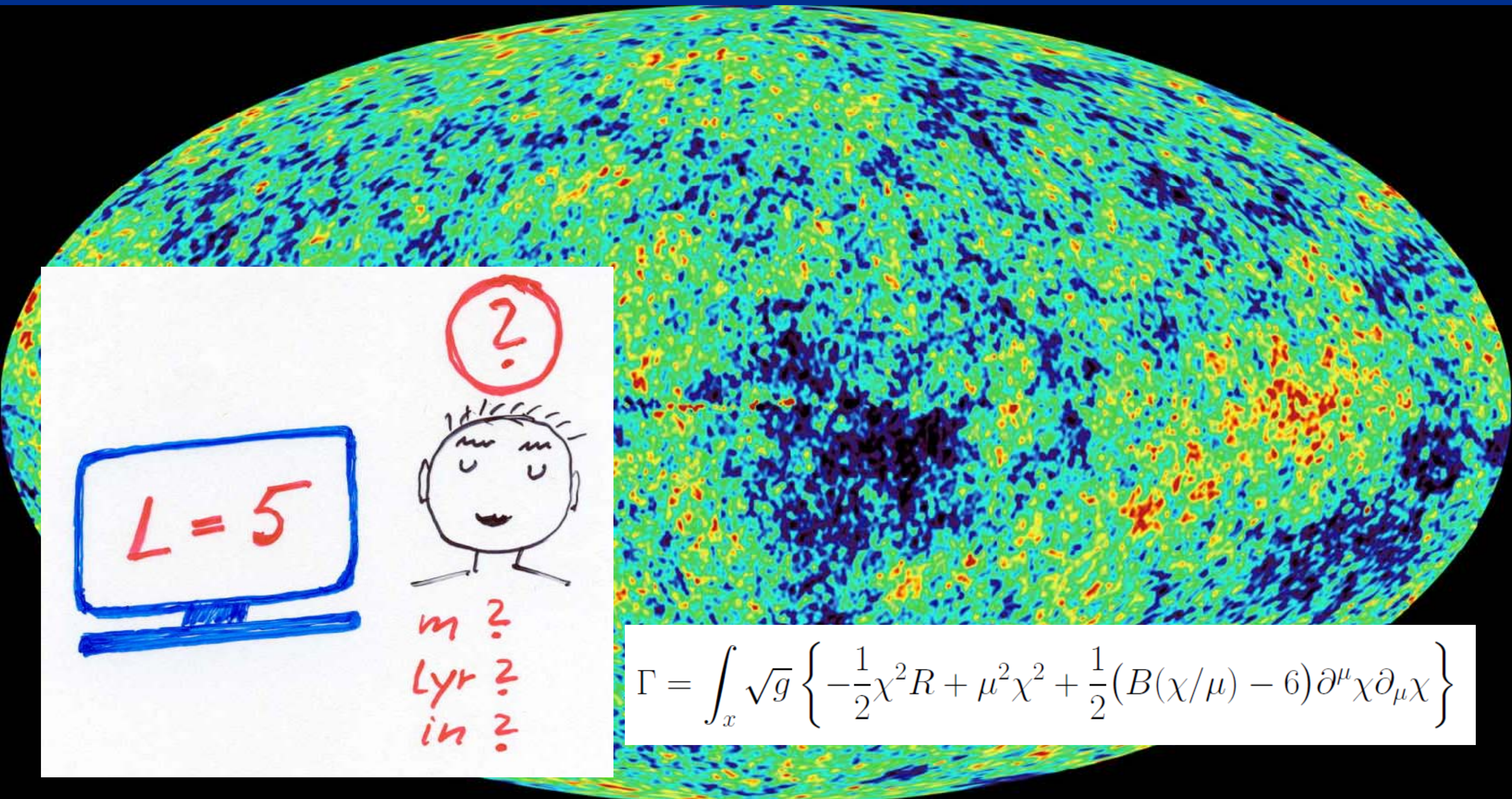
Scale symmetry – a link from quantum gravity to cosmology

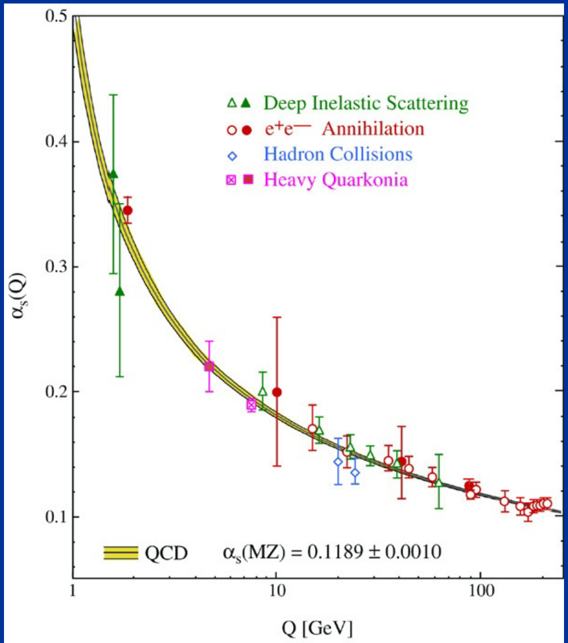


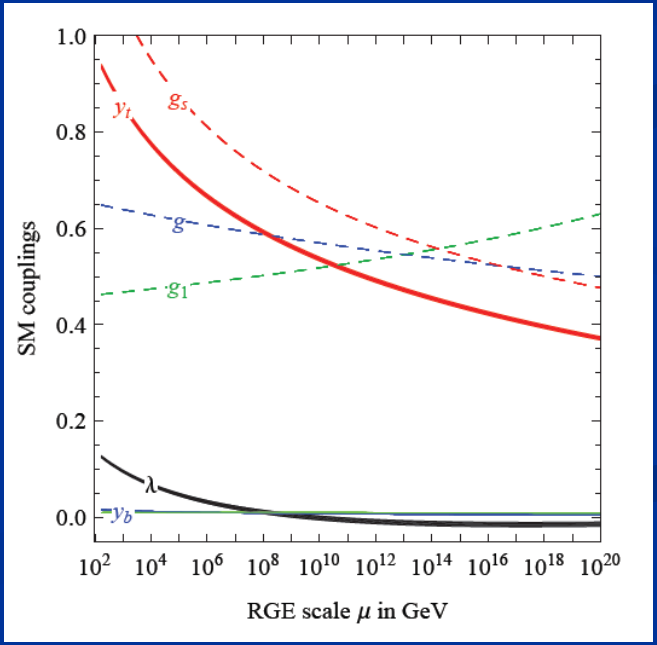
$$\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\big(B(\chi/\mu)-6\big)\partial^\mu\chi\,\partial_\mu\chi\right\}$$

scale symmetry

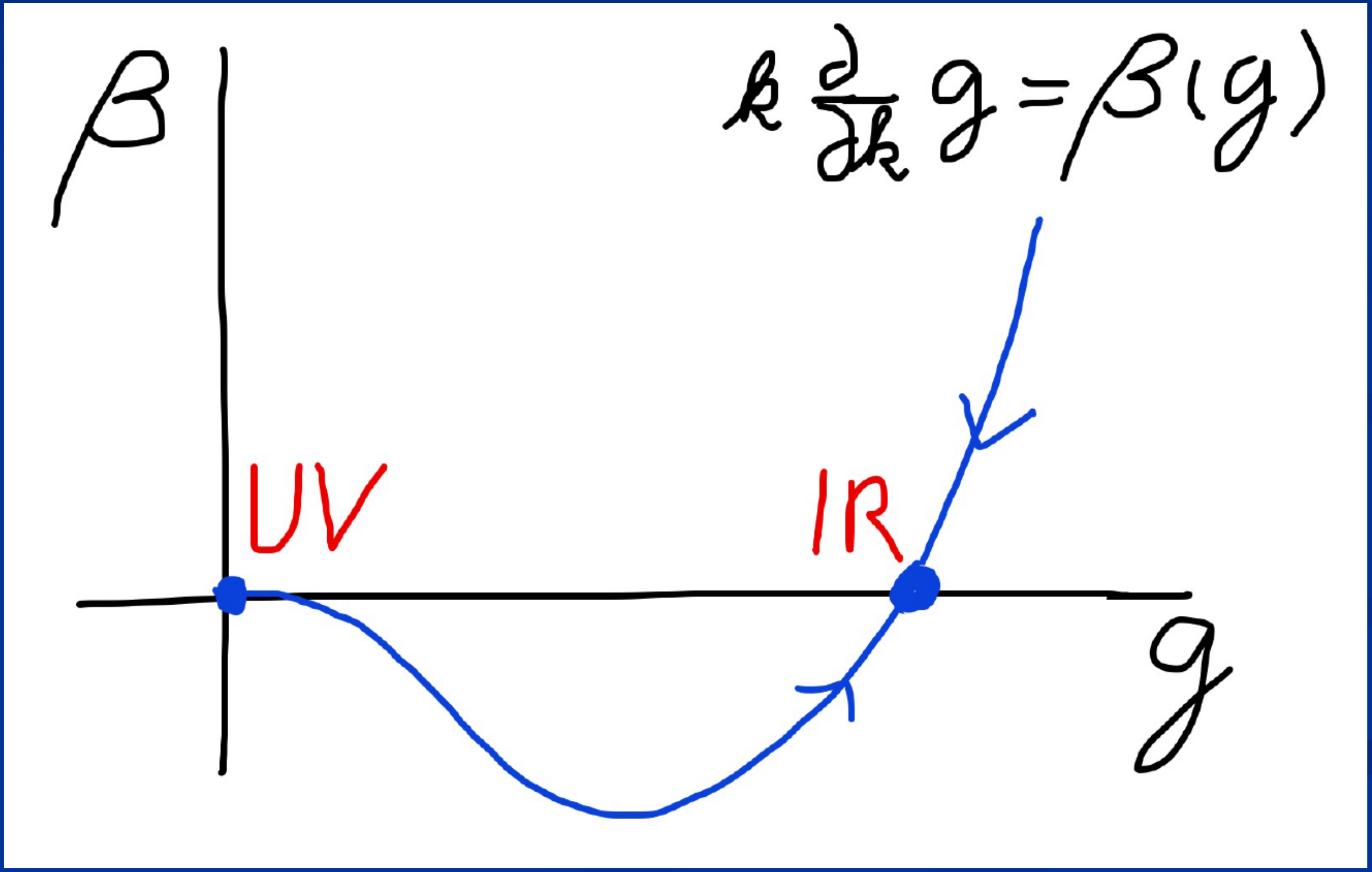
fluctuations induce running couplings

- violation of scale symmetry
- well known in QCD or standard model





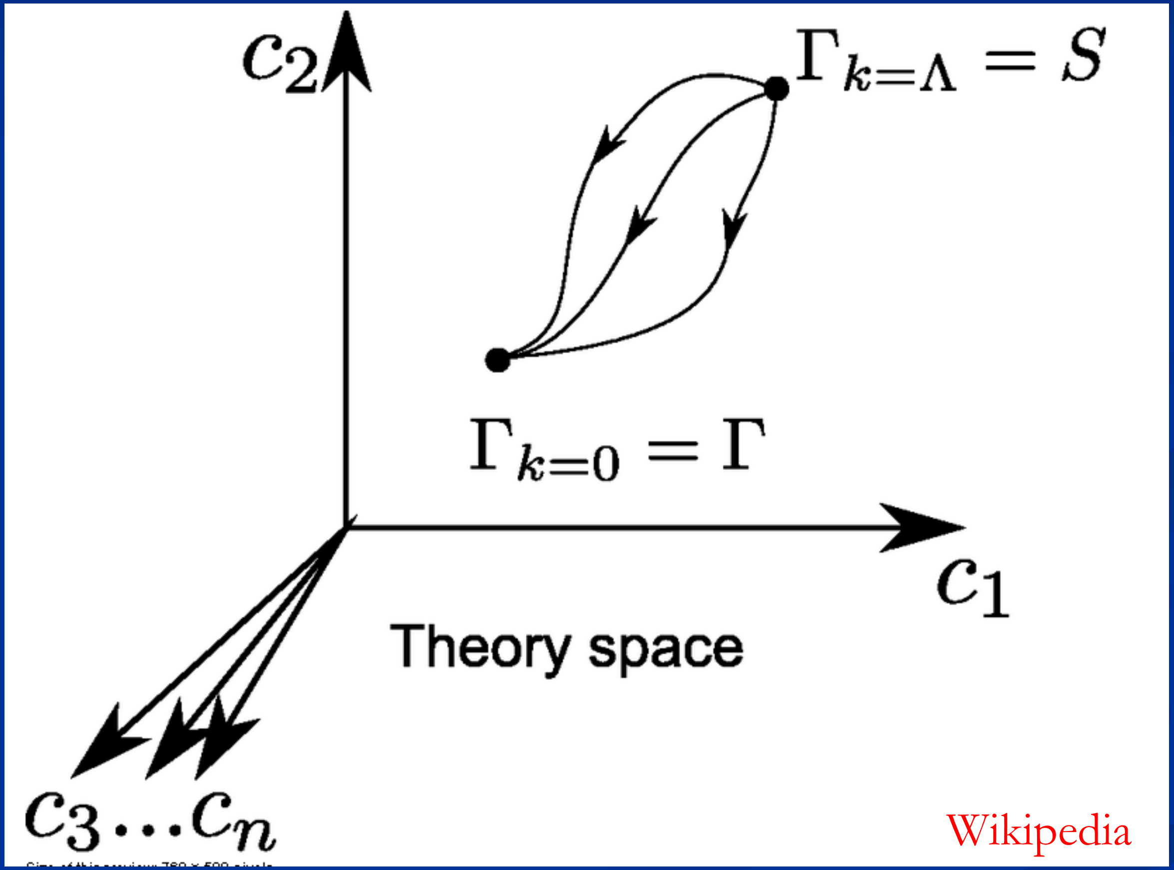
Fixed Points



Quantum scale symmetry

- quantum fluctuations violate scale symmetry
- running dimensionless couplings
- at fixed points , scale symmetry is exact !

functional renormalization : flowing action



quantum gravity –
the role of scale symmetry

Flowing action

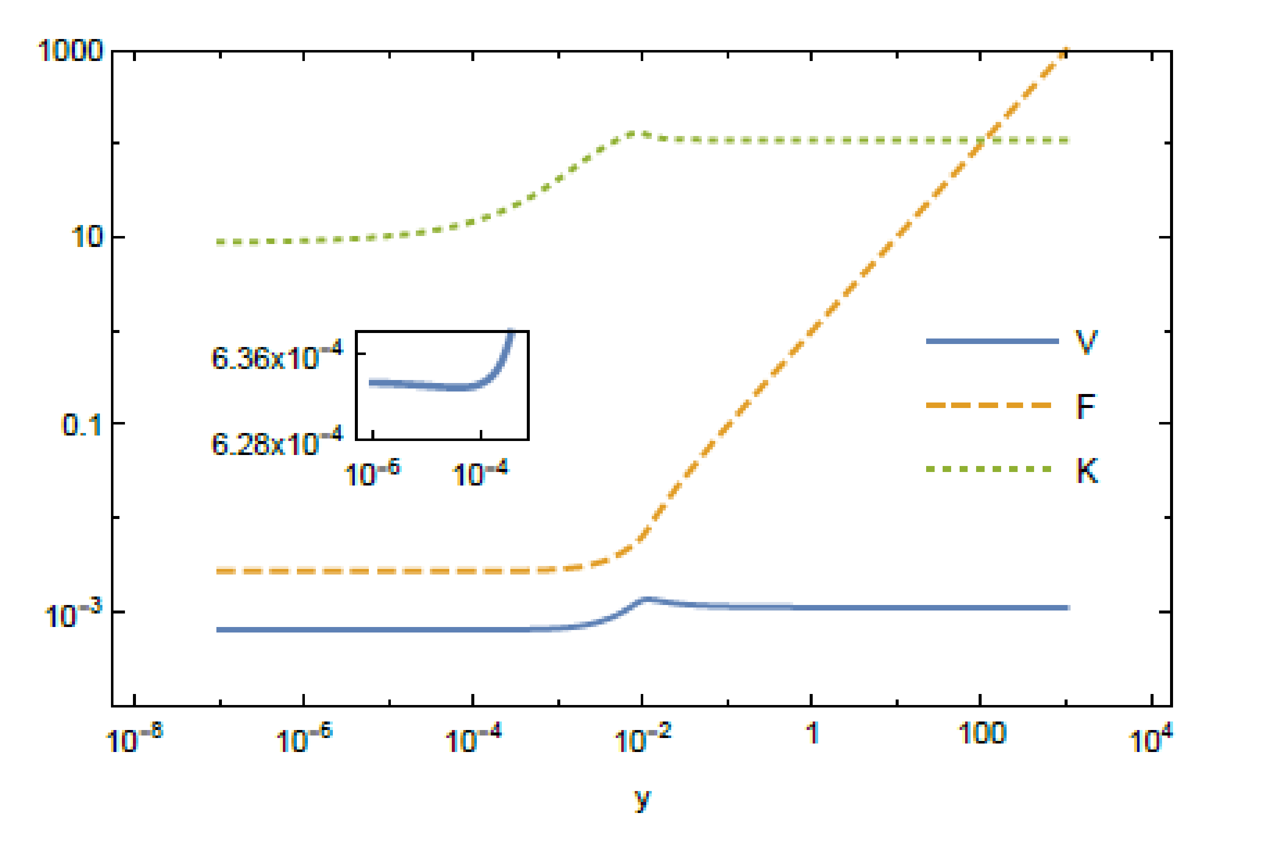
second order derivative expansion

$$\Gamma = \int_x \sqrt{g} \left(V(\chi^2) - \frac{1}{2} F(\chi^2)\, R + \frac{1}{2} K(\chi^2) g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi \right)$$

variable gravity
metric and scalar field

Scaling solutions for Dilaton Quantum Gravity

T. Henz, J. M. Pawlowski, and C. Wetterich

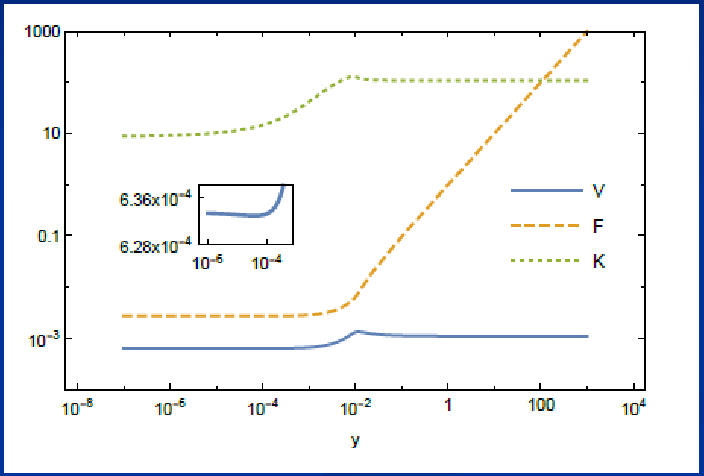


$$\Gamma = \int_x \sqrt{g}\left(V(\chi^2) - \frac{1}{2}F(\chi^2)\,R + \frac{1}{2}K(\chi^2)g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi\right)$$

$$y = \chi^2/k^2.$$

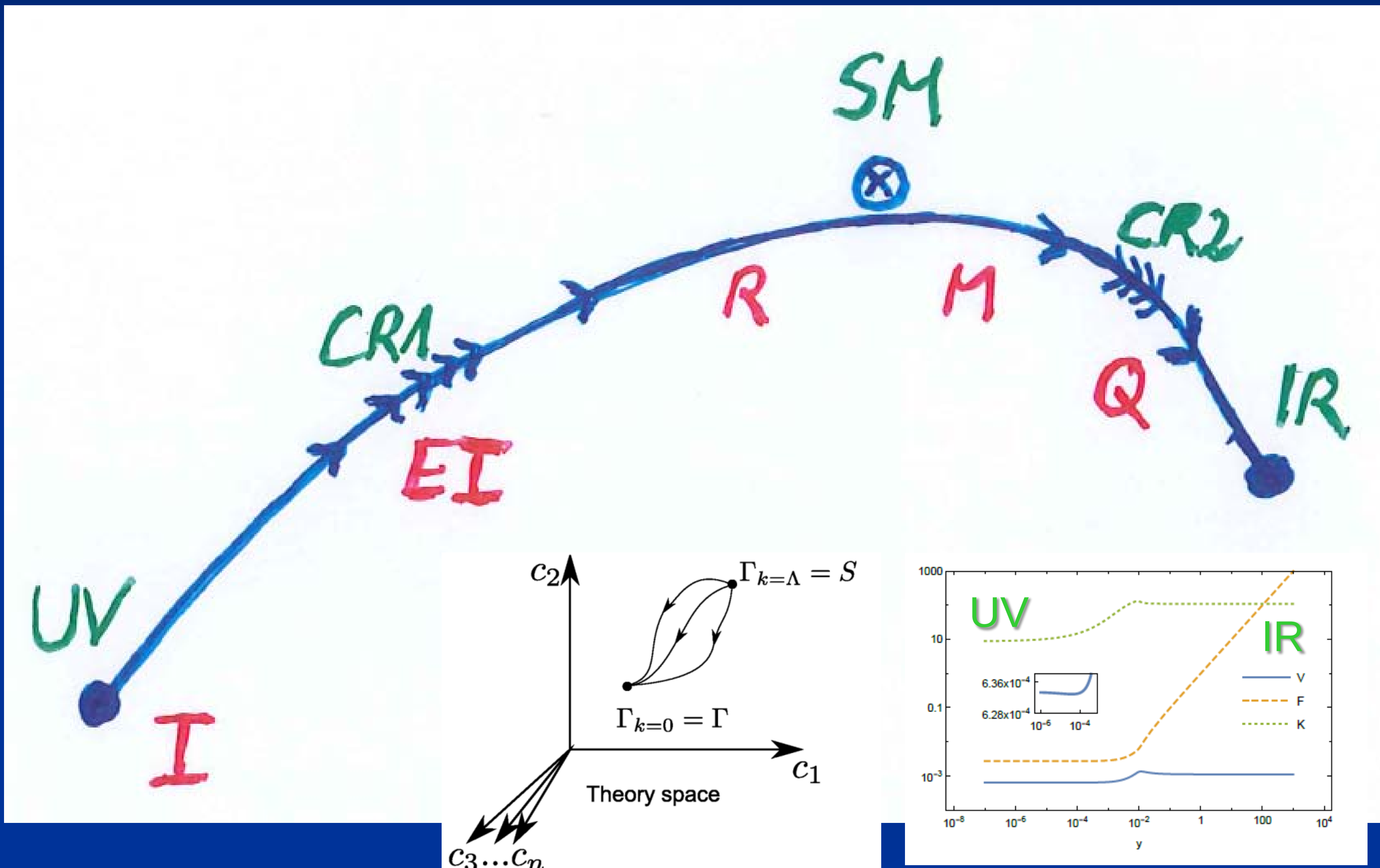
Anomalous dimension

solution with vanishing anomalous dimension



candidates with anomalous dimension more interesting

Crossover in quantum gravity



Origin of mass

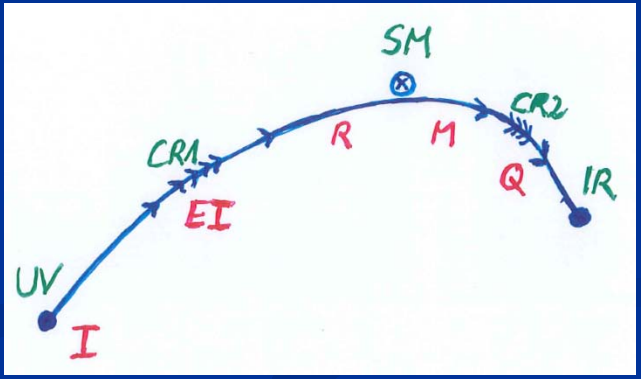
- UV fixed point : scale symmetry unbroken

  all particles are massless

- IR fixed point :

  scale symmetry spontaneously broken,

  massive particles , massless dilaton



- crossover : explicit mass scale μ important

- approximate SM fixed point : approximate scale symmetry spontaneously broken, massive particles , almost massless cosmon, tiny cosmon potential

Spontaneous breaking of scale symmetry

- expectation value of scalar field breaks scale symmetry spontaneously
- massive particles are compatible with scale symmetry
- in presence of massive particles : sign of exact scale symmetry is exactly massless Goldstone boson – the dilaton

Approximate scale symmetry near fixed points

- UV : approximate scale invariance of primordial fluctuation spectrum from inflation

- IR : cosmon is pseudo Goldstone boson of
  spontaneously broken scale symmetry,
  tiny mass,
  responsible for dynamical Dark Energy

Asymptotic safety

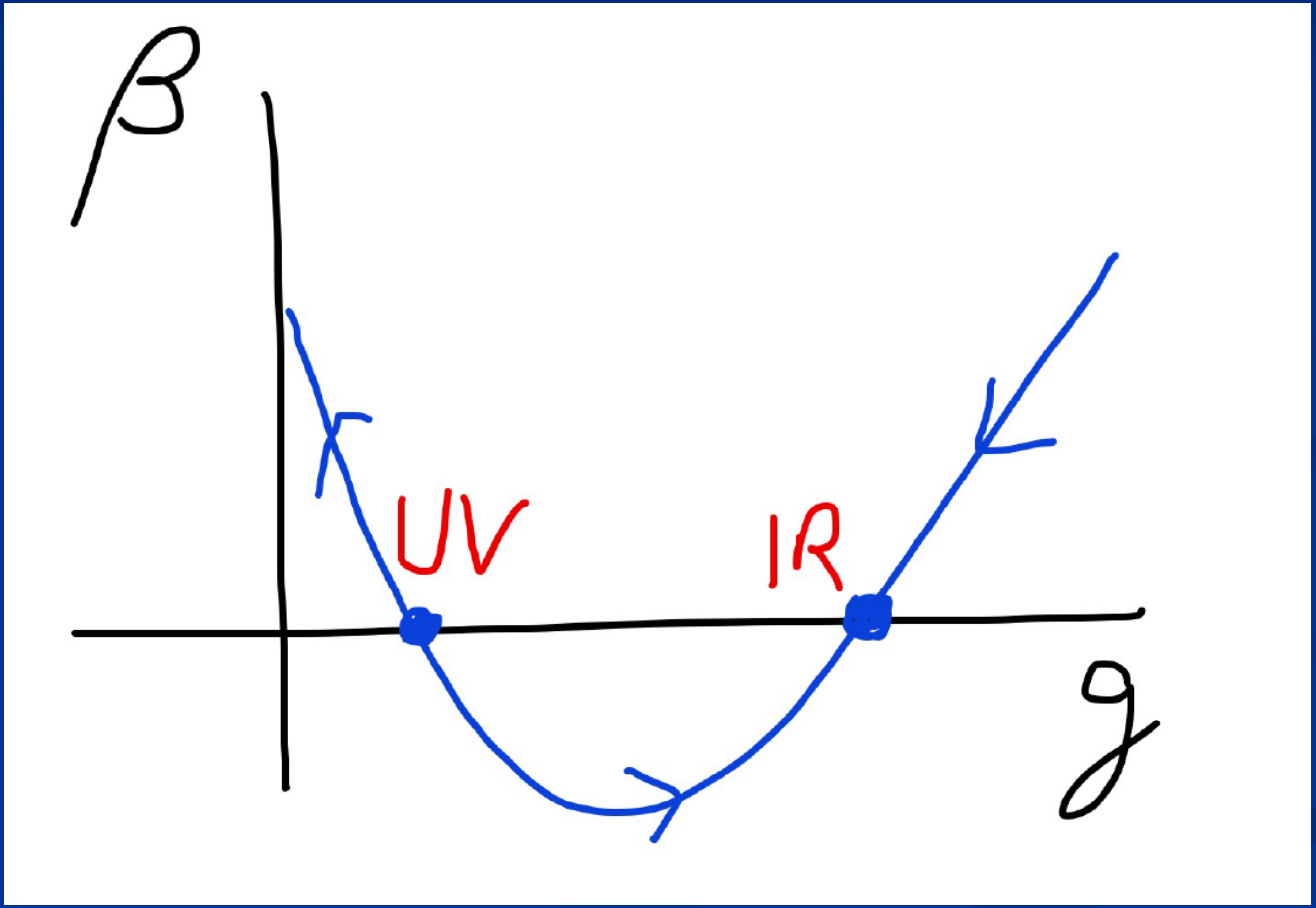
if UV fixed point exists :

quantum gravity is

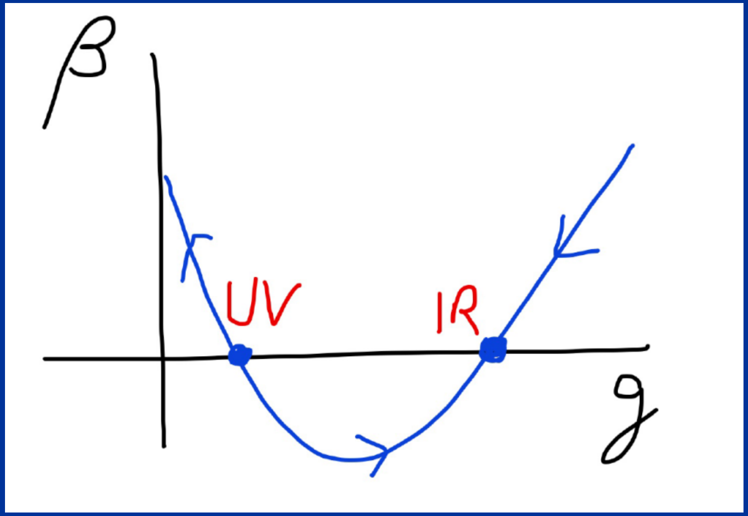
non-perturbatively renormalizable !

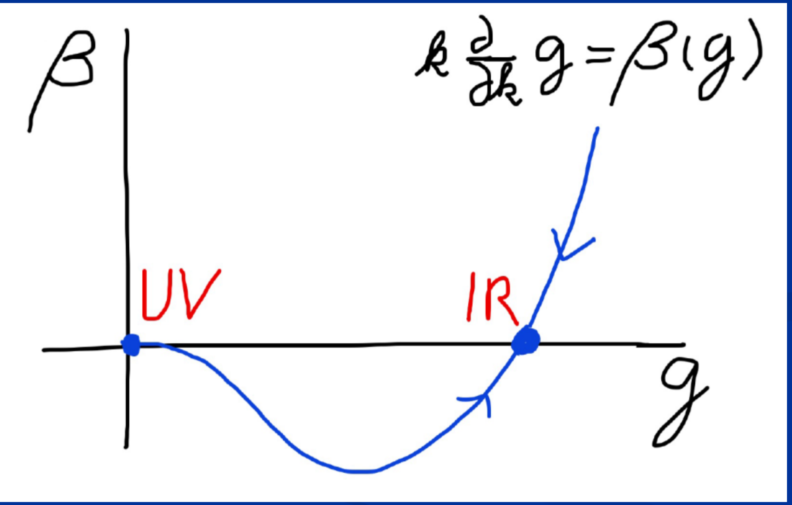
S. Weinberg , M. Reuter

Asymptotic safety



Asymptotic safety Asymptotic freedom





a prediction…

Asymptotic safety of gravity and the Higgs boson mass

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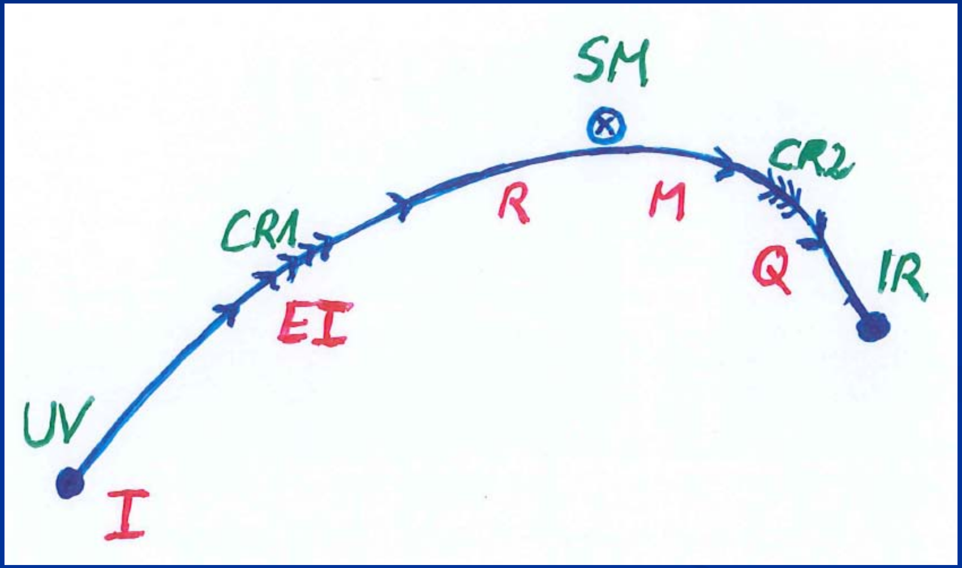
12 January 2010

Abstract

There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalous dimension $A_\lambda > 0$ the running of the quartic scalar self interaction λ at scales beyond the Planck mass is determined by a fixed point at zero. This results in $m_H = m_{\rm min} = 126$ GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well.

s in $m_H = m_{\rm min} = 126$ GeV, with o

Possible consequences of crossover in quantum gravity



Realistic model for inflation and dark energy with single scalar field

Variable Gravity

$$\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi\right\}$$

quantum effective action,
variation yields field equations

Einstein gravity : $\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}\right.$ M^2 R $\left.\right\}$

Variable Gravity

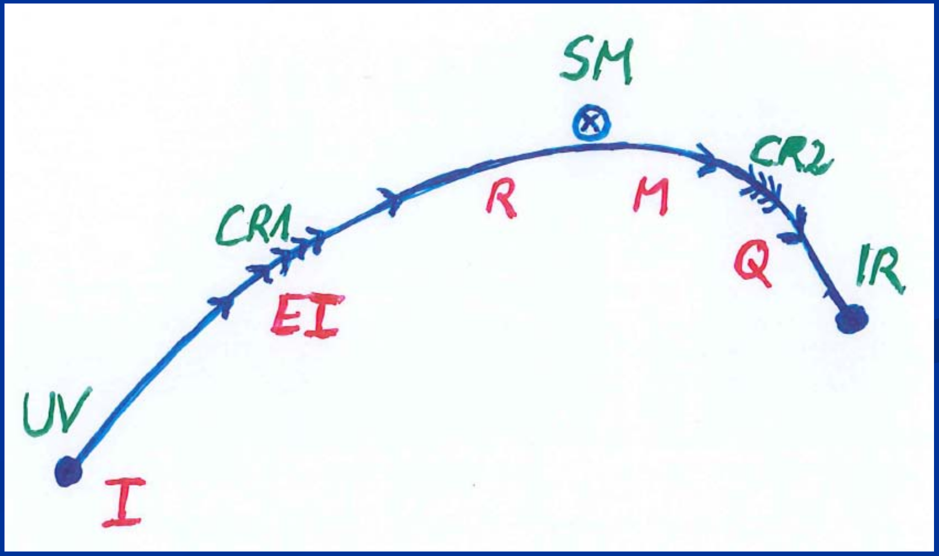
- Scalar field coupled to gravity
- Effective Planck mass depends on scalar field
- Simple quadratic scalar potential involves intrinsic mass μ
- Nucleon and electron mass proportional to dynamical Planck mass

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\big(B(\chi/\mu) - 6\big)\partial^\mu\chi\partial_\mu\chi \right\}$$

Cosmological solution : crossover from UV to IR fixed point

- Dimensionless functions as B depend only on ratio μ/χ .
- IR: $\mu \to 0$, $\chi \to \infty$
- UV: $\mu \to \infty$, $\chi \to 0$

Cosmology makes crossover between fixed points by variation of χ .



renormalization flow and cosmological evolution

- renormalization flow as function of μ

  is mapped by dimensionless functions to

- field dependence of effective action on scalar field χ

  translates by solution of field equation to

- dependence of cosmology an time t or η

Simplicity

simple description of all cosmological epochs

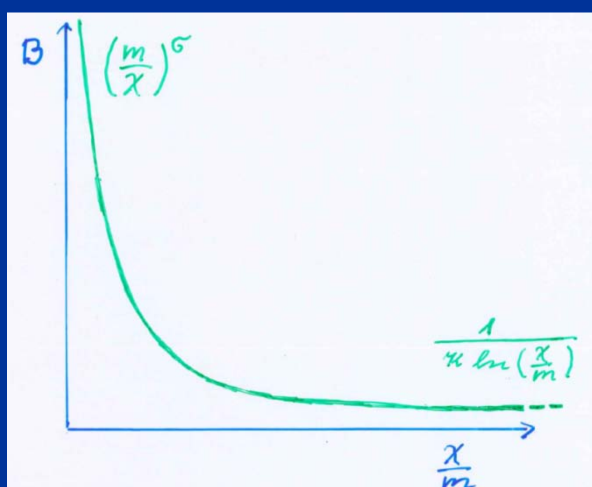
natural incorporation of Dark Energy :

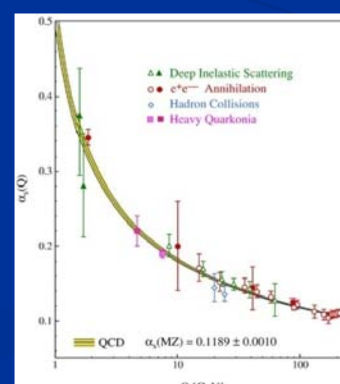
- inflation
- Early Dark Energy
- present Dark Energy dominated epoch

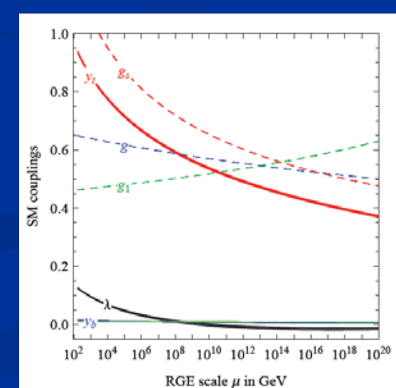
running coupling B

$$\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi\right\}$$

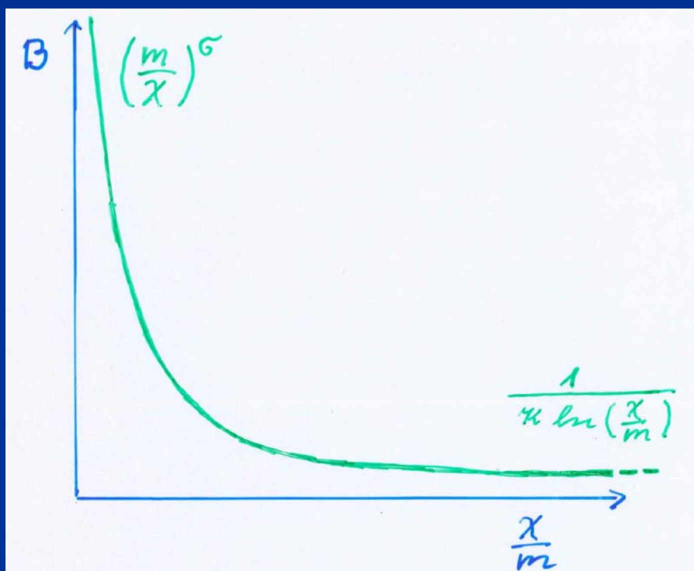
- B varies if intrinsic scale μ changes
- similar to QCD or standard model

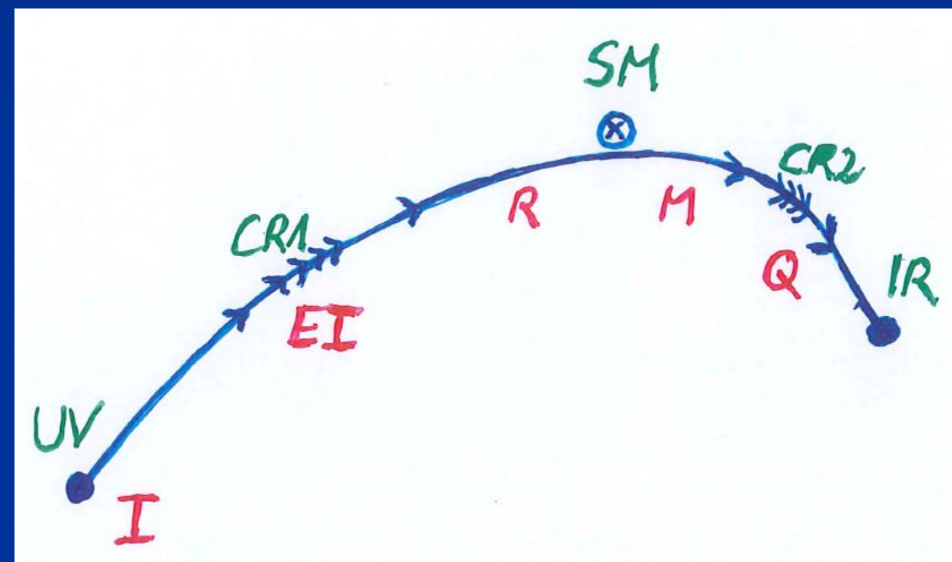




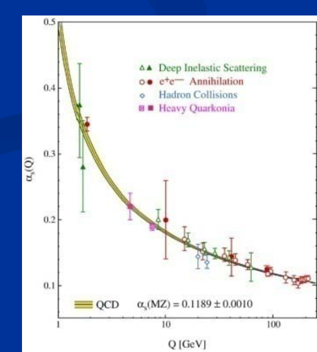


Kinetial B :
Crossover between two fixed points

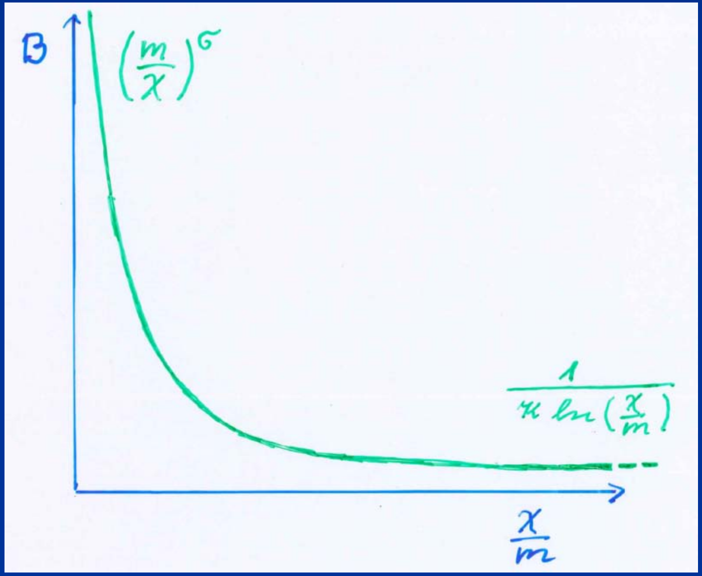




$$\mu \frac{\partial B}{\partial \mu} = \frac{\kappa \sigma B^2}{\sigma + \kappa B}$$



Kinetial B :
Crossover between two fixed points



running coupling obeys flow equation

$$\mu \frac{\partial B}{\partial \mu} = \frac{\kappa \sigma B^2}{\sigma + \kappa B}$$

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[\ln \left(\frac{\chi}{\mu} \right) - c_t \right] = \kappa \ln \left(\frac{\chi}{m} \right)$$

m : scale of crossover

can be exponentially larger than intrinsic scale μ

Cosmological solution

- derive field equation from effective action of variable gravity
- solve them for homogenous and isotropic metric and scalar field
- scalar field χ vanishes in the infinite past
- scalar field χ diverges in the infinite future

No tiny dimensionless parameters
(except gauge hierarchy)

- one mass scale $\mu = 2 \cdot 10^{-33}$ eV
- one time scale $\mu^{-1} = 10^{10}$ yr
- Planck mass does not appear as parameter
- Planck mass grows large dynamically

Particle masses change with time

At SM fixed point :

- All particle masses (except for neutrinos) are proportional to scalar field χ .
- Scalar field varies with time – so do particle masses.
- Ratios of particle masses are independent of χ and therefore remain constant.
- Compatibility with observational bounds on time dependence of particle mass ratios.
- Dimensionless couplings are independent of χ .

Four-parameter model

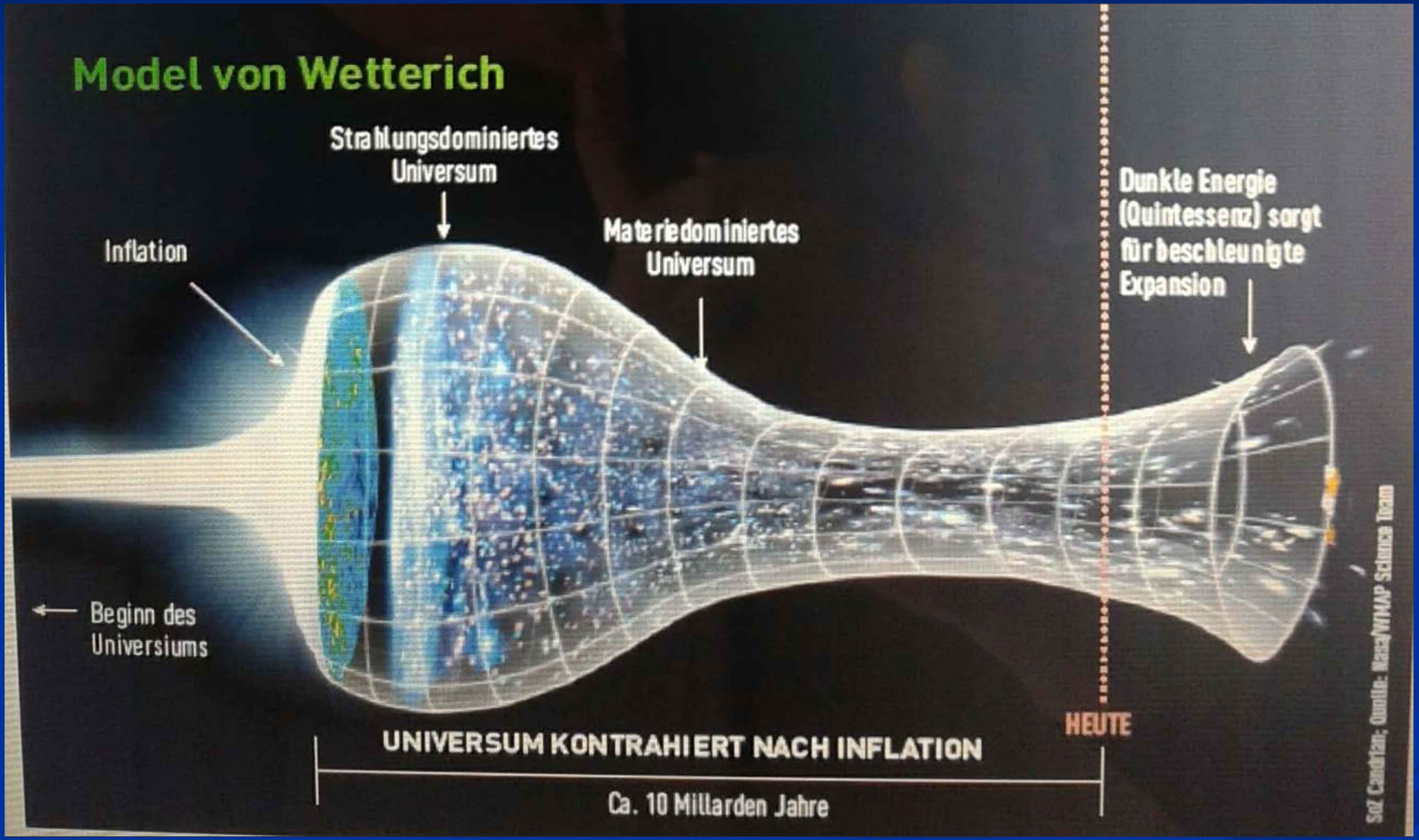
- model has four dimensionless parameters
- three in kinetial B :

  $\sigma \sim 2.5$

  $\varkappa \sim 0.5$

  $c_t \sim 14$ (or m/μ)
- one parameter for present growth rate of neutrino mass over electron mass : $\gamma \sim 8$
- + standard model particles and dark matter : sufficient for realistic cosmology from inflation to dark energy
- no more free parameters than ΛCDM

Strange evolution of Universe



Sonntagszeitung Zürich , Laukenmann

Slow Universe

Asymptotic solution :

$$H = \frac{\mu}{\sqrt{3}} \, , \ \chi = \frac{3^{\frac{1}{4}} m}{2\sqrt{\mu}} (t_c - t)^{-\frac{1}{2}}$$

$$\mu = 2 \cdot 10^{-33} \, \text{eV}$$

Expansion or shrinking always slow ,
characteristic time scale of the order of the age of the Universe : $t_{ch} \sim \mu^{-1} \sim$ 10 billion years !

Hubble parameter of the order of present Hubble parameter for all times , including inflation and big bang !

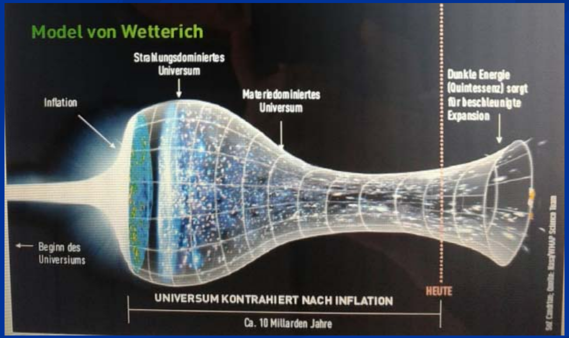
Slow increase of particle masses !

Eternal Universe

Asymptotic solution in freeze frame :

$$H = \frac{\mu}{\sqrt{3}} \ , \ \chi = \frac{3^{\frac{1}{4}} m}{2\sqrt{\mu}} (t_c - t)^{-\frac{1}{2}}$$

- solution valid back to the infinite past in physical time
- no singularity



- physical time to infinite past is infinite

asymptotically vanishing cosmological „constant“

- What matters : Ratio of potential divided by fourth power of Planck mass

$$\frac{V}{\chi^4} = \frac{\mu^2 \chi^2}{\chi^4} = \frac{\mu^2}{\chi^2}$$

- vanishes for $\chi \to \infty$!

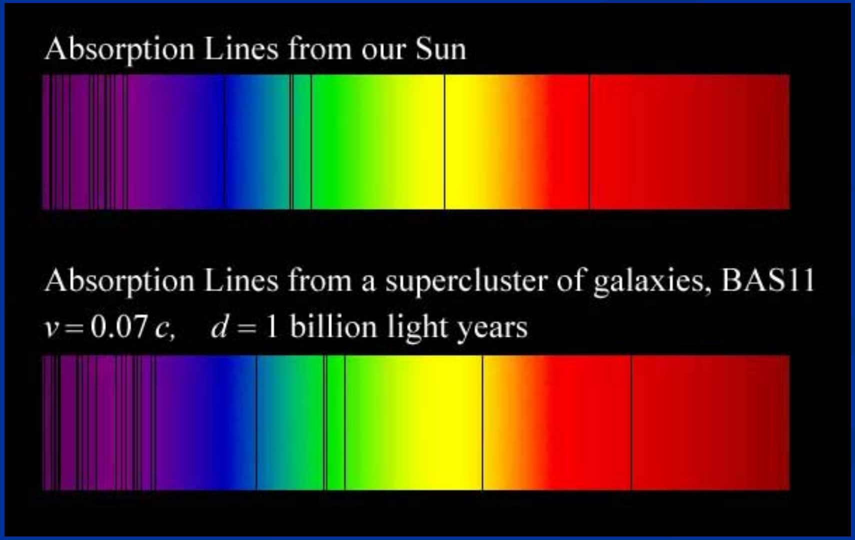
small dimensionless number ?

- needs two intrinsic mass scales
- V and M (cosmological constant and Planck mass)
- variable Planck mass moving to infinity , with fixed V: ratio vanishes asymptotically !

Do we know that the Universe expands ?

instead of redshift due to expansion :

smaller frequencies have been emitted in the past, because electron mass was smaller !



What is increasing ?

Ratio of distance between galaxies
over size of atoms !

atom size constant : expanding geometry

alternative : shrinking size of atoms

Hot plasma ?

- Temperature in radiation dominated Universe : $T \sim \chi^{1/2}$ smaller than today
- Ratio temperature / particle mass : $T / m_p \sim \chi^{-1/2}$ larger than today
- T/m_p counts ! This ratio decreases with time.
- Nucleosynthesis , CMB emission as in standard cosmology !

Model is compatible with present observations

Together with variation of neutrino mass over electron mass in present cosmological epoch :

model is compatible with all present observations

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi \right\}$$

$$B^{-1} - \frac{\kappa}{\sigma}\ln B = \kappa\left[\ln\left(\frac{\chi}{\mu}\right) - c_t\right] = \kappa\ln\left(\frac{\chi}{m}\right)$$

Einstein frame

- "Weyl scaling" maps variable gravity model to Universe with fixed masses and standard expansion history.
- Exact equivalence of different frames !
- Standard gravity coupled to scalar field.
- Only neutrino masses are growing.

Einstein frame

Weyl scaling :

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} \ , \ \varphi = \frac{2M}{\alpha} \ln\left(\frac{\chi}{\mu}\right)$$

effective action in Einstein frame :

$$\Gamma = \int_x \sqrt{g'} \left\{ -\frac{1}{2} M^2 R' + V'(\varphi) + \frac{1}{2} k^2(\varphi) \partial^\mu \varphi \partial_\mu \varphi \right\}$$

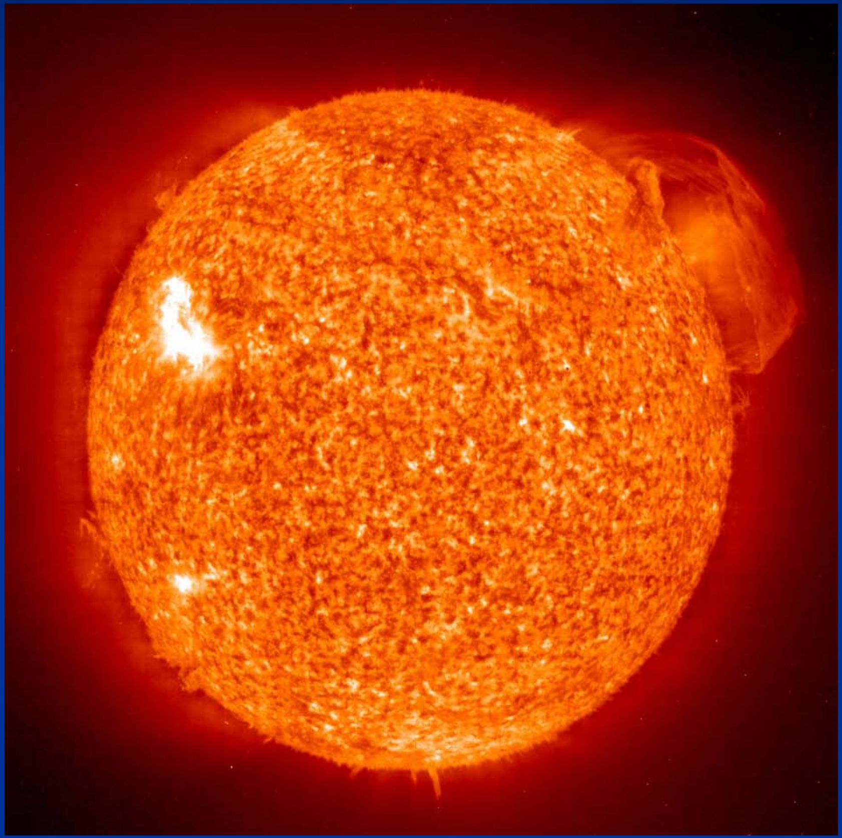
$$V'(\varphi) = M^4 \exp\left(-\frac{\alpha\varphi}{M}\right)$$

$$k^2 = \frac{\alpha^2 B}{4}$$

Field relativity : different pictures of cosmology

- same physical content can be described by different pictures
- related by field – redefinitions , e.g. Weyl scaling , conformal scaling of metric
- which picture is usefull ?

Big bang or freeze ?




Big bang or freeze ?
just two ways of looking at same physics

Infinite past : slow inflation

$\sigma = 2$: field equations

$$\ddot{\chi} + \left(3H + \frac{1}{2}\frac{\dot{\chi}}{\chi}\right)\dot{\chi} = \frac{2\mu^2\chi^2}{m}$$

$$H = \sqrt{\frac{\mu^2}{3} + \frac{m\dot{\chi}^2}{6\chi^3}} - \frac{\dot{\chi}}{\chi}$$

approximate solution

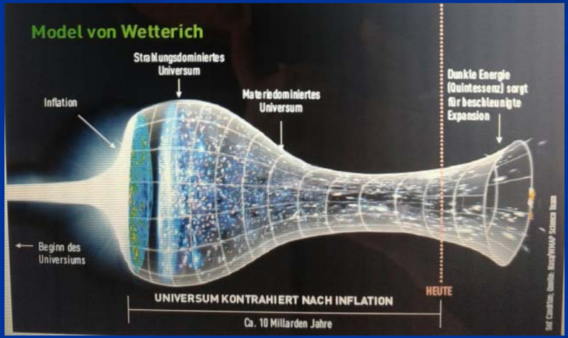
$$H = \frac{\mu}{\sqrt{3}}\ , \ \ \chi = \frac{3^{\frac{1}{4}}m}{2\sqrt{\mu}}(t_c - t)^{-\frac{1}{2}}$$

Eternal Universe

Asymptotic solution in freeze frame :

$$H = \frac{\mu}{\sqrt{3}} \; , \; \chi = \frac{3^{\frac{1}{4}} m}{2\sqrt{\mu}} (t_c - t)^{-\frac{1}{2}}$$

- solution valid back to the infinite past in physical time
- no singularity



- physical time to infinite past is infinite

Physical time

field equation for scalar field mode

$$(\partial_\eta^2 + 2Ha\partial_\eta + k^2 + a^2m^2)\varphi_k = 0$$

$$\varphi_k = \frac{\tilde{\varphi}_k}{a} \qquad \left\{\partial_\eta^2 + k^2 + a^2\left(m^2 - \frac{R}{6}\right)\right\}\tilde{\varphi}_k = 0$$

determine physical time by counting number of oscillations

$$\tilde{t}_p = n_k \qquad n_k = \frac{k\eta}{\pi}$$

(m=0)

Big bang singularity in Einstein frame is field singularity !

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} \ , \ \varphi = \frac{2M}{\alpha} \ln\left(\frac{\chi}{\mu}\right)$$

choice of frame with constant particle masses is not well suited if physical masses go to zero !

conclusions

Fixed points and scale symmetry crucial

Big bang singularity is artefact
of inappropriate choice of field variables –
no physical singularity

conclusions (2)

- crossover in quantum gravity is reflected in crossover in cosmology
- quantum gravity becomes testable by cosmology
- quantum gravity plays a role not only for primordial cosmology
- crossover scenario explains different cosmological epochs
- simple model is compatible with present observations
- no more parameters than ΛCDM : tests possible

end

Inflation

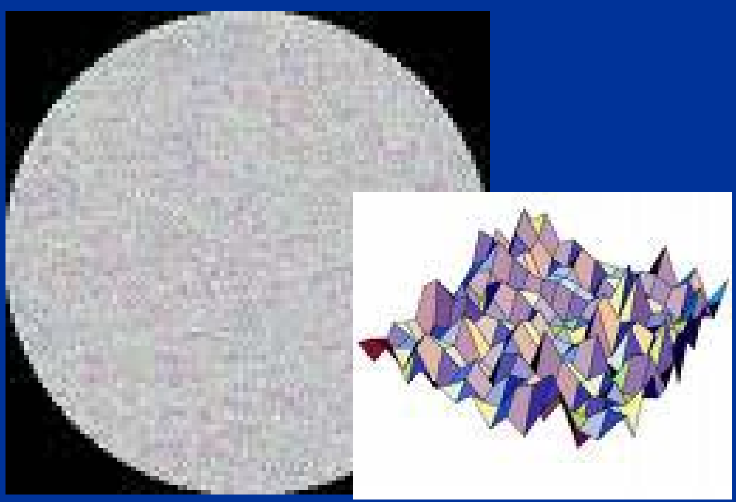
solution for small χ : inflationary epoch

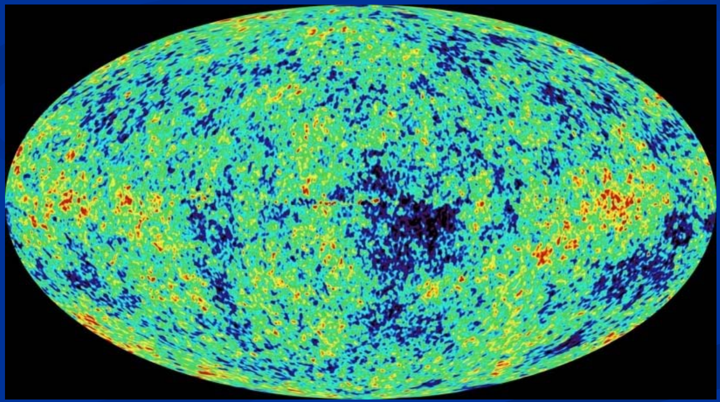
kinetial characterized by anomalous dimension σ

$$B = b\left(\frac{\mu}{\chi}\right)^{\sigma} = \left(\frac{m}{\chi}\right)^{\sigma}$$

Primordial fluctuations

- inflaton field : χ
- primordial fluctuations of inflaton become observable in cosmic microwave background





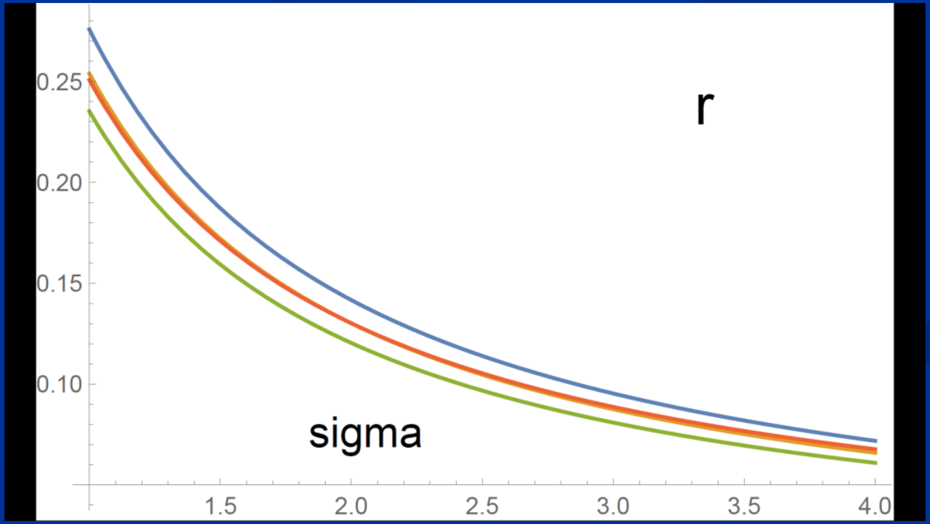
Anomalous dimension determines spectrum of primordial fluctuations

$$r = \frac{0.26}{\sigma}$$

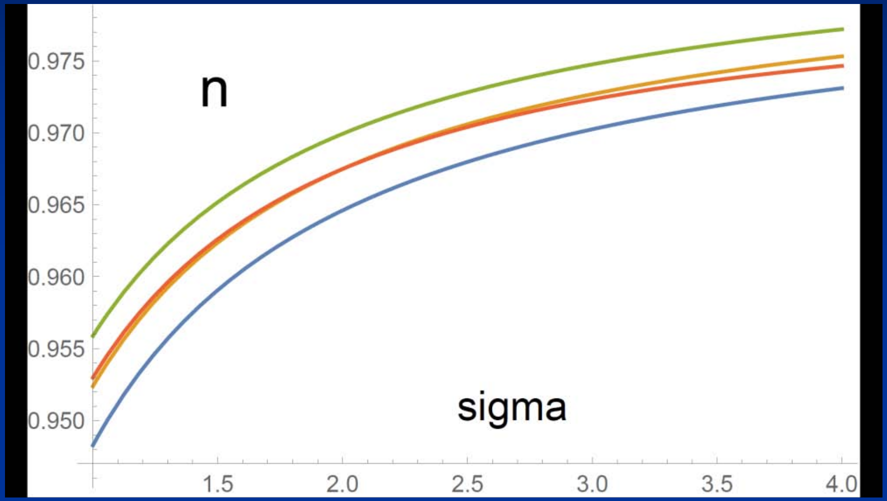
$$n = 1 - \frac{0.065}{\sigma} \cdot \left(1 + \frac{\sigma - 2}{4}\right)$$

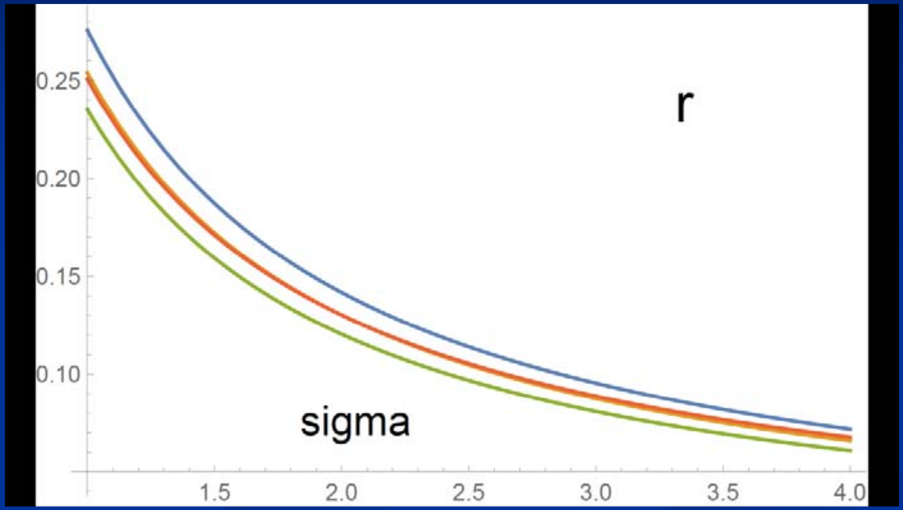
spectral index n

tensor amplitude r



relation between n and r





$$r = 8.19\,(1 - n) - 0.1365$$

Amplitude of density fluctuations

small because of logarithmic running near UV fixed point !

$$\mathcal{A} = \frac{(N+3)^3}{4} e^{-2c_t}$$

$$c_t = \ln\left(\frac{m}{\mu}\right) = 14.1.$$

σ=1

$$\frac{m}{\mu} = \frac{(N+3)^{\frac{3}{2}}}{2\sqrt{\mathcal{A}}} = 1.32 \cdot 10^6 \left(\frac{N}{60}\right)^{\frac{3}{2}}$$

$$B^{-1} - \frac{\kappa}{\sigma}\ln B = \kappa\left[\ln\left(\frac{\chi}{\mu}\right) - c_t\right] = \kappa \ln\left(\frac{\chi}{m}\right)$$

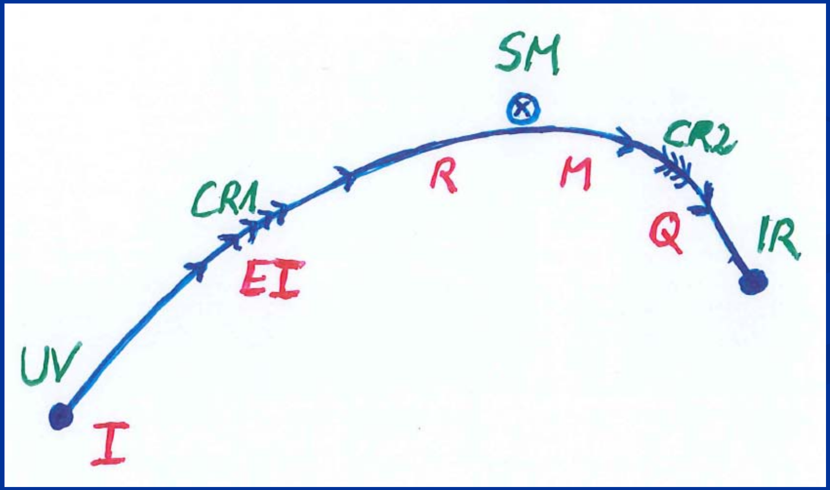
N : number of e – foldings at horizon crossing

First step of crossover ends inflation

- induced by crossover in B

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[\ln\left(\frac{\chi}{\mu}\right) - c_t \right] = \kappa \ln\left(\frac{\chi}{m}\right)$$

- after crossover B changes only very slowly



Scaling solution

- Heating of the Universe after inflation
- Scaling solution with almost fixed fraction of Early Dark Energy

Cosmon inflation

Unified picture of inflation and dynamical dark energy

Cosmon and inflaton are the same scalar field

Quintessence

Dynamical dark energy ,

generated by scalar field (cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87

P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Prediction :

homogeneous dark energy influences recent cosmology

- of same order as dark matter -

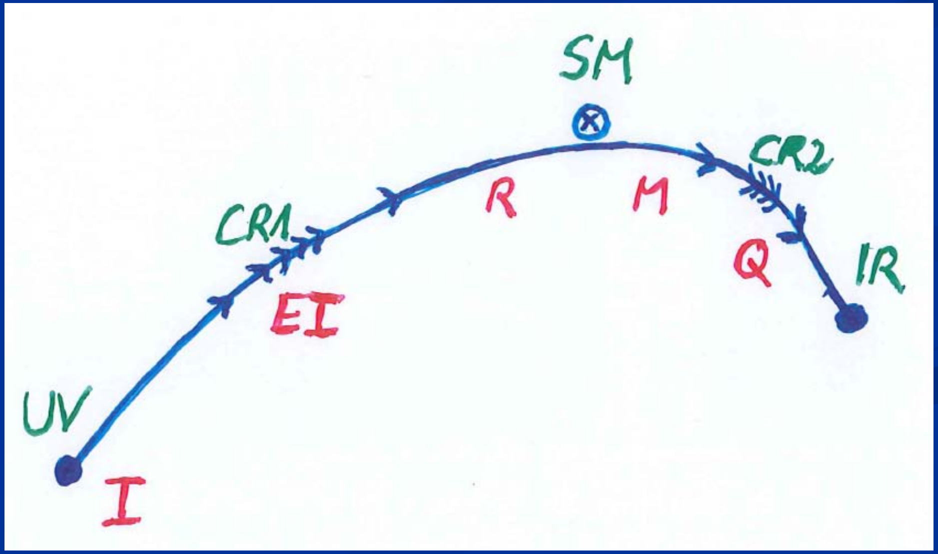
Original models do not fit the present observations

…. modifications

(different growth of neutrino mass)

Second stage of crossover

- from SM to IR
- in sector Beyond Standard Model
- affects neutrino masses first (seesaw or cascade mechanism)



Varying particle masses at onset of second crossover

- All particle masses **except for neutrinos** are proportional to χ .
- Ratios of particle masses remain constant.
- Compatibility with observational bounds on time dependence of particle mass ratios.
- Neutrino masses show stronger increase with χ , such that **ratio neutrino mass over electron mass grows** .

connection between dark energy and neutrino properties

$$[\rho_h(t_0)]^{\frac{1}{4}} = 1.27 \left(\frac{\gamma m_\nu(t_0)}{eV}\right)^{\frac{1}{4}} 10^{-3} eV$$

L.Amendola, M.Baldi, …

present dark energy density given by neutrino mass

present equation of state given by neutrino mass !

$$w_0 \approx -1 + \frac{m_\nu(t_0)}{12\text{eV}}$$

Compatibility with observations and possible tests

- Realistic inflation model
- Almost same prediction for radiation, matter, and Dark Energy domination as ΛCDM
- Presence of small fraction of Early Dark Energy
- Large neutrino lumps

Simplicity

simple description of all cosmological epochs

natural incorporation of Dark Energy :

- inflation
- Early Dark Energy
- present Dark Energy dominated epoch

conclusions (3)

- Variable gravity cosmologies can give a simple and realistic description of Universe
- Compatible with tests of equivalence principle and bounds on variation of fundamental couplings if nucleon and electron masses are proportional to variable Planck mass
- Cosmon dependence of ratio neutrino mass/ electron mass can explain why Universe makes a transition to Dark Energy domination now
- characteristic signal : neutrino lumps

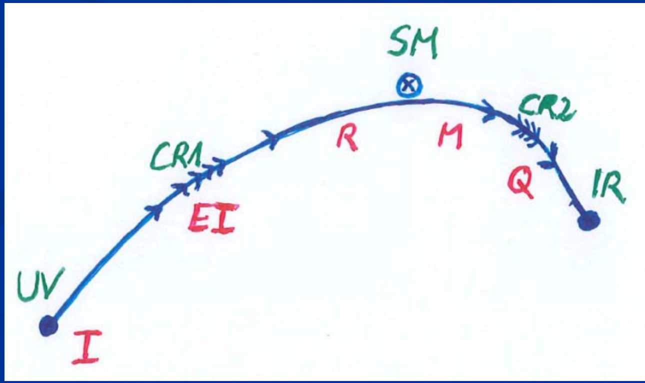
Infrared fixed point

- μ→ 0
- B→ 0

$$\mu \partial_\mu B = \kappa B^2 \quad \text{for} \quad B \to 0$$

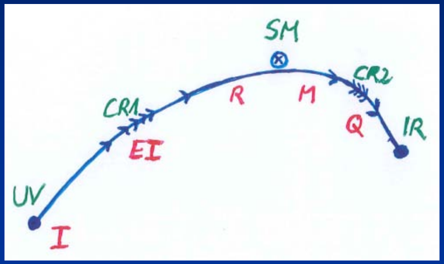
$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2 \chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right) \partial^\mu \chi \partial_\mu \chi \right\}$$

- no intrinsic mass scale
- scale symmetry



Ultraviolet fixed point

- $\mu \longrightarrow \infty$



- kinetial diverges

$$B = b\left(\frac{\mu}{\chi}\right)^{\sigma} = \left(\frac{m}{\chi}\right)^{\sigma}$$

- scale symmetry with anomalous dimension σ

$$g_{\mu\nu} \to \alpha^2 g_{\mu\nu}\ , \ \chi \to \alpha^{-\frac{2}{2-\sigma}}\chi$$

Renormalized field at UV fixed point

$$\chi_R = b^{\frac{1}{2}} \left(1 - \frac{\sigma}{2}\right)^{-1} \mu^{\frac{\sigma}{2}} \chi^{1-\frac{\sigma}{2}}$$

$1 < \sigma$

$$\Gamma_{UV} = \int_x \sqrt{g} \left\{ \frac{1}{2} \partial^\mu \chi_R \partial_\mu \chi_R - \frac{1}{2} C R^2 + D R^{\mu\nu} R_{\mu\nu} \right\}$$

no mass scale

$$\Delta\Gamma_{UV} = \int_x \sqrt{g} E \left(\mu^2 - \frac{R}{2}\right) \mu^{-\frac{2\sigma}{2-\sigma}} \chi_R^{\frac{4}{2-\sigma}},$$

$$E = b^{-\frac{2}{2-\sigma}} \left(1 - \frac{\sigma}{2}\right)^{\frac{4}{2-\sigma}}$$

deviation from fixed point vanishes for

$\mu \longrightarrow \infty$