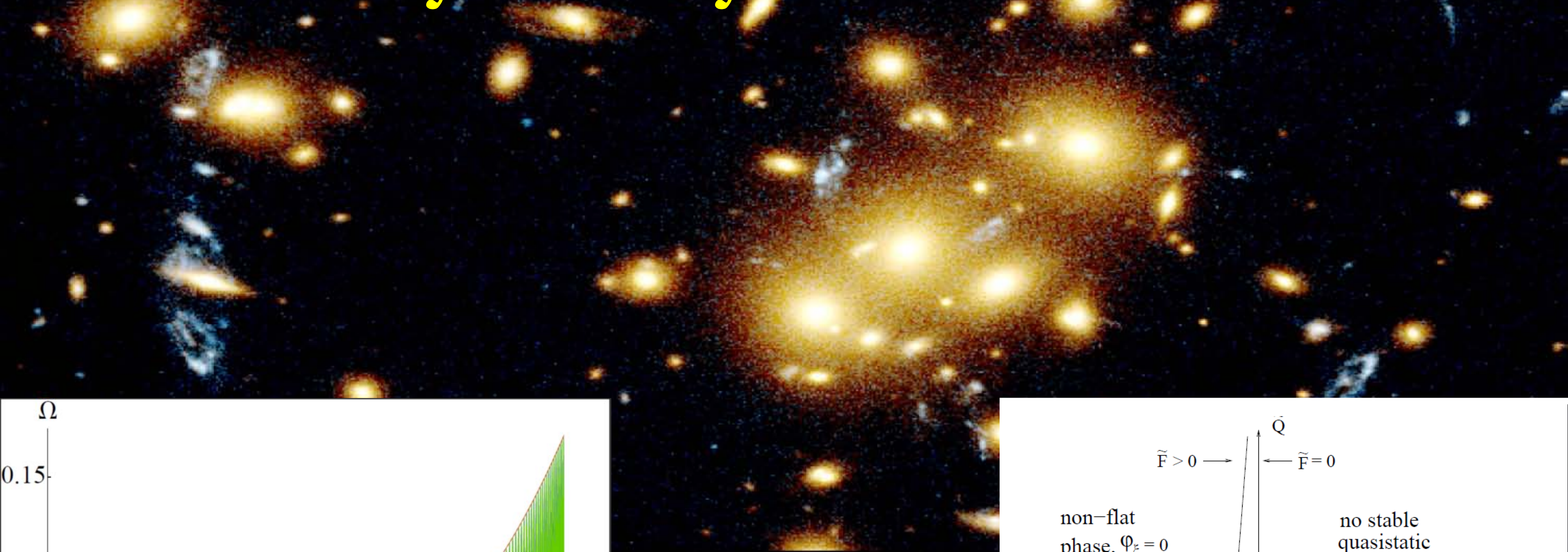
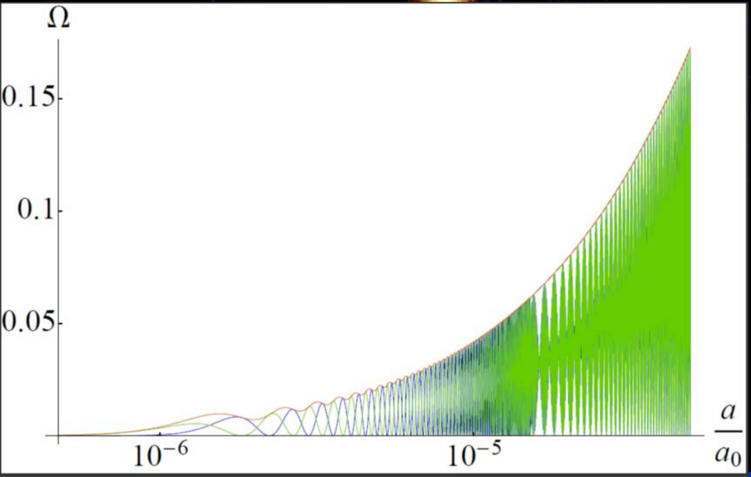
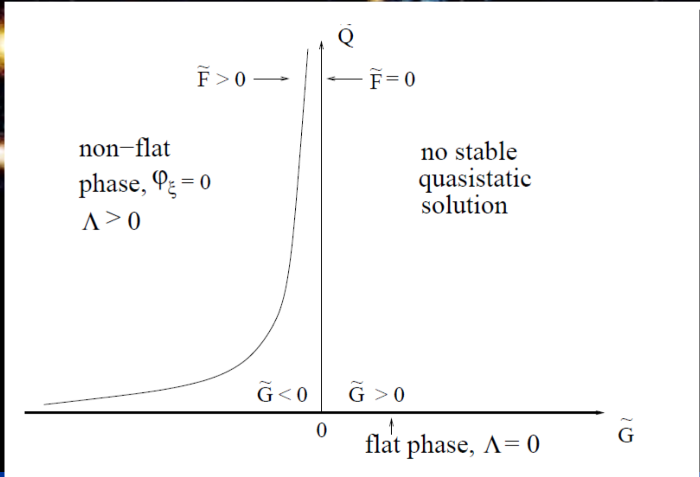
Coupled Dark Energy and Dark Matter from dilatation symmetry







Cosmological Constant
- Einstein -

- Constant λ compatible with all symmetries
- Constant λ compatible with all observations
- No time variation in contribution to energy density

- Why so small ? $\lambda/M^4 = 10^{-120}$

- Why important just today ?

Cosmological mass scales

- Energy density

$$\varrho \sim (\,2.4\times10^{-3}\ \mathrm{eV}\,)^{-4}$$

- Reduced Planck mass

  $M=2.44\times10^{18}\mathrm{GeV}$

- Newton's constant

  $G_N=(8\pi M^2)$

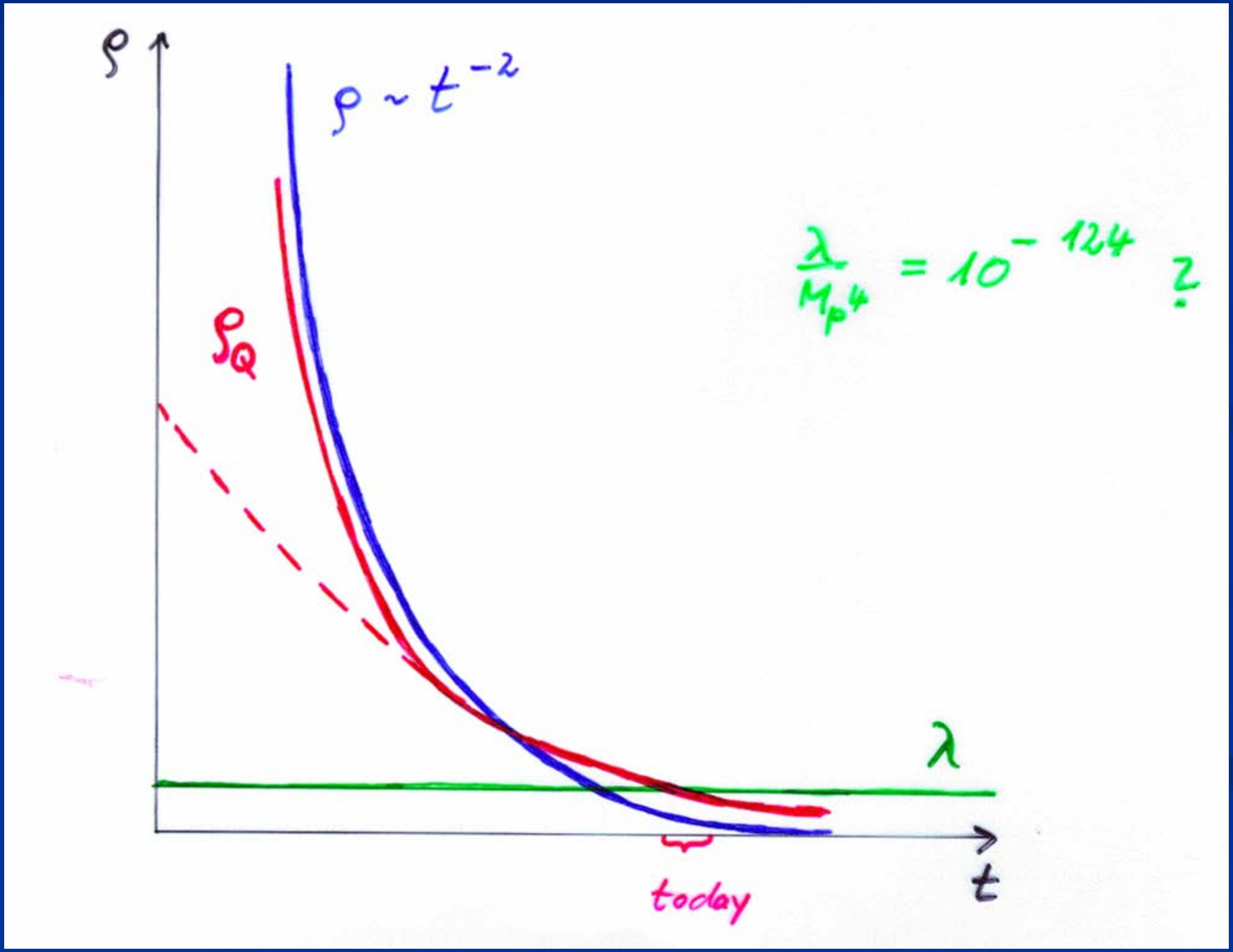
Only ratios of mass scales are observable !

homogeneous dark energy: $\rho_h/M^4 = 7 \cdot 10^{-121}$

matter: $\rho_m/M^4 = 3 \cdot 10^{-121}$

Cosm. Const | Quintessence

static | dynamical



Quintessence

Dynamical dark energy , generated by scalar field (cosmon)

Prediction :

homogeneous dark energy influences recent cosmology

- of same order as dark matter -

Original models do not fit the present observations …. modifications

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87

P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Cosmon

- *Scalar field changes its value even in the* ***present*** *cosmological epoch*
- *Potential und kinetic energy of cosmon contribute to the energy density of the Universe*

$$3M^2H^2 = V + \frac{1}{2}\dot{\phi}^2 + \rho$$

- *Time - variable dark energy :*

  $\rho_h(t)$ decreases with time !

$V(\varphi) = M^4 \exp(- \alpha\varphi/M)$

Two key features for realistic cosmology

1) Exponential cosmon potential and scaling solution

$$V(\varphi) = M^4 \exp(- \alpha\varphi/M)$$

$$V(\varphi \rightarrow \infty) \rightarrow 0 \ !$$

2) Stop of cosmon evolution by cosmological trigger
e.g. growing neutrino quintessence

Evolution of cosmon field

Field equations

$$\ddot{\phi} + 3H\dot{\phi} = -dV/d\phi$$

$$3M^2H^2 = V + \frac{1}{2}\dot{\phi}^2 + \rho$$

Potential $V(\varphi)$ determines details of the model

$$V(\varphi) = M^4 \exp(-\alpha\varphi/M)$$

for increasing φ the potential decreases towards zero !

exponential potential ⇒ constant fraction in dark energy

$$\Omega_h = 3(4)/\alpha^2$$

can explain order of magnitude of dark energy !

Asymptotic solution

explain $V(\varphi \rightarrow \infty) = 0$!

effective field equations should have generic solution of this type

setting : quantum effective action , all quantum fluctuations included: investigate generic form

realized by fixed point
of runaway solution
in higher dimensions :

dilatation symmetry

Cosmon and bolon

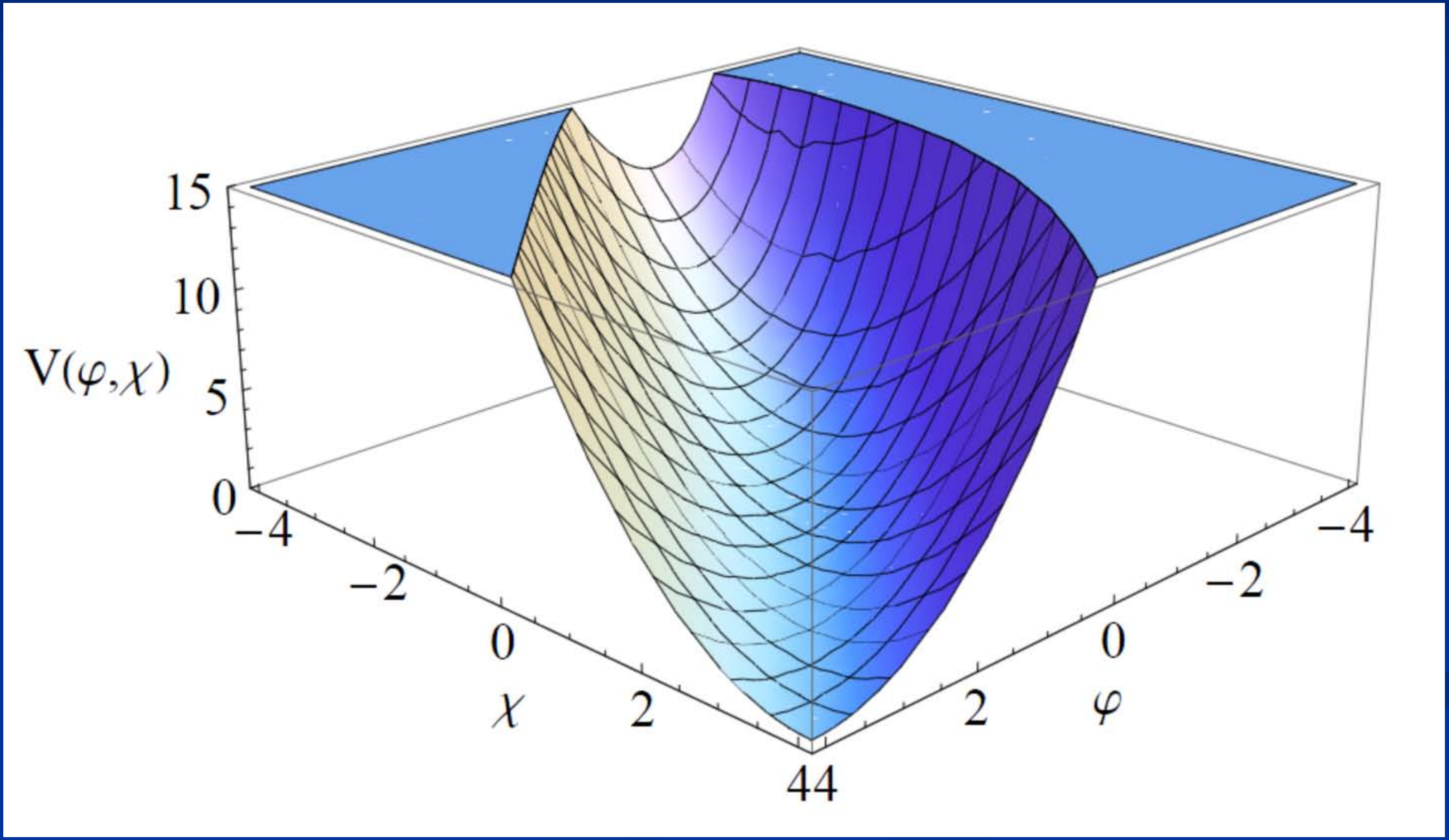
$$V = M^4 \left[\left(\frac{\mu}{M}\right)^A e^{-\alpha\varphi/M} + \left(\frac{\mu}{M}\right)^B e^{-2\beta\varphi/M} \left(\frac{\chi}{M}\right)^2 \right]$$

Two scalar fields : common origin from dilatation symmetric fixed point of higher dimensional theory

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + M^4 e^{-\alpha\varphi/M}$$

$$\rho_\chi = \frac{1}{2}\dot{\chi}^2 + M^4 \left(\frac{\mu}{M}\right)^{\tilde{B}} e^{-2\beta\varphi/M} \left(\frac{\chi}{M}\right)^2$$

Cosmon – bolon - potential



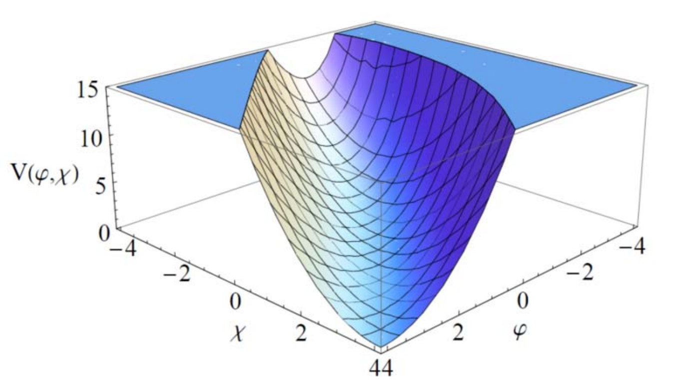
Two characteristic behaviors

- Bolon oscillates -

  if mass larger than H

- Bolon is frozen -

  if mass smaller that H



Cosmon and bolon

$$V = M^4 \left[\left(\frac{\mu}{M}\right)^A e^{-\alpha\varphi/M} + \left(\frac{\mu}{M}\right)^B e^{-2\beta\varphi/M} \left(\frac{\chi}{M}\right)^2 \right]$$

Dark Energy

$$\rho_\varphi \; = \; \frac{1}{2}\dot{\varphi}^2 + M^4 e^{-\alpha\varphi/M}$$

Dark Matter

$$\rho_\chi \; = \; \frac{1}{2}\dot{\chi}^2 + M^4 \left(\frac{\mu}{M}\right)^{\tilde{B}} e^{-2\beta\varphi/M} \left(\frac{\chi}{M}\right)^2$$

Early scaling solution

$$\varphi = -\frac{M}{\alpha} \ln\left(\frac{4H^2}{M^2\alpha^2}\right)$$

$$\rho_\varphi = \frac{12}{\alpha^2} H^2 M^2$$

dominated by cosmon
bolon frozen and negligible
bolon mass increases during scaling solution

$$m_\chi^2 = 2M^2 \left(\frac{\mu}{M}\right)^{\widetilde{B}} \Omega_\varphi^{2\beta/\alpha} \left(\frac{H}{M}\right)^{4\beta/\alpha}$$

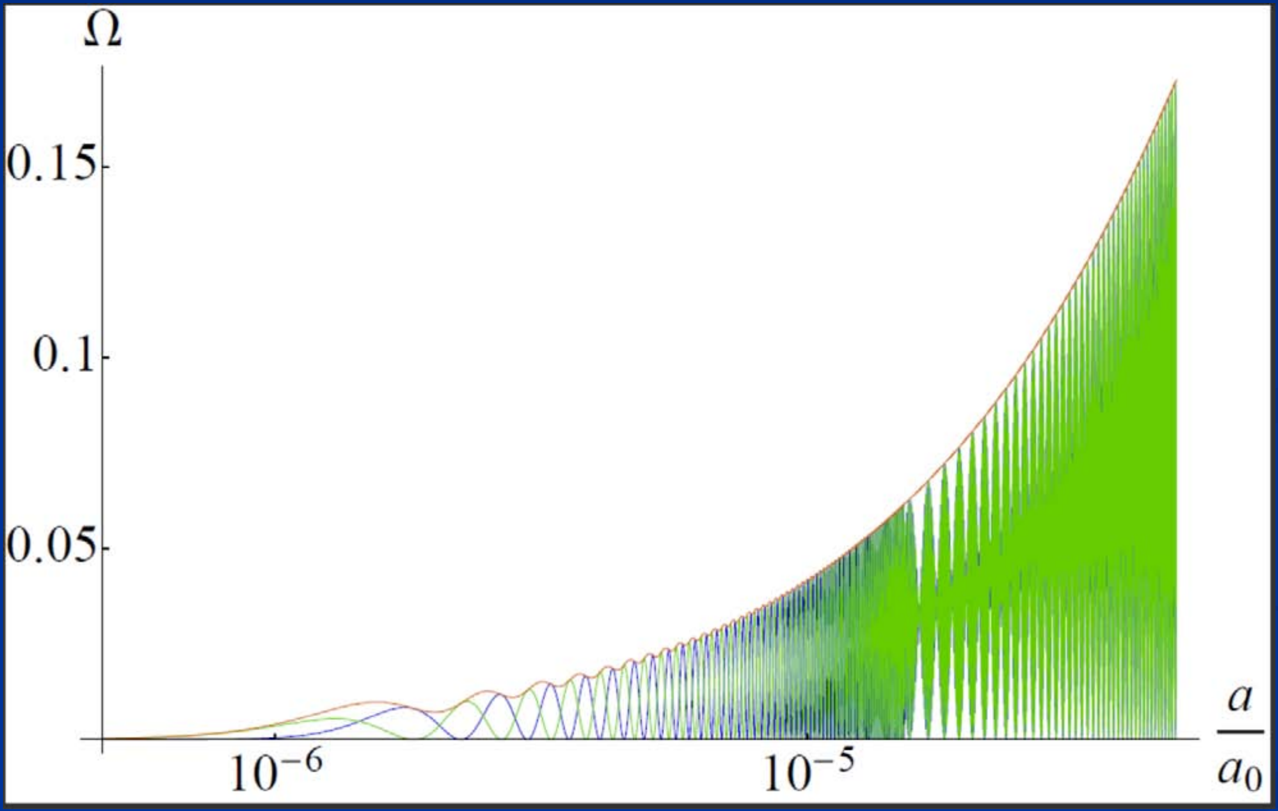
Bolon oscillations

- ratio bolon mass / H increases
- bolon starts oscillating once mass larger than H
- subsequently bolon behaves as Dark Matter
- matter radiation equality around beginning of oscillations

$$\frac{H_{\rm eq}}{M} \sim \left(\frac{\mu}{M}\right)^{\widetilde{B}/2} \left(\frac{\chi_{\rm eq}}{M}\right)$$

$$\frac{\chi_{\rm eq}}{M} \sim \left(\frac{\chi_0}{M}\right)^4$$

Bolon oscillations



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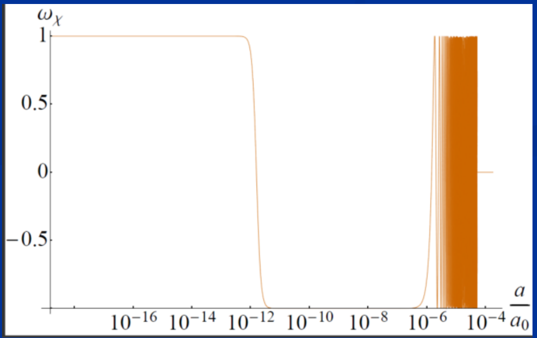
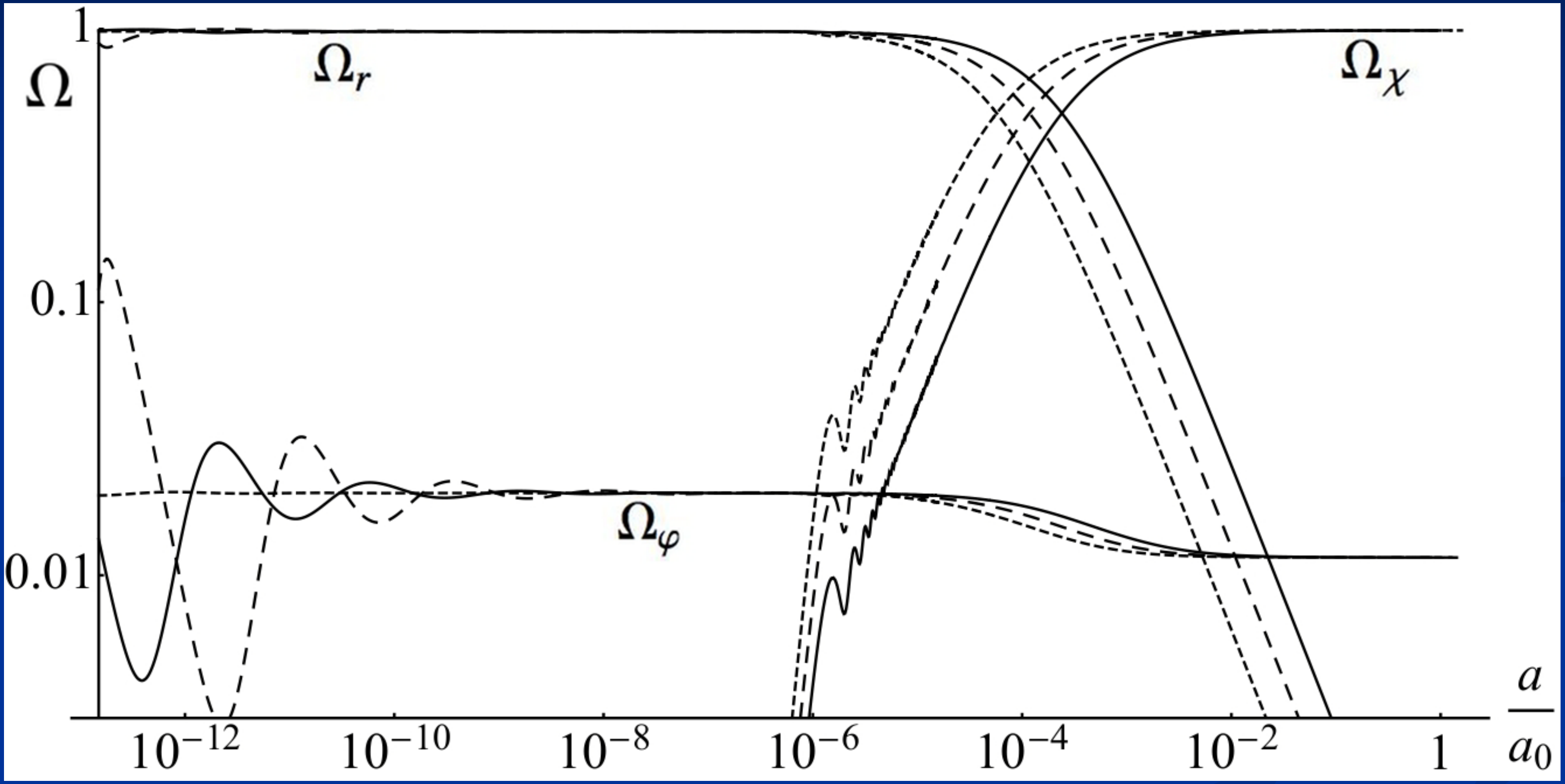


Transition to matter domination



precise timing depends at this stage on initial value of bolon

$$\frac{H_{\mathrm{eq}}}{M} \sim \left(\frac{\mu}{M}\right)^{\widetilde{B}/2} \left(\frac{\chi_{\mathrm{eq}}}{M}\right) \qquad \frac{\chi_{\mathrm{eq}}}{M} \sim \left(\frac{\chi_0}{M}\right)^4$$

Effective coupling between Dark Energy and Dark matter

$$\ddot{\varphi} + 3H\dot{\varphi} - \alpha M^3 e^{-\alpha\varphi/M} = \frac{\beta}{M}\rho_\chi$$

$$\dot{\rho}_\chi + 3H\rho_\chi = -\frac{\beta}{M}\rho_\chi\dot{\varphi}$$

$$\dot{\rho}_\gamma + 4H\rho_\gamma = 0$$

$$3M^2H^2 = (\rho_r + \rho_\chi + \rho_\varphi)$$

Scaling solution for coupled Dark Energy

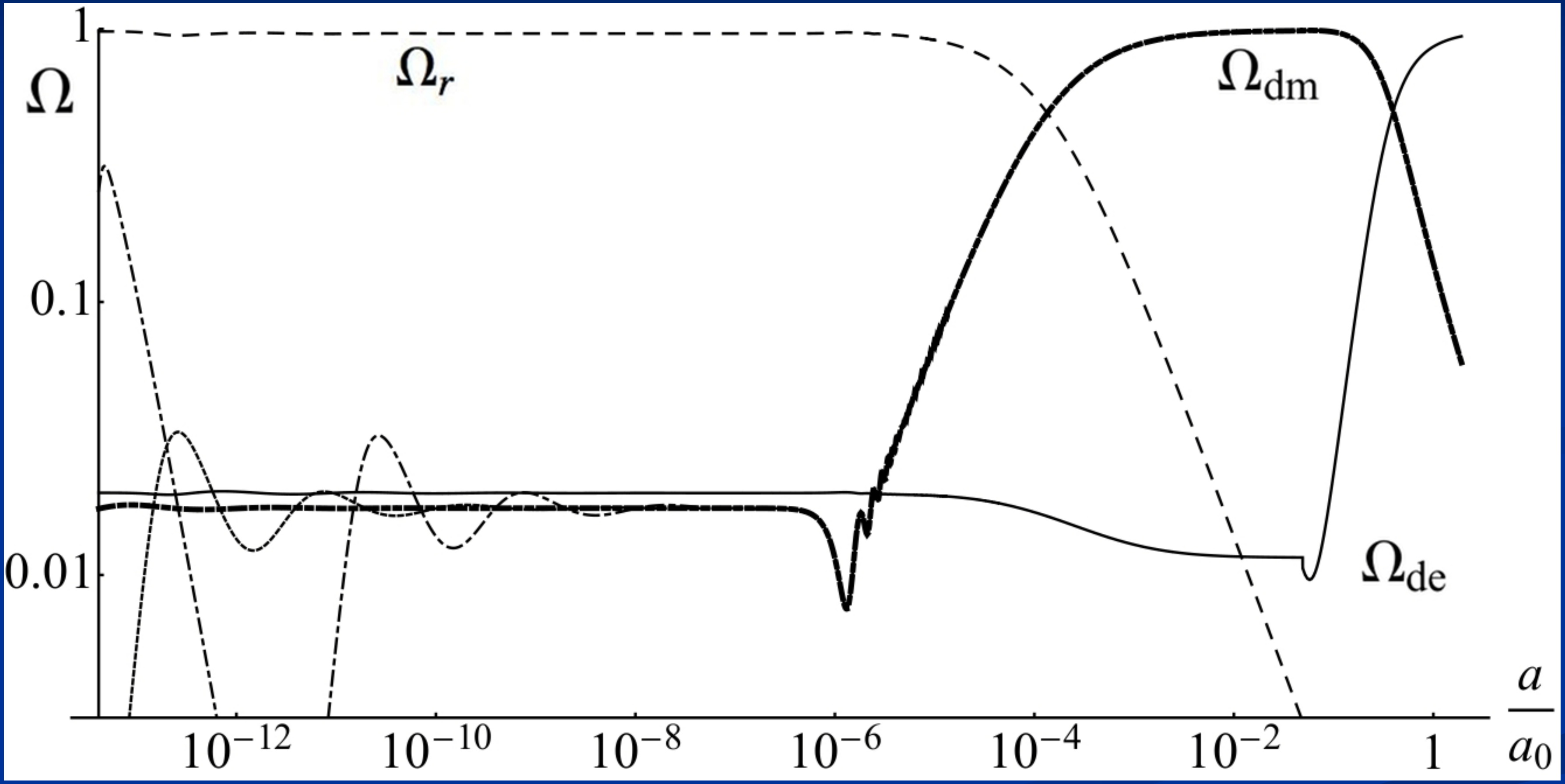
$$a \propto t^{1-\beta/\alpha}$$

$$\rho_\varphi = 3M^2H^2 f(\alpha,\beta)$$

$$f \;=\; (18 + 6\beta^2 - 6\beta\alpha)/(6(\alpha - \beta)^2)$$

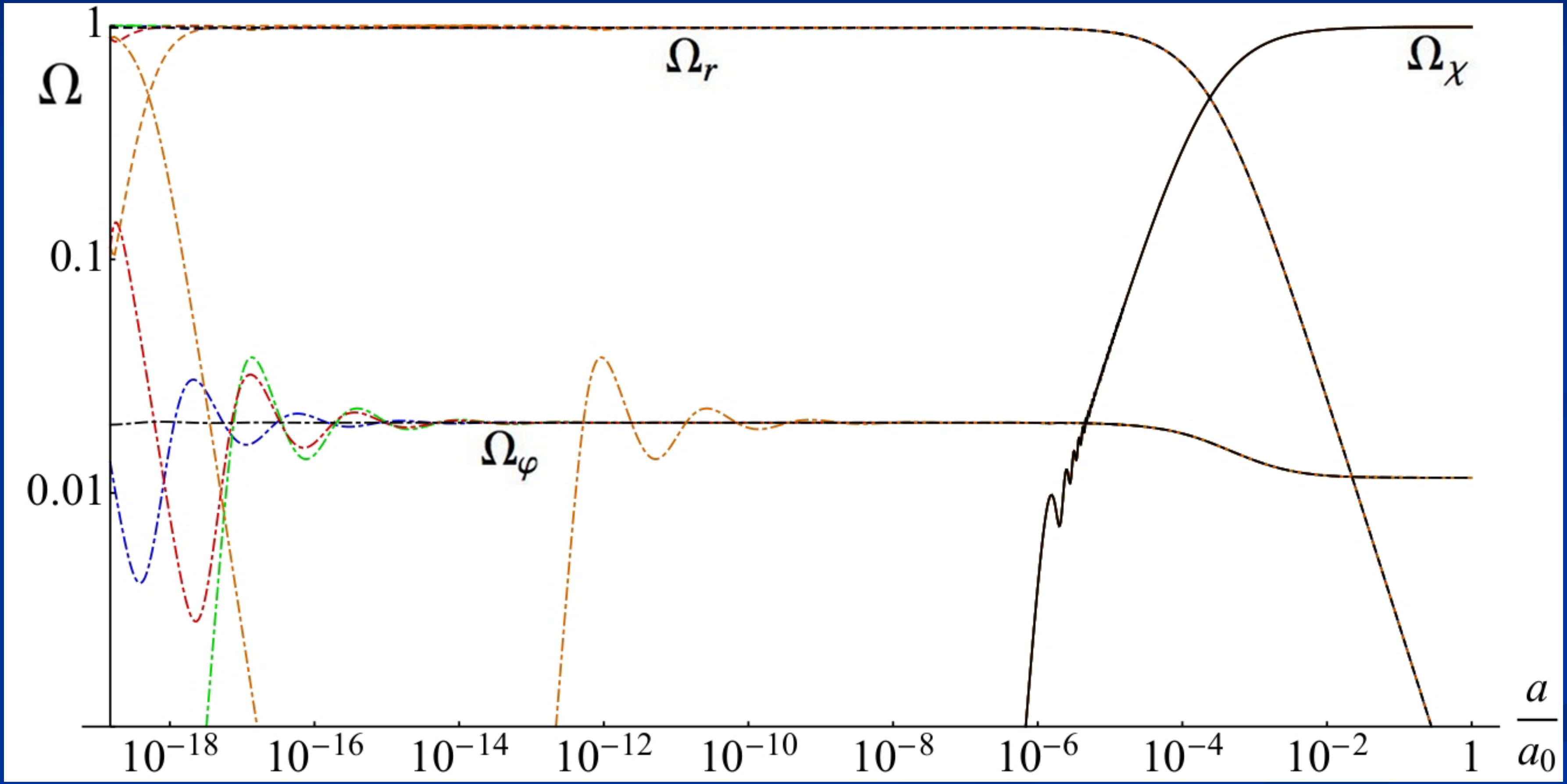
$$\Omega_\varphi = \frac{3}{\alpha^2} - \frac{\beta}{\alpha}\left(1 - \frac{6}{\alpha^2}\right) + \mathcal{O}(\beta^2/\alpha^2)$$

Realistic quintessence needs late modification



modification of cosmon – bolon potential
or growing neutrinos or …

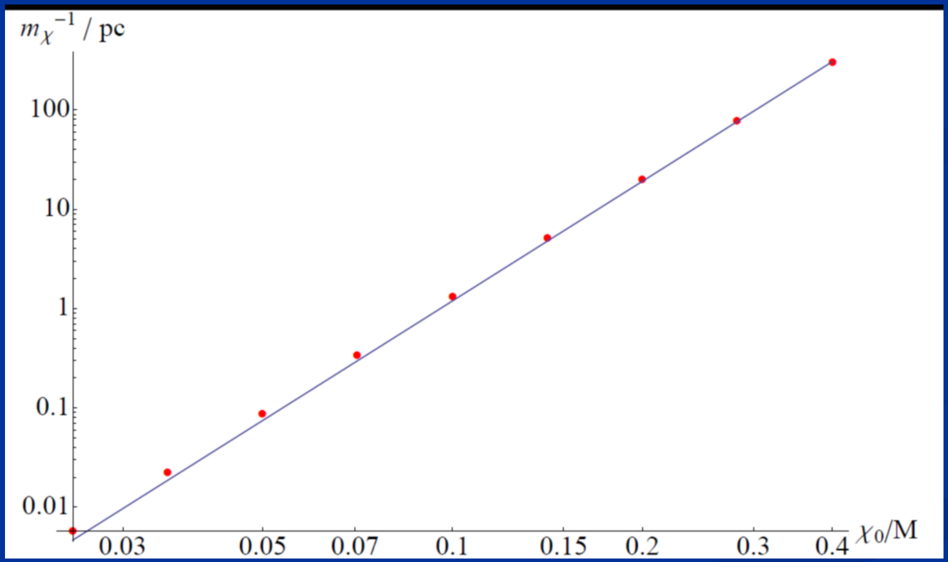
Modification of potential for large χ : independence of initial conditions



matter - radiation equality depends now on parameters of potential

Present bolon mass corresponds to range on subgalactic scales

$$m_\chi^{-1} = \sqrt{\frac{1}{3}\frac{\chi_{\rm eq}}{M}}\, H_{\rm eq}^{-1}\, e^{\beta\Delta\varphi/M} \approx \left(\frac{10\,\chi_0}{M}\right)^4 \,{\rm pc}$$

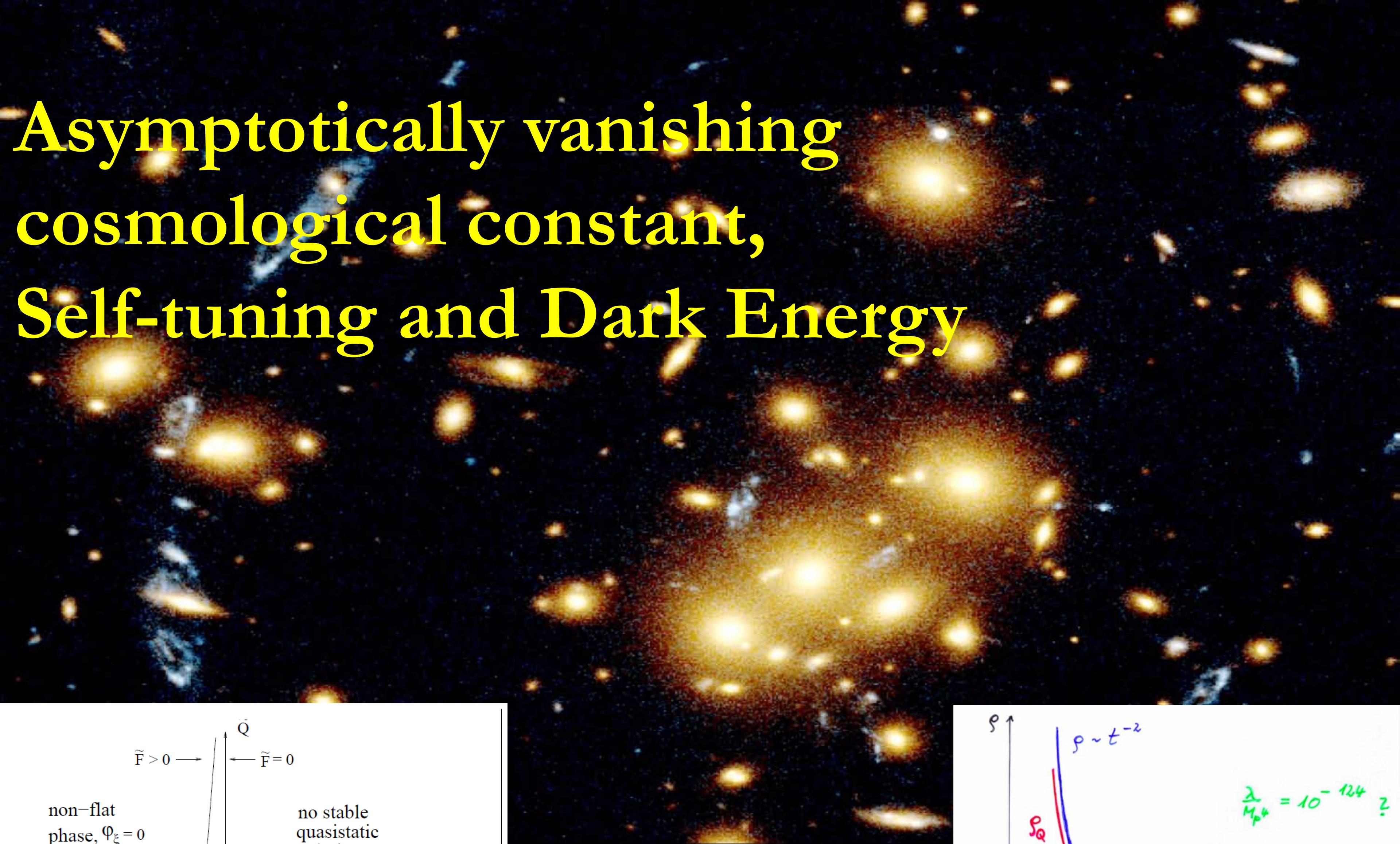


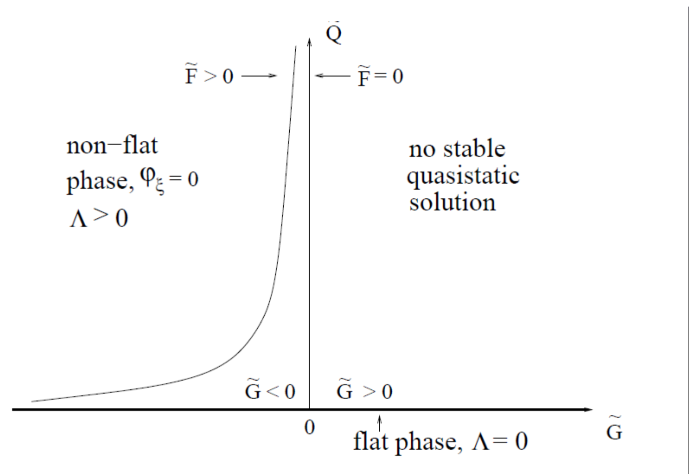
suppression of small scale Dark Matter structures ?

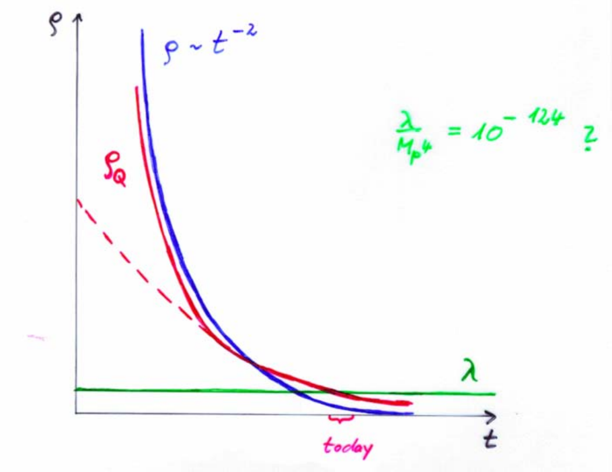
conclusions (1)

- Bolon : new Dark Matter candidate
- not detectable by local observations –
  direct or indirect dark matter searches
- perhaps observation by influence on subgalactic dark matter structures

Asymptotically vanishing cosmological constant, Self-tuning and Dark Energy







Higher –dimensional
dilatation symmetry
solves
cosmological constant problem

graviton and dilaton

dilatation symmetric effective action

$$\Gamma = \int_{\hat{x}} \hat{g}^{1/2} \Big\{ -\frac{1}{2}\xi^2 \hat{R} + \frac{\zeta}{2}\partial^{\hat{\mu}}\xi \partial_{\hat{\mu}}\xi + F(\hat{R}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}) \Big\}$$

simple example $F = \tau \hat{R}^{\frac{d}{2}}$

in general : many dimensionless parameters characterize effective action

dilatation transformations

$$\Gamma = \int_{\hat{x}} \hat{g}^{1/2} \mathcal{L}.$$

$$\hat{g}_{\hat{\mu}\hat{\nu}} \to \alpha^2 \hat{g}_{\hat{\mu}\hat{\nu}} \ , \ \hat{g}^{1/2} \to \alpha^d \hat{g}^{1/2},$$
$$\xi \to \alpha^{-\frac{d-2}{2}} \xi \ , \ \mathcal{L} \to \alpha^{-d} \mathcal{L}.$$

$$\Gamma = \int_{\hat{x}} \hat{g}^{1/2} \mathcal{L}.$$

is invariant

flat phase

generic existence of solutions of
higher dimensional field equations with

effective four –dimensional gravity and

vanishing cosmological constant

torus solution

example :

Minkowski space x D-dimensional torus

ξ = const

- solves higher dimensional field equations
- extremum of effective action

$$\Gamma = \int_{\hat{x}} \hat{g}^{1/2} \Big\{ -\frac{1}{2}\xi^2 \hat{R} + \frac{\zeta}{2}\partial^{\hat{\mu}}\xi \partial_{\hat{\mu}}\xi + F(\hat{R}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}) \Big\}$$

- finite four- dimensional gauge couplings
- dilatation symmetry spontaneously broken

generically many more solutions in flat phase !

massless scalars

- dilaton

- geometrical scalars (moduli)

  variation of circumference of tori

  change of volume of internal space

  bolon is associated to one such scalar

Higher dimensional dilatation symmetry

- for arbitrary values of effective couplings within a certain range : higher dimensional dilatation symmetry implies existence of a large class of solutions with vanishing four –dimensional cosmological constant
- all stable quasi-static solutions of higher dimensional field equations , which admit a finite four-dimensional gravitational constant and non-zero value for the dilaton , have V=0
- self-tuning mechanism

look for extrema of effective action
for general field configurations

warping

$$\hat{g}_{\hat{\mu}\hat{\nu}}(x,y) = \begin{pmatrix} \sigma(y) g^{(4)}_{\mu\nu}(x) & , & 0 \\ 0 & , & g^{(D)}_{\alpha\beta}(y) \end{pmatrix}$$

most general metric with maximal four – dimensional symmetry

general form of quasi – static solutions (non-zero or zero cosmological constant)

effective four – dimensional action

$$W(x) = \int_y (g^{(D)}(y))^{1/2} \sigma^2(y) \mathcal{L}(x,y).$$

$$\Gamma = \int_x (g^{(4)})^{1/2} W.$$

flat phase : extrema of W

in higher dimensions , those exist generically !

extrema of W

- provide large class of solutions with vanishing four – dimensional constant
- dilatation transformation

$$W \to \alpha^{-4} W.$$

- extremum of W must occur for W=0 !
- effective cosmological constant is given by W

$$\Gamma = \int_x (g^{(4)})^{1/2} W.$$

$$W(x) = \int_y (g^{(D)}(y))^{1/2} \sigma^2(y) \mathcal{L}(x,y)$$

extremum of W must occur for W = 0

for any given solution : rescaled metric and dilaton is again a solution

$$\hat{g}_{\hat{\mu}\hat{\nu}} \to \alpha^2 \hat{g}_{\hat{\mu}\hat{\nu}}$$

$$\xi \to \alpha^{-\frac{d-2}{2}} \xi$$

for rescaled solution : $W \to \alpha^{-4} W.$

use $\alpha = 1 + \epsilon$

extremum condition :

$$\partial_\epsilon (1+\epsilon)^{-4} W_0 = 0 \quad \Rightarrow \quad W_0 = 0$$

extremum of W is extremum of effective action

$$\delta\Gamma = \int_{\hat{x}} (\hat{g}_0^{1/2} \delta W + \delta \hat{g}^{1/2} W_0) = 0$$

effective four – dimensional cosmological constant vanishes for extrema of W

expand effective 4 – d - action $\Gamma = \int_x (g^{(4)})^{1/2} W.$

in derivatives : $\Gamma^{(4)} = \int_x (g^{(4)})^{1/2} \left\{ V - \frac{\chi^2}{2} R^{(4)} + \dots \right\}$

4 - d - field equation $\chi^2 \left(R^{(4)}_{\mu\nu} - \frac{1}{2} R^{(4)} g^{(4)}_{\mu\nu} \right) = -V g^{(4)}_{\mu\nu}$

$$\Gamma = \Gamma^{(4)} = -V \int_x (g^{(4)})^{1/2}$$

$$\Gamma_0 = -\int_x (g^{(4)})^{1/2} \chi^2 \Lambda$$

Quasi-static solutions

- for arbitrary parameters of dilatation symmetric effective action :
- large classes of solutions with extremum of W and $W_{ext} = 0$ are explicitly known (flat phase)

  example : Minkowski space x D-dimensional torus
- only for certain parameter regions : further solutions without extremum of W exist :

(non-flat phase)

sufficient condition for vanishing cosmological constant

extremum of W exists

self tuning in higher dimensions

- involves infinitely many degrees of freedom !
- for arbitrary parameters in effective action :
  flat phase solutions are present
- extrema of W exist $$\bar{W} = \int_y (g^{(D)}(y))^{1/2} \sigma^2 \mathcal{L}(y).$$
- for flat 4-d-space : W is functional
  of internal geometry, independent of x $$\hat{g}_{\hat{\mu}\hat{\nu}}(y) = \begin{pmatrix} \sigma(y)\eta_{\mu\nu} & , & 0 \\ 0 & , & g^{(D)}_{\alpha\beta}(y) \end{pmatrix}$$
- solve field equations for internal metric and σ and ξ

Dark energy

if cosmic runaway solution has not yet reached fixed point :

dilatation symmetry of field equations

not yet exact

“ dilatation anomaly “

non-vanishing effective potential V in reduced four –dimensional theory

conclusions (2)

cosmic runaway towards fixed point may

solve the cosmological constant problem

and

account for dynamical Dark Energy

effective dilatation symmetry in full quantum theory

realized for fixed points

Cosmic runaway

- large class of cosmological solutions which never reach a static state : runaway solutions
- some characteristic scale χ changes with time
- effective dimensionless couplings flow with χ
  (similar to renormalization group)
- couplings either diverge or reach fixed point
- for fixed point : exact dilatation symmetry of full quantum field equations and corresponding quantum effective action

approach to fixed point

- dilatation symmetry not yet realized
- dilatation anomaly
- effective potential **V(φ)**
- exponential potential reflects anomalous dimension for vicinity of fixed point

$$V(\varphi) = M^4 \exp(- \alpha\varphi/M)$$

cosmic runaway and the problem of time varying constants

- It is not difficult to obtain quintessence potentials from higher dimensional (or string ?) theories
- Exponential form rather generic

  (after Weyl scaling)
- Potential goes to zero for $\varphi \rightarrow \infty$
- But most models show too strong time dependence of constants !

higher dimensional dilatation symmetry

generic class of solutions with

vanishing effective four-dimensional cosmological constant

and

constant effective dimensionless couplings

effective four – dimensional theory

characteristic length scales

l : scale of internal space

$$\int_y (g^{(D)})^{1/2} \sigma^2 = l^D.$$

$\bar{\xi}$: dilaton scale

$$\int_y (g^{(D)})^{1/2} \sigma^2 \xi^2 = l^D \bar{\xi}^2.$$

effective Planck mass

$$\Gamma^{(4)} = \int_x (g^{(4)})^{1/2} \left\{ V - \frac{\chi^2}{2} R^{(4)} + \dots \right\}$$

$$\chi^2 = l^D \bar{\xi}^2 - 2\tilde{G} l^{-2}$$

$$\tilde{G} = l^2 \int_y (g^{(D)})^{1/2} \sigma G.$$

dimensionless ,
depends on internal geometry ,
from expansion of F in R

effective potential

$$V = \tilde{Q}\bar{\xi}^2 l^{D-2} + \tilde{F} l^{-4}$$

$$\tilde{F} = l^4 \int_y (g^{(D)})^{1/2} \sigma^2 F(R^{(int)}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}})$$

$$\tilde{Q} = \frac{1}{2}\bar{\xi}^{-2} l^{2-D} \int_y (g^{(D)})^{1/2} \sigma^2 (\zeta \partial^\alpha \xi \partial_\alpha \xi - \xi^2 R^{(int)})$$

canonical scalar fields

consider field configurations with rescaled internal length scale and dilaton value

$$\varphi_\xi = \bar{\xi} l^{\frac{D}{2}} \; , \; \varphi_l = l^{-1}$$

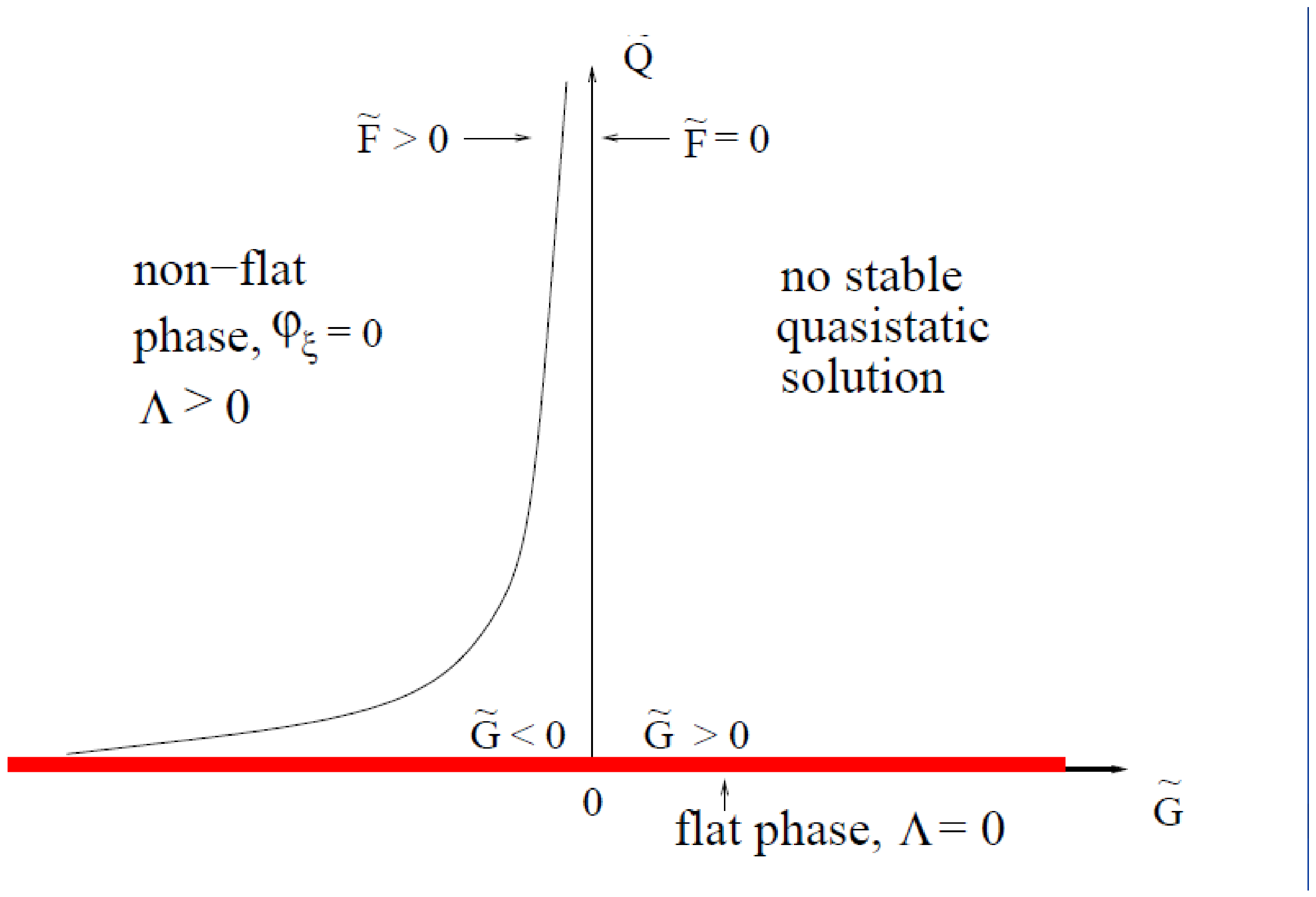
potential and effective Planck mass depend on scalar fields

$$V = \tilde{Q}\varphi_\xi^2\varphi_l^2 + \tilde{F}\varphi_l^4$$

$$\chi^2 = \varphi_\xi^2 - 2\tilde{G}\varphi_l^2$$

$$W = \tilde{Q}\varphi_\xi^2\varphi_l^2 + \tilde{F}\varphi_l^4 - 2\Lambda\varphi_\xi^2 + 4\tilde{G}\Lambda\varphi_l^2$$

phase diagram



stable solutions

phase structure of solutions

- solutions in flat phase exist for arbitrary values of effective parameters of higher dimensional effective action

- question : how "big" is flat phase

  (which internal geometries and warpings are possible beyond torus solutions)

- solutions in non-flat phase only exist for restricted parameter ranges

self tuning

for all solutions in flat phase :

self tuning of cosmological constant to zero !

self tuning

for simplicity : no contribution of F to V

$$V = \tilde{Q}\bar{\xi}^2 l^{D-2} + \tilde{F} l^{-4}$$

assume Q depends on parameter α , which characterizes internal geometry:

tuning required : $\frac{\partial \tilde{Q}(\alpha)}{\partial \alpha}_{|\alpha_0} = 0$ **and** $\tilde{Q}(\alpha_0) = 0$

self tuning in higher dimensions

Q depends on higher dimensional fields $\tilde{Q} = \tilde{R}[\alpha(y)]$

extremum condition
amounts to field equations $\frac{\delta \tilde{R}}{\delta \alpha(y)} = 0$

typical solutions depend on integration constants γ

solutions obeying boundary condition exist :

$$\tilde{R}[\alpha_0(y;\gamma_i)] = 0$$

self tuning in higher dimensions

- involves infinitely many degrees of freedom !
- for arbitrary parameters in effective action :
  flat phase solutions are present
- extrema of W exist

$$\bar{W} = \int_y (g^{(D)}(y))^{1/2} \sigma^2 \mathcal{L}(y).$$

- for flat 4-d-space : W is functional
  of internal geometry, independent of x

$$\hat{g}_{\hat{\mu}\hat{\nu}}(y) = \begin{pmatrix} \sigma(y)\eta_{\mu\nu} & , & 0 \\ 0 & , & g^{(D)}_{\alpha\beta}(y) \end{pmatrix}$$

- solve field equations for internal metric and σ and ξ

Dark energy

if cosmic runaway solution has not yet reached fixed point :

dilatation symmetry of field equations

not yet exact

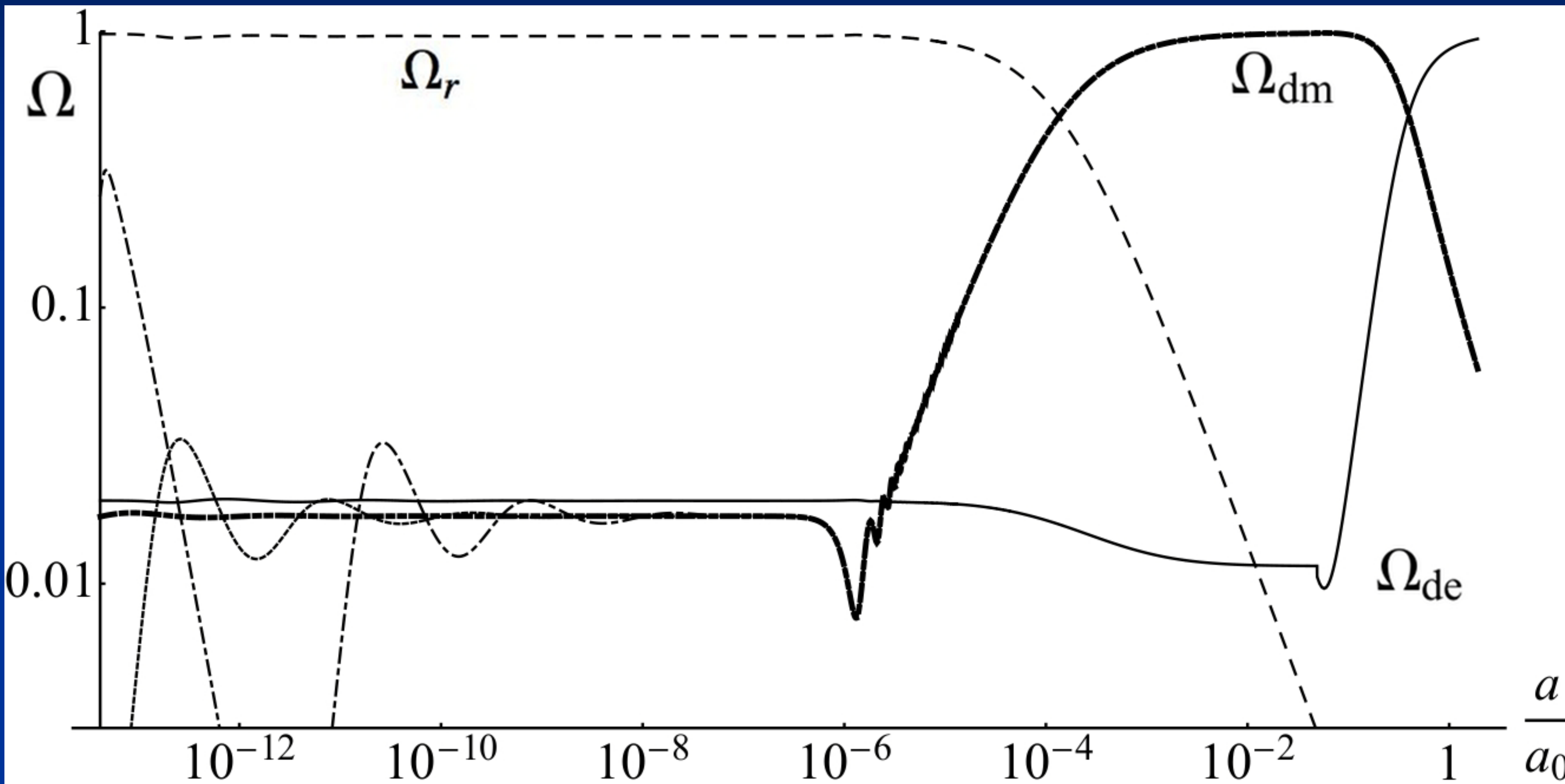
“ dilatation anomaly “

non-vanishing effective potential V in reduced four –dimensional theory

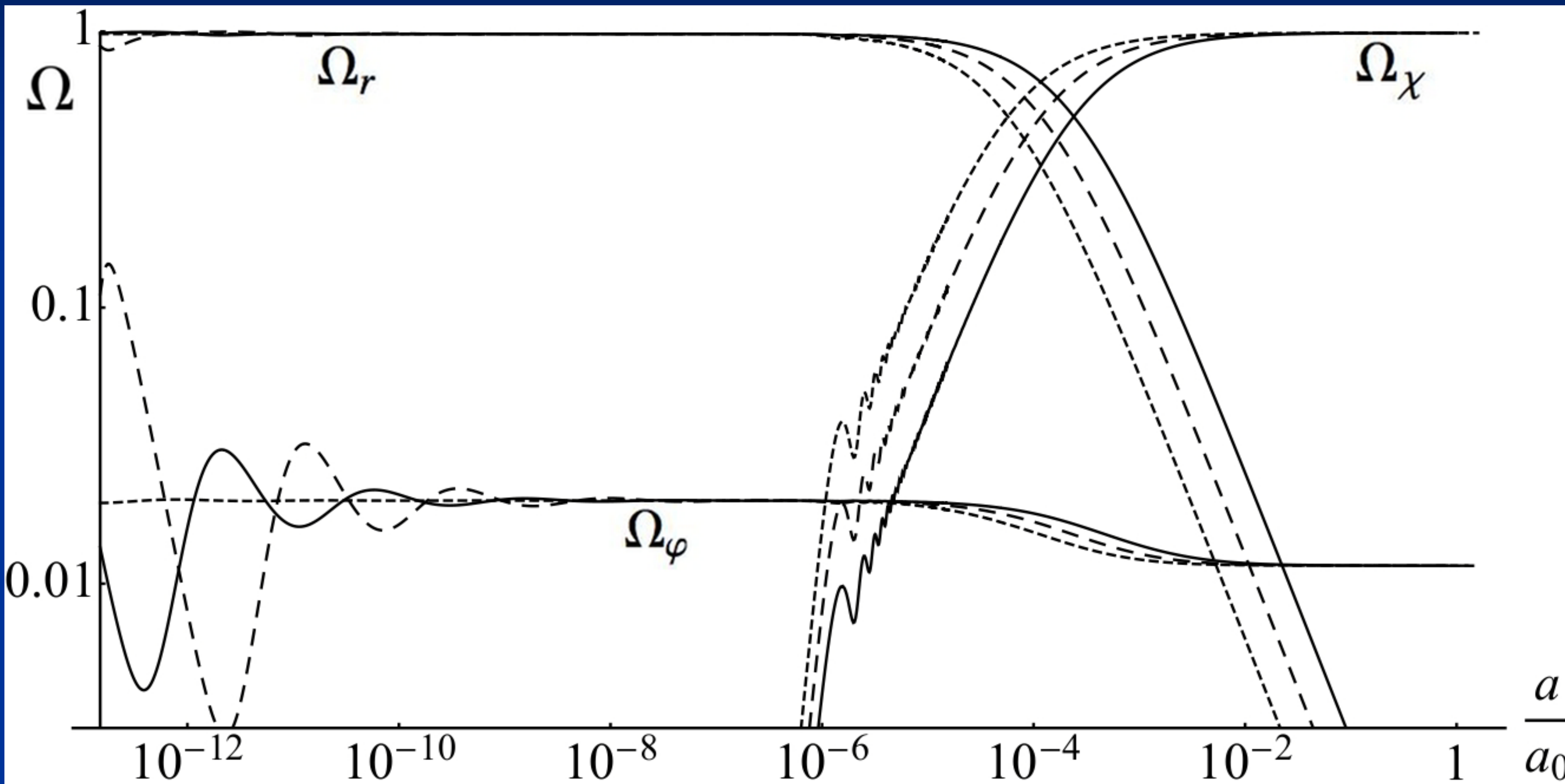
End

$$\rho_{dm} = \frac{1}{1+c^2}(\dot{\chi} - c\dot{\varphi})^2 + V(\varphi, \chi) - V(\varphi, g(\varphi))$$

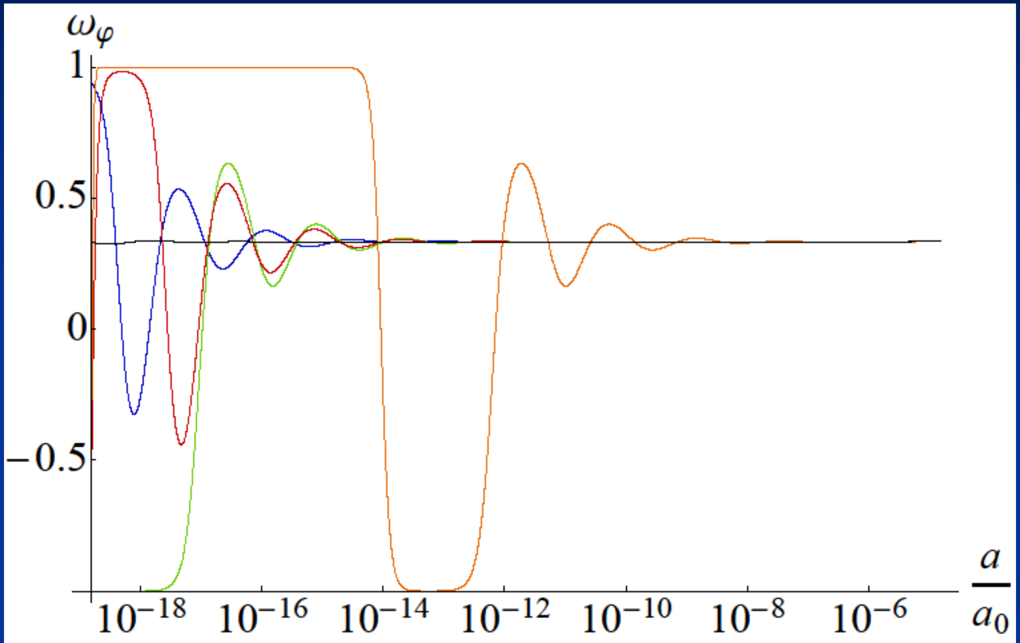
$$\rho_{de} = \frac{1}{1+c^2}(c\dot{\chi} + \dot{\varphi})^2 + V(\varphi, g(\varphi))$$



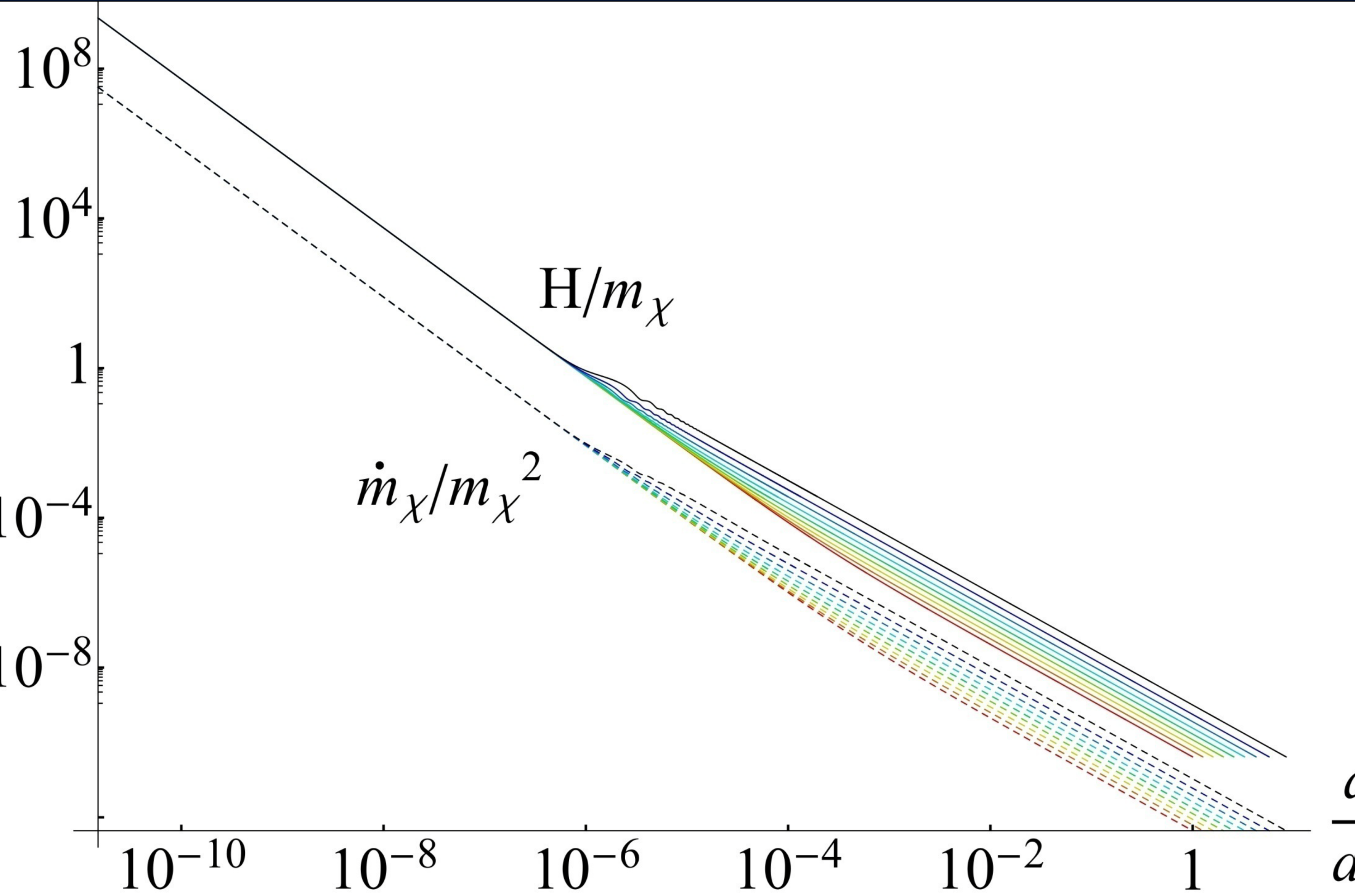
1
Ω
Ω_r
Ω_{dm}
0.1
0.01
Ω_{de}
$\frac{a}{a_0}$
10^{-12}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
1



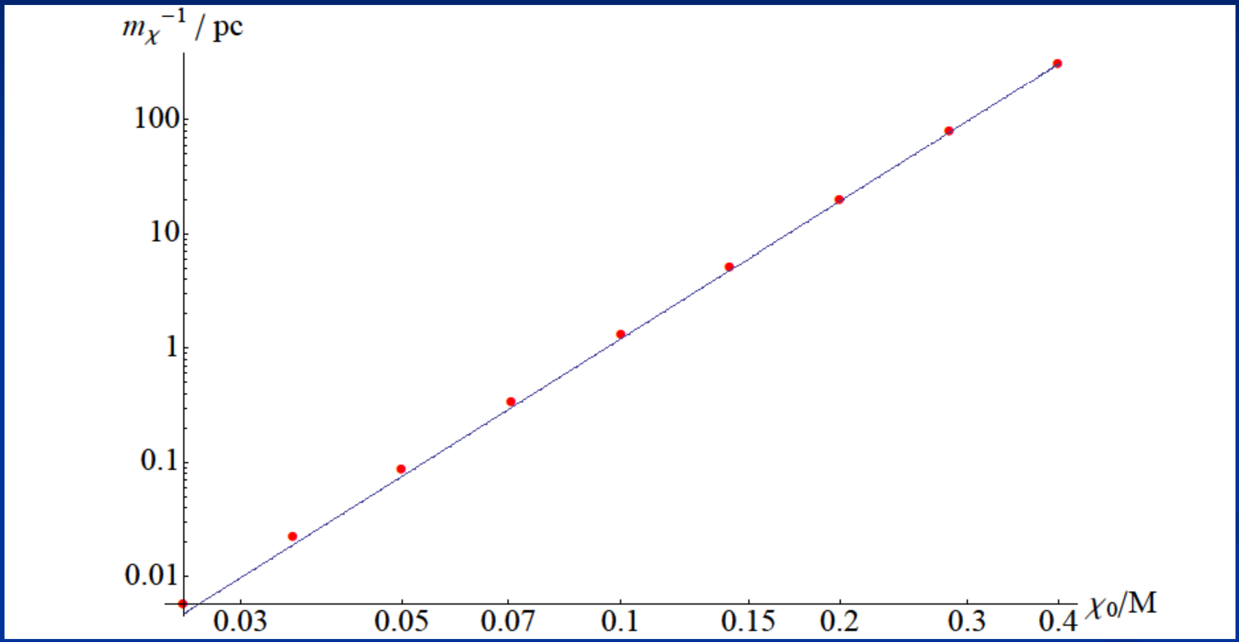
1
Ω
Ω_r
Ω_χ
0.1
Ω_φ
0.01
10^{-12}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
1
$\frac{a}{a_0}$



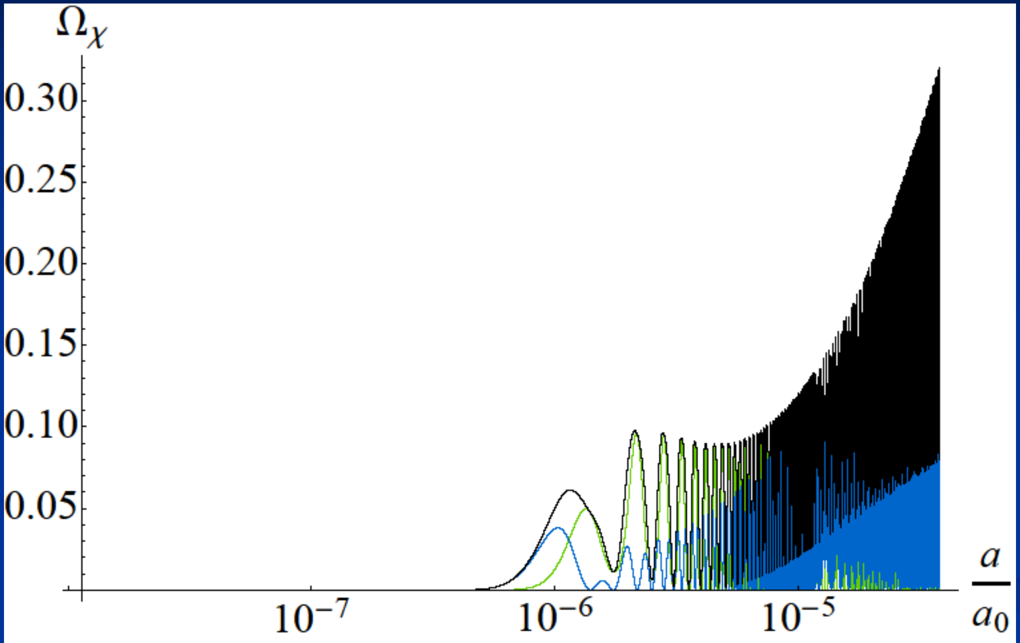
ω_φ
1
0.5
0
−0.5
10^{-18} 10^{-16} 10^{-14} 10^{-12} 10^{-10} 10^{-8} 10^{-6}
$\frac{a}{a_0}$



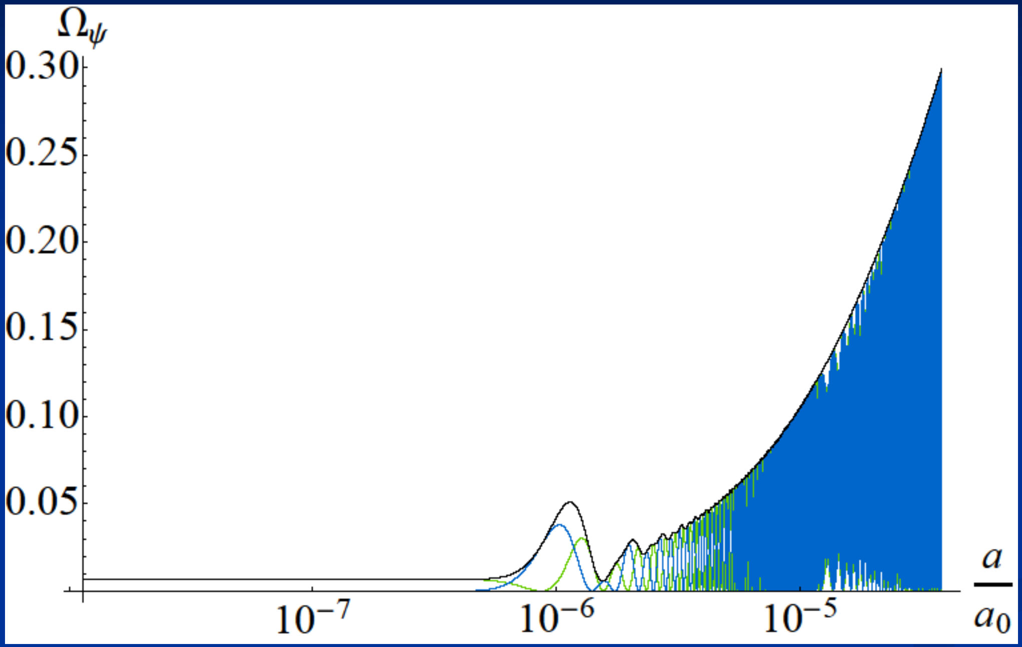
10^8
10^4
1
10^{-4}
10^{-8}
H/m_χ
$\dot{m}_\chi/m_\chi{}^2$
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
1



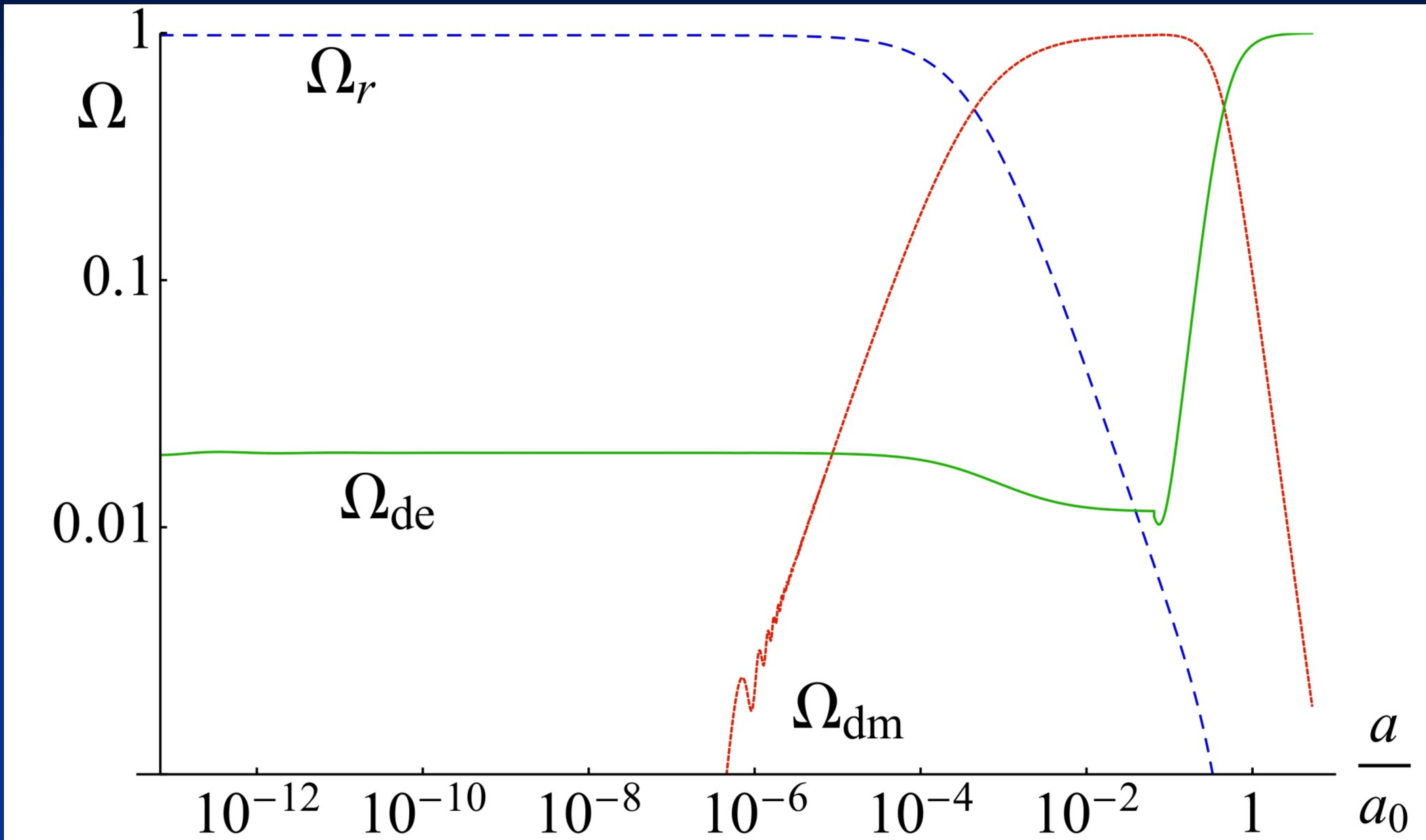
m_χ^{-1} / pc
100
10
1
0.1
0.01
0.03
0.05
0.07
0.1
0.15
0.2
0.3
0.4
χ_0/M

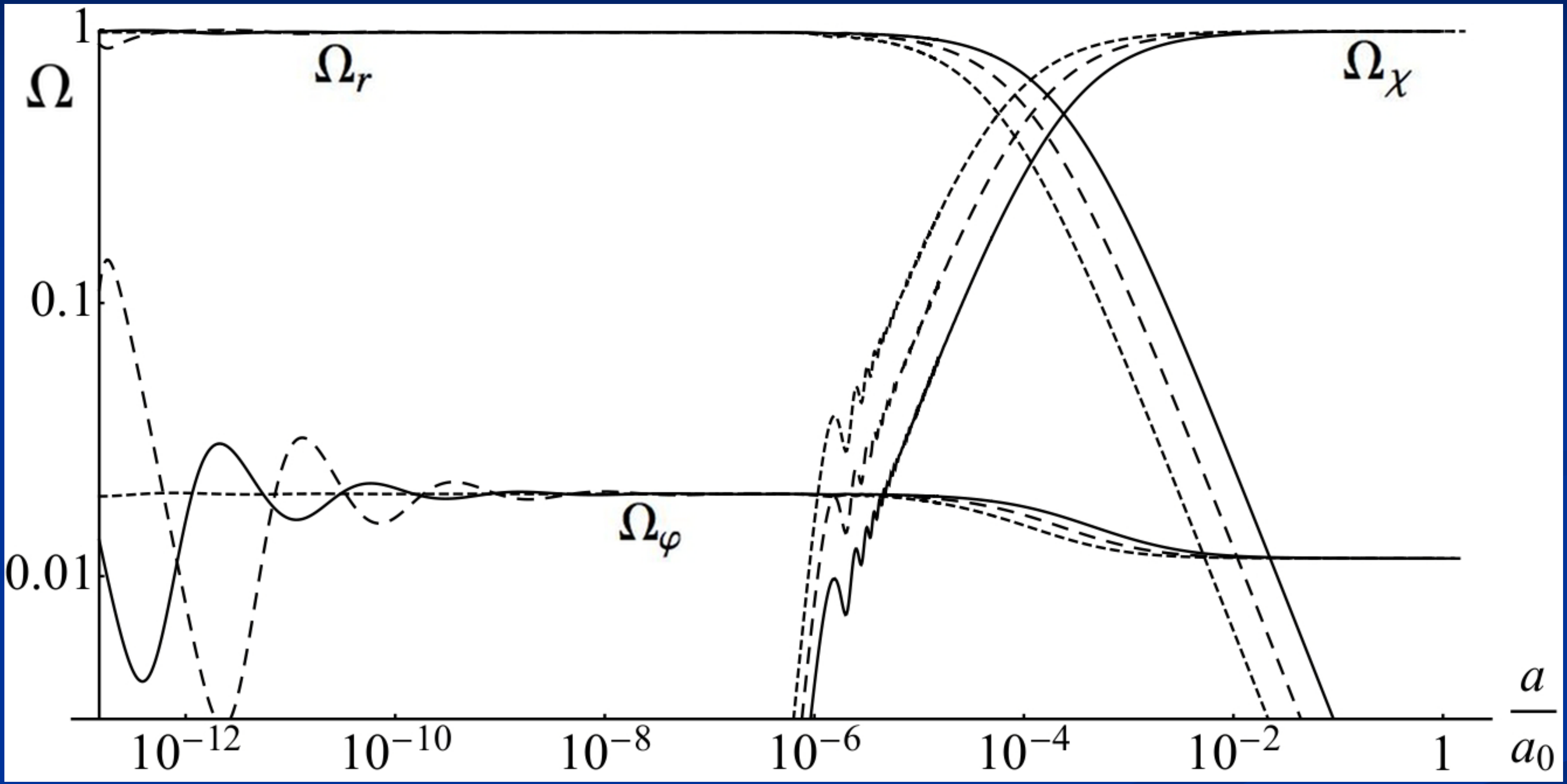


Ω_χ
0.30
0.25
0.20
0.15
0.10
0.05
10^{-7}
10^{-6}
10^{-5}
$\frac{a}{a_0}$



Ω_ψ
0.30
0.25
0.20
0.15
0.10
0.05
10^{-7}
10^{-6}
10^{-5}
$\frac{a}{a_0}$


1
0.1
0.01
Ω
Ω_r
Ω_de
Ω_dm
10^−12
10^−10
10^−8
10^−6
10^−4
10^−2
1
a/a_0



1
Ω
Ω_r
Ω_χ
0.1
Ω_φ
0.01
10^{-12}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
1
$\frac{a}{a_0}$