta \hat{S} \approx 1.4 \cdot 10^{-3}$$

composite scalars

- two composite Higgs doublets expected
- mass 400 -500 GeV
- loop effects ?

mixing of chiral tensors with ϱ - meson

$$\tilde{P}_{\nu\rho} = \frac{\partial_\nu \partial_\rho}{\partial^2}, \quad \tilde{P}_{\nu\rho} \tilde{P}^\rho{}_\mu = \tilde{P}_{\nu\mu}$$

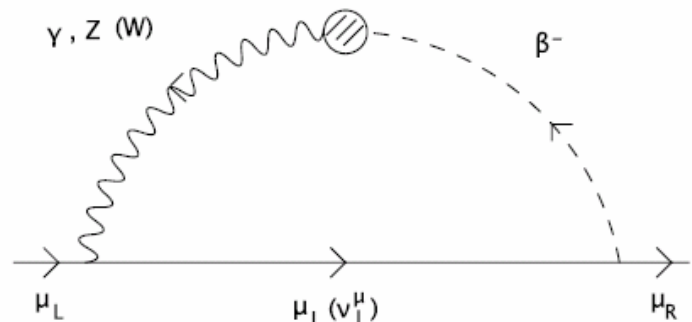
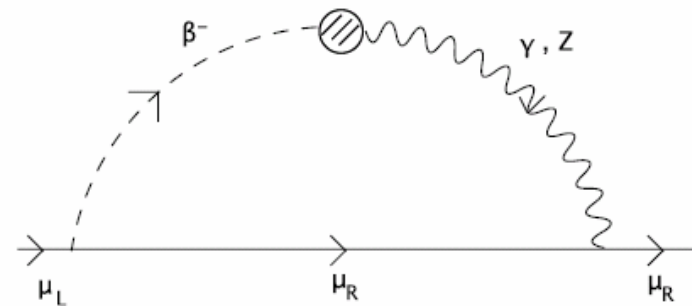
$$-\mathcal{L}_{4F2}^{(\rho)} = -\kappa^{(\rho)} \partial_\nu (\bar{\nu}_L \sigma^{\mu\nu} e_R) (\bar{d} \gamma_\mu u) + c.c.$$

$$\frac{\kappa^{(\rho)} q}{G_F} \sim \frac{\nu_\rho f_e g_\rho M_W^3 M_\pi}{g^3 M_{ch}^2 M_\rho^2}$$

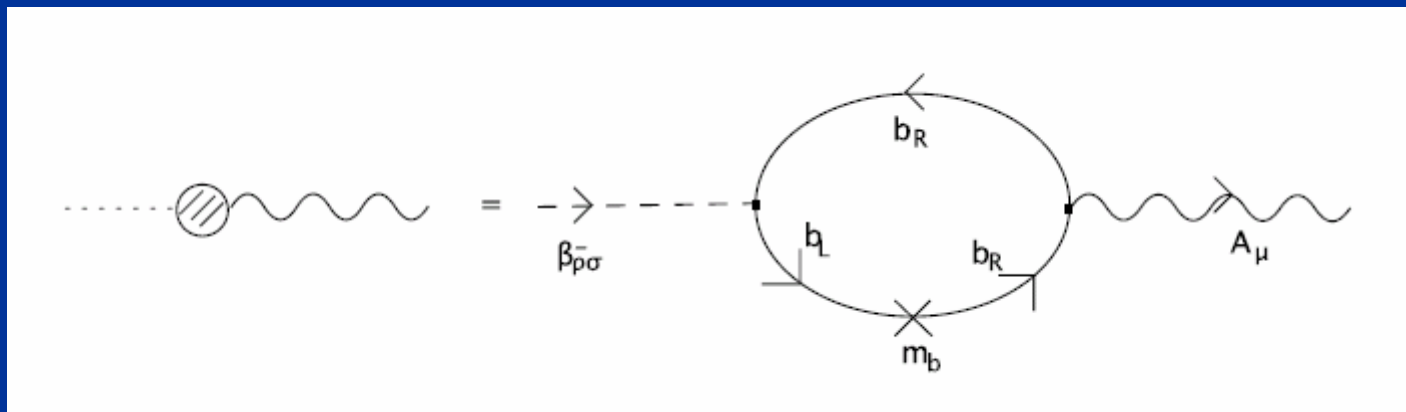
could contribute to anomaly
in radiative pion decays

generation of light fermion masses

involves
chiral couplings
and
chiral – gauge boson
mixing



chiron – photon - mixing



effective tensor vertex of photon

$$\Gamma_{\gamma, \text{ch1}} = \int_p \int_q \alpha_\gamma(p, q) \{ \bar{e}_R(q + p) F_L \sigma^{\mu\nu} e_L(q) \\ + \bar{d}_R(q + p) F_D \sigma^{\mu\nu} d_L(q) \} F_{\mu\nu}(p) + \text{h. c.}$$

contributes to g-2

determination of chiral couplings

$$\frac{m_\mu}{m_b} = 2 \cdot 10^{-3} f_\mu f_b \frac{\kappa_\mu}{\bar{\kappa}_\mu}$$

$$f_\mu f_b \approx 5 \frac{\bar{\kappa}_\mu}{\kappa_\mu}$$

restricts g-2

$$\Delta(g-2) = -\frac{8m_\mu^2}{H\kappa_\mu e^2 m_-^2(0)} \approx -2.5 \cdot 10^{-6} \frac{\bar{H} \bar{\kappa}_\mu 1TeV^2}{H \kappa_\mu m_-^2(0)}$$

for characteristic value ...

$$\Delta(g - 2) = 6 \cdot 10^{-9}$$


and neglecting chiron – mixing



large chiron mass above LHC range

$$m_{-}(0) = 20TeV$$

conclusions

- chiral tensor model has good chances to be consistent
- mass generation needs to be understood quantitatively
- interesting solution of gauge hierarchy problem
- phenomenology needs to be explored !
- if quartic couplings play no major role: 
less couplings than in standard model
predictivity !



end

effective interactions from chiral tensor exchange

$$\begin{aligned}
 -\mathcal{L} = & (J^{+\mu})^\dagger S_\mu^+ + (J^{-\mu})^\dagger S_\mu^- + h.c. \\
 & + (\partial^\mu S^{+\nu})^* \partial_\mu S_\nu^+ + (\partial^\mu S^{-\nu})^* \partial_\mu S_\nu^- \\
 & + m_+^2 (S_+^\mu)^* S_{+\mu} + m_-^2 (S_-^\mu)^* S_{-\mu} \\
 & + \hat{m}^2 ((S_+^\mu)^* S_{-\mu} + (S_-^\mu)^* S_{+\mu})
 \end{aligned}$$

$$(J^{+\mu})^\dagger = \epsilon_+ \sqrt{\partial^2} W^{\mu*} + \frac{\partial_\nu}{\sqrt{\partial^2}} \bar{u}_R F_U \sigma^{\nu\mu} d_L$$

$$\begin{aligned}
 (J^{-\mu})^\dagger = & \epsilon_- \sqrt{\partial^2} W^{\mu*} \\
 & + \frac{\partial_\nu}{\sqrt{\partial^2}} (\bar{u}_L F_D^\dagger \sigma^{\nu\mu} d_R + \bar{\nu}_L F_L^\dagger \sigma^{\nu\mu} e_R)
 \end{aligned}$$

- solve for S_μ in presence of other fields
- reinsert solution

general solution

$$-\mathcal{L} = -(J^{\beta\mu})^\dagger G^{\beta\alpha} J_\mu^\alpha$$

$$\begin{aligned}(J^{+\mu})^\dagger &= \epsilon_+ \sqrt{\partial^2} W^{\mu*} + \frac{\partial_\nu}{\sqrt{\partial^2}} \bar{u}_R F_U \sigma^{\nu\mu} d_L \\(J^{-\mu})^\dagger &= \epsilon_- \sqrt{\partial^2} W^{\mu*} \\ &\quad + \frac{\partial_\nu}{\sqrt{\partial^2}} (\bar{u}_L F_D^\dagger \sigma^{\nu\mu} d_R + \bar{\nu}_L F_L^\dagger \sigma^{\nu\mu} e_R)\end{aligned}$$

propagator for charged chiral tensors

$$G = \begin{pmatrix} (-\partial^2 + m_{R1}^2)^{-1} (-\partial^2 + m_{R2}^2)^{-1} \\ \left(\begin{array}{cc} -\partial^2 + m_-^2 & -\hat{m}^2 \\ -\hat{m}^2 & -\partial^2 + m_+^2 \end{array} \right) \end{pmatrix}$$

effective propagator correction

$$\begin{aligned} -\mathcal{L} &= -\frac{1}{2}G^{(1)}j_\mu^\dagger j^\mu - \frac{1}{2}j_\mu^\dagger \vec{G}^{(3)} \vec{\tau} j^\mu \\ &= \frac{q^2}{2}|\varphi|^2 \{G^{(1)}[|\nu_y|^2 Y^\mu Y_\mu + |\nu_w|^2 \vec{W}^\mu \vec{W}_\mu \\ &\quad - (\nu_y \nu_w^* + \nu_y^* \nu_w) Y^\mu W_{3\mu}] \end{aligned}$$

$$j_\mu = \sqrt{-q^2}(\nu_y^* Y_\mu \varphi + \nu_w^* \vec{W}_\mu \vec{\tau} \varphi)$$

$$|\varphi| = 174\text{GeV}, G^{(1)} \approx M_c^{-2}$$

$$|\nu\varphi| \approx 0.2m_t$$

new four fermion interactions

$$\begin{aligned}
 -\mathcal{L}_{4Fch} = & - \{ \bar{u}_R F_U \sigma^{\nu\mu} d_L \} G^{++} (-\partial^2) \\
 & \tilde{P}_{\nu\rho} \{ \bar{d}_L F_U^\dagger \sigma^\rho{}_\mu u_R \} \\
 & - \{ \bar{u}_L F_D^\dagger \sigma^{\nu\mu} d_R + \bar{\nu}_L F_L^\dagger \sigma^{\nu\mu} e_R \} G^{--} (-\partial^2) \\
 & \tilde{P}_{\nu\rho} \{ \bar{d}_R F_D \sigma^\rho{}_\mu u_L + \bar{e}_R F_L \sigma^\rho{}_\mu \nu_L \} \\
 & - \{ \bar{u}_R F_U \sigma^{\nu\mu} d_L \} G^{+-} (-\partial^2) \\
 & \tilde{P}_{\nu\rho} \{ \bar{d}_R F_D \sigma^\rho{}_\mu u_L + \bar{e}_R F_L \sigma^\rho{}_\mu \nu_L \} \\
 & - \{ \bar{u}_L F_D^\dagger \sigma^{\nu\mu} d_R + \bar{\nu}_L F_L^\dagger \sigma^{\nu\mu} e_R \} G^{-+} (-\partial^2) \\
 & \tilde{P}_{\nu\rho} \{ \bar{d}_L F_U^\dagger \sigma^\rho{}_\mu u_R \}
 \end{aligned}$$

typically rather small effect for lower generations
 more substantial for bottom , top !

mixing of charged spin one fields

$$-\mathcal{L}_{F\beta} = -2\sqrt{2}W_d^{-\mu}\sqrt{\partial^2} \left(\frac{\nu_{w+\varphi_t^*}}{\sqrt{Z_+}} S_{\mu}^{+,+} + \frac{\nu_{w-\varphi_b^*}}{\sqrt{Z_-}} S_{\mu}^{-,+} \right) + c.c.$$

$$\Gamma_C^{(2)} = \begin{pmatrix} q^2 + m_+^2, & \hat{m}^2, & \varepsilon_+^* \sqrt{-q^2} \\ \hat{m}^2, & q^2 + m_-^2, & \varepsilon_-^* \sqrt{-q^2} \\ \varepsilon_+ \sqrt{-q^2}, & \varepsilon_- \sqrt{-q^2}, & q^2 + \bar{M}_w^2 \end{pmatrix}$$

- modification of W-boson mass
- similar for Z – boson
- **watch LEP – precision tests !**

momentum dependent Weinberg angle

$$\frac{g^2}{M_W^2 + q^2} \rightarrow \frac{g^2}{\bar{M}_W^2 + q^2(1 + p_W(q^2))} = \frac{g_{eff}^2(q^2)}{M_W^2 + q^2}$$