Universality in ultra-cold fermionic atom gases

Universality in ultra-cold fermionic atom gases

with

S. Diehl , H.Gies , J.Pawlowski

BEC – BCS crossover

Bound molecules of two atoms
on microscopic scale:

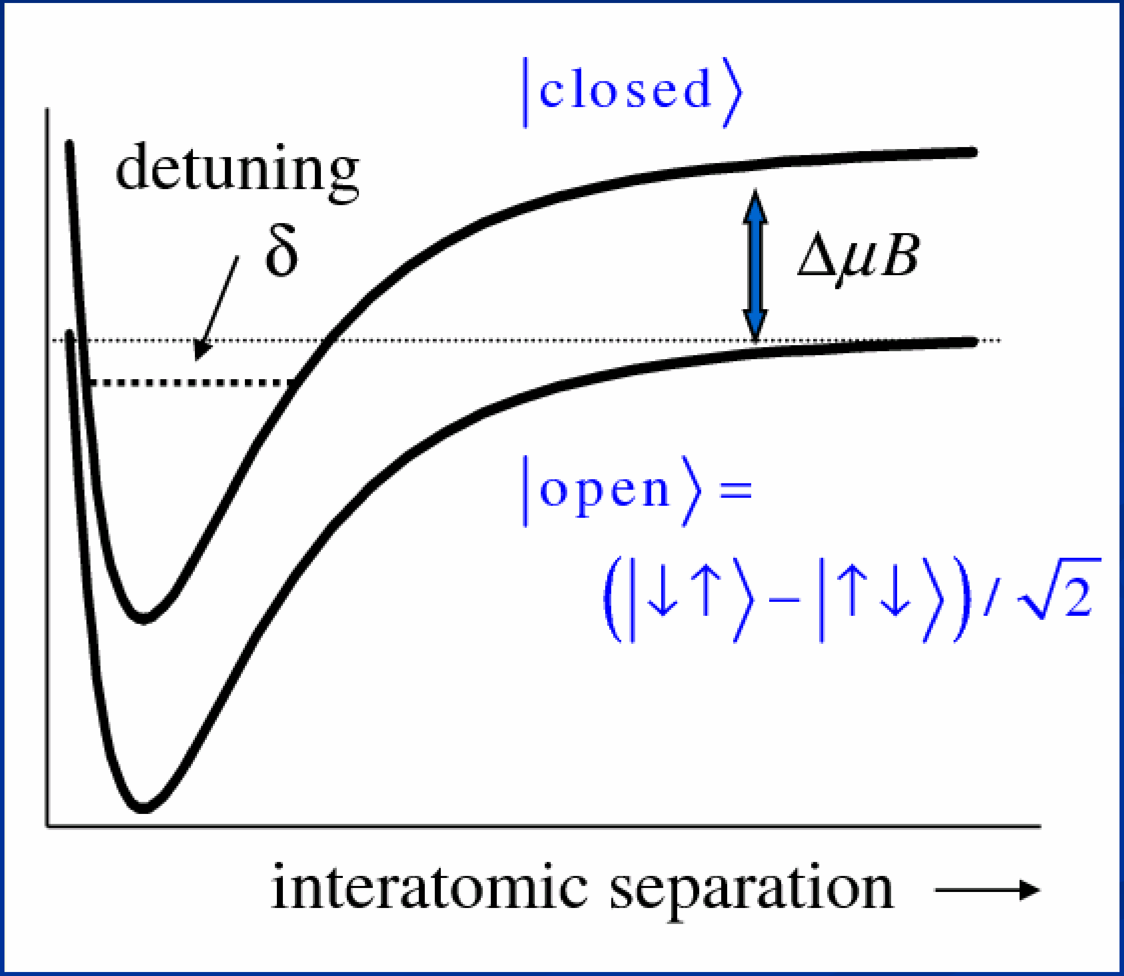
Bose-Einstein condensate (BEC) for low T

Fermions with attractive interactions
(molecules play no role) :

BCS – superfluidity at low T
by condensation of Cooper pairs

Crossover by Feshbach resonance
as a transition in terms of external magnetic field

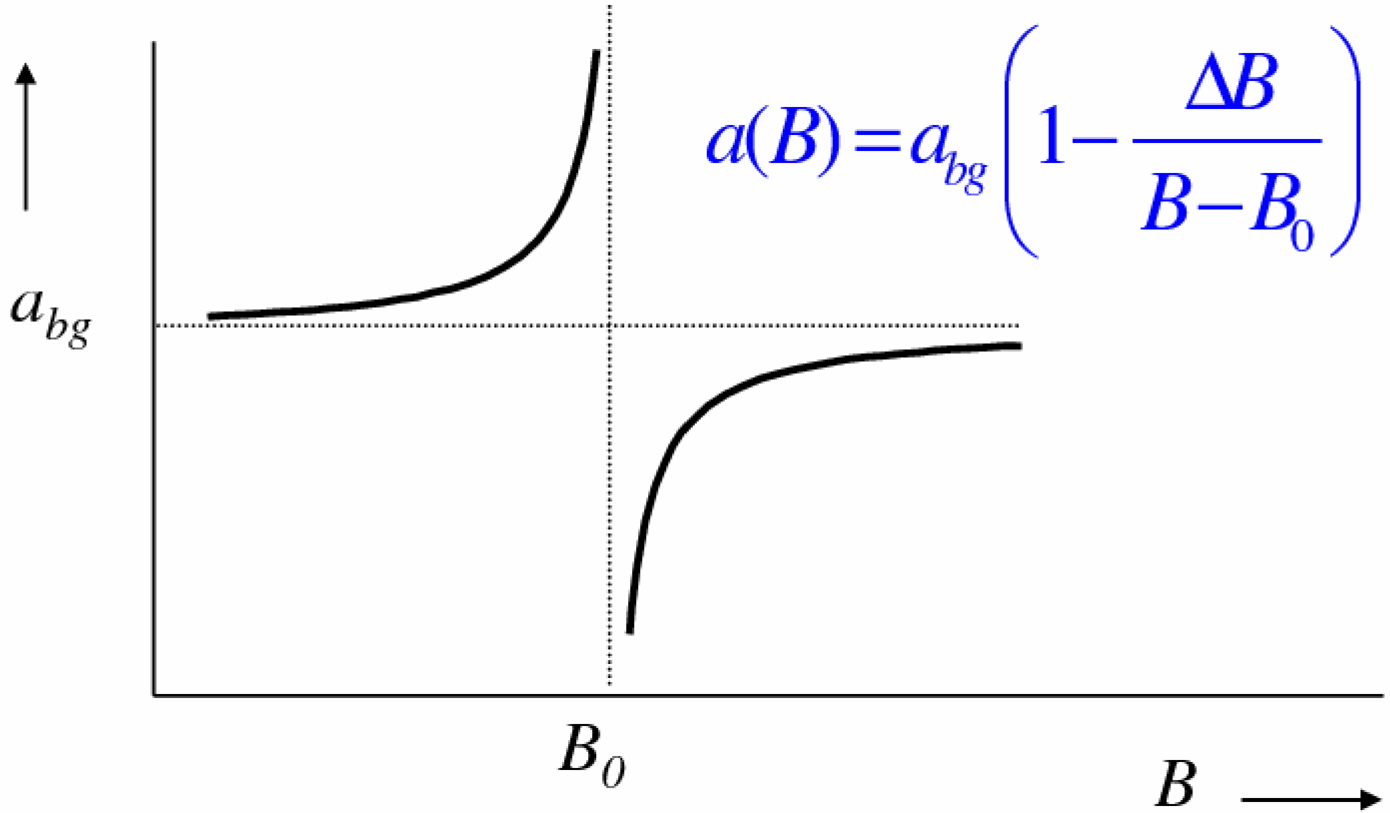
Feshbach resonance



H.Stoof

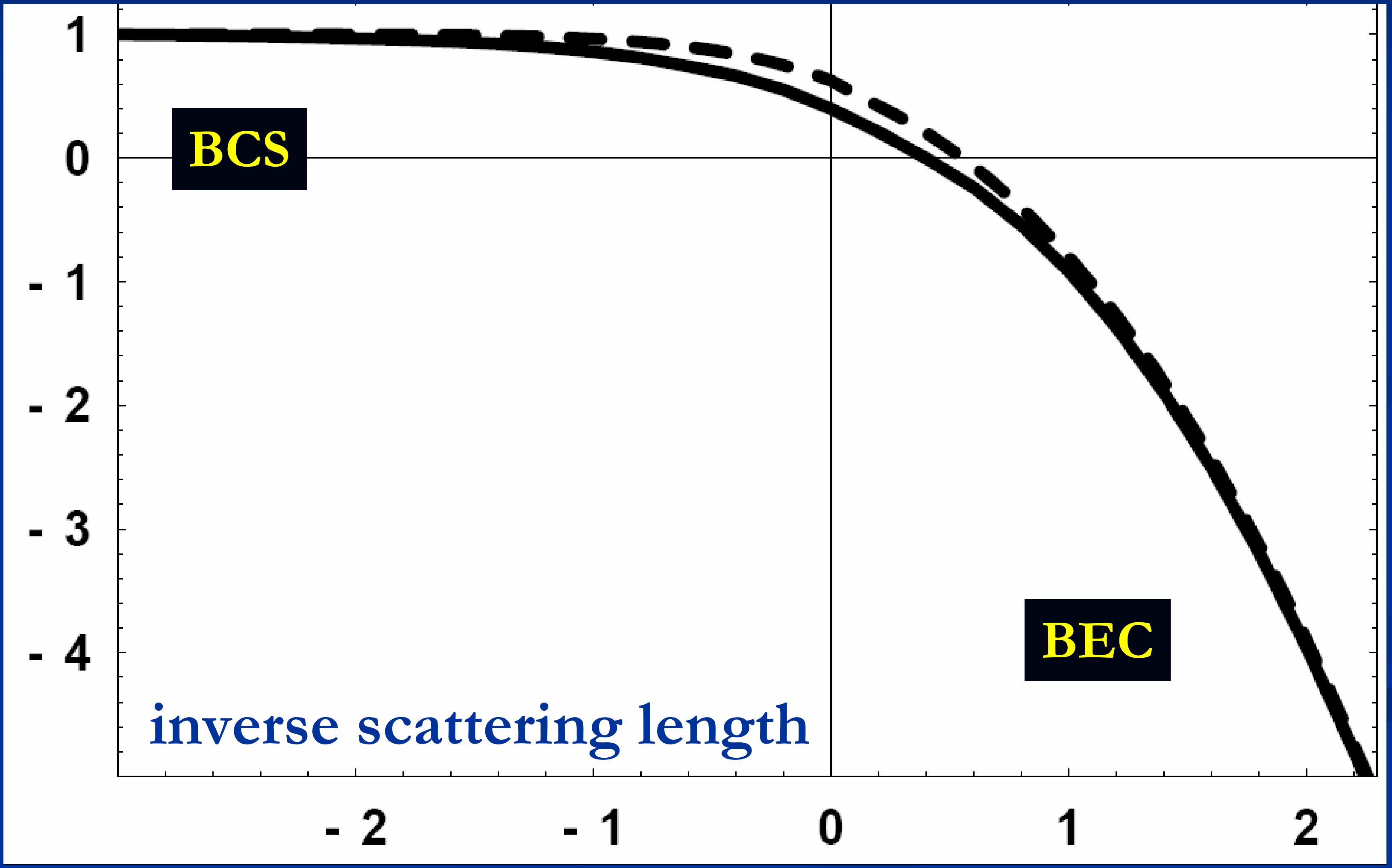
scattering length

$$a(B) = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$



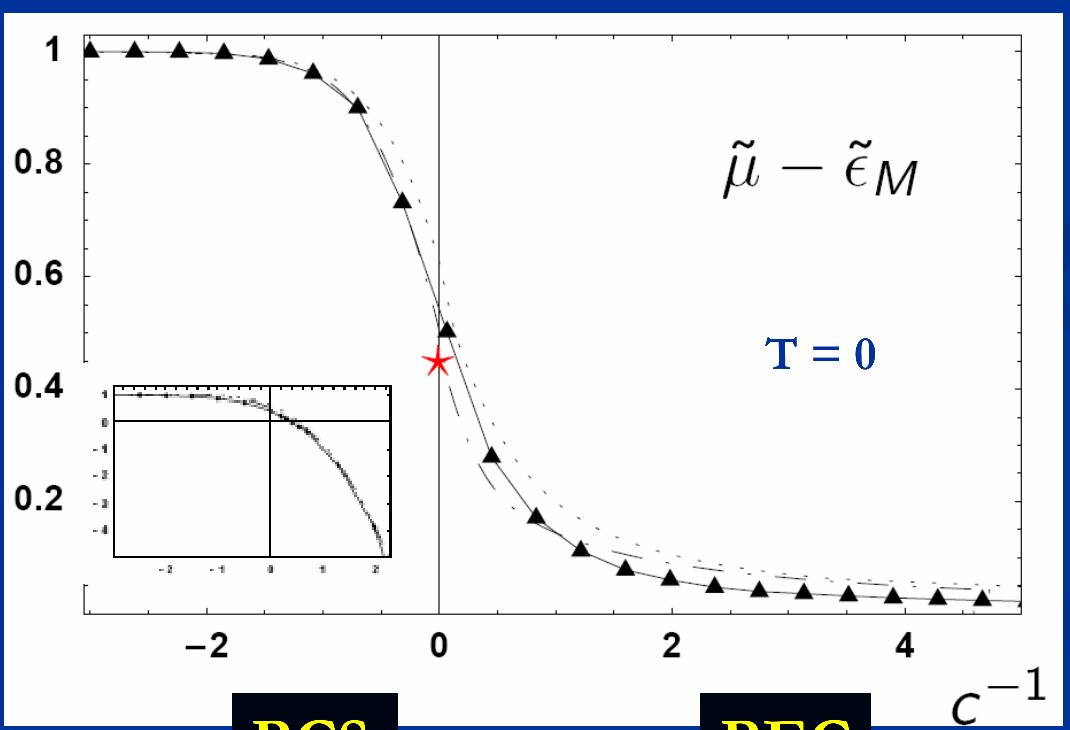
BEC **BCS**

chemical potential



BEC – BCS crossover

- qualitative and partially quantitative theoretical understanding
- mean field theory (MFT) and first attempts beyond



BCS **BEC**

concentration : $\mathbf{c = a\, k_F}$

reduced chemical potential : $\tilde{\sigma} = \mu/\varepsilon_F$

Fermi momemtum : $\mathbf{k_F}$

Fermi energy : $\boldsymbol{\varepsilon_F}$

binding energy :

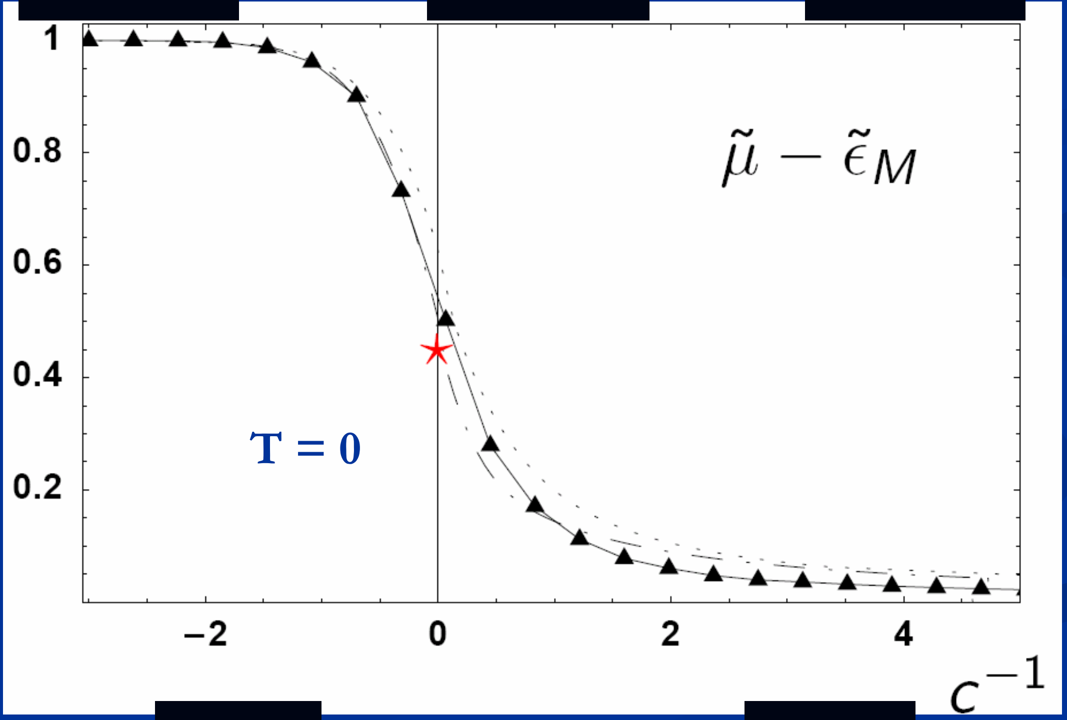
$$\tilde{\epsilon}_M = -\theta(c^{-1})c^{-2}$$

concentration

- $c = a\,k_F$, **a(B)** : scattering length
- needs computation of density $n=k_F^3/(3\pi^2)$

dilute | dense | dilute

non-interacting Fermi gas

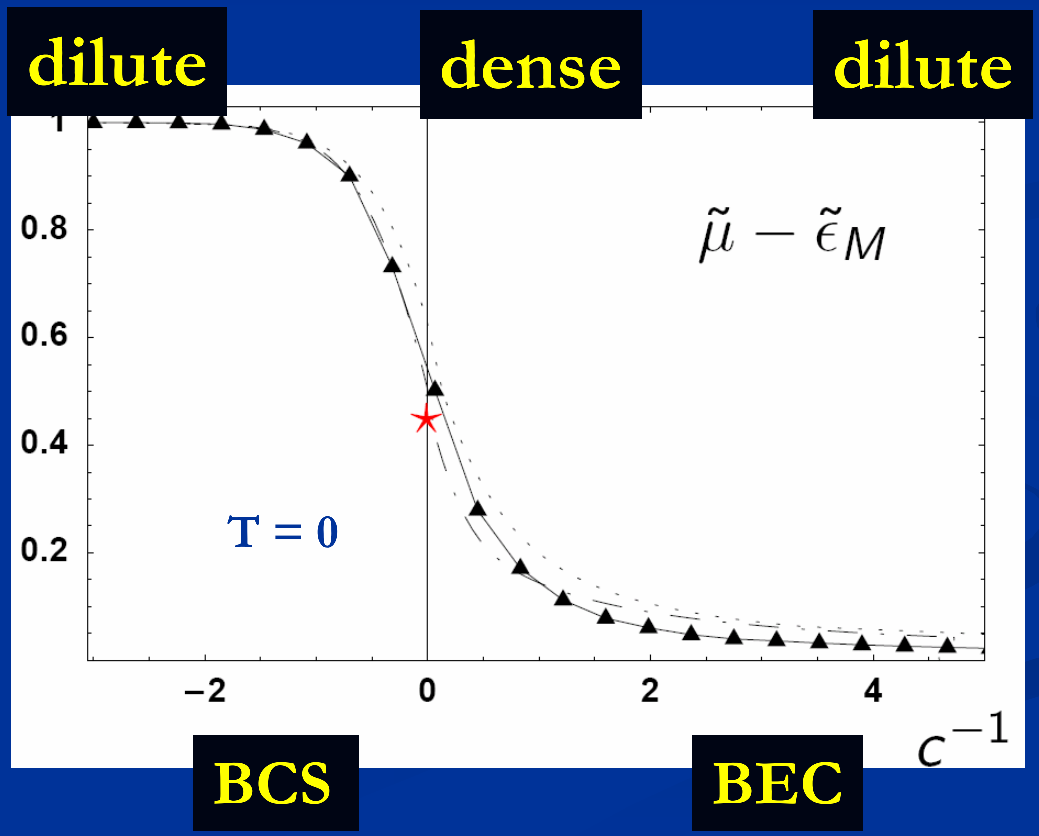


non-interacting Bose gas

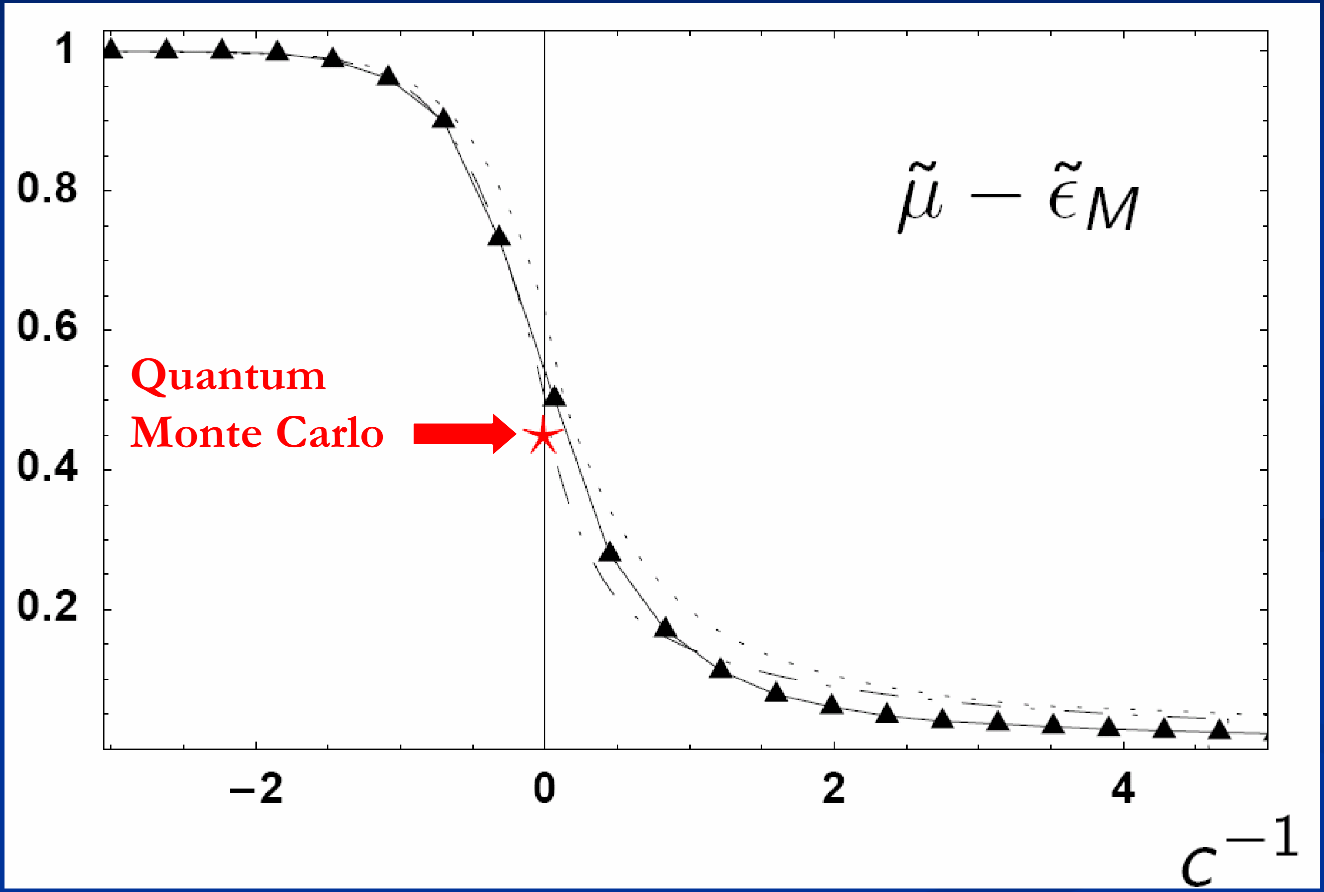
BCS | BEC

universality

same curve for Li and K atoms ?



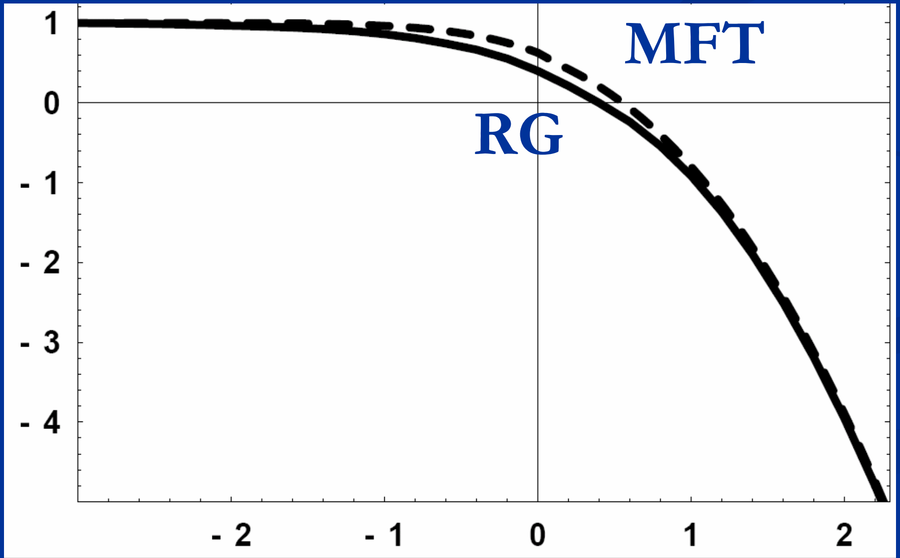
different methods



- Compare RGE (diamonds), SDE (dashed-dotted) and MFT (dashed) approximation schemes.

who cares about details ?

a theorists game ...?



precision many body theory
- quantum field theory -

so far :

- particle physics : perturbative calculations

  magnetic moment of electron :

  g/2 = 1.001 159 652 180 85 (76) (Gabrielse et al.)

- statistical physics : universal critical exponents for second order phase transitions : $\nu = 0.6308\ (10)$

  renormalization group

- lattice simulations for bosonic systems in particle and statistical physics (e.g. QCD)

QFT with fermions

needed:

universal theoretical tools for complex fermionic systems

wide applications :

electrons in solids ,

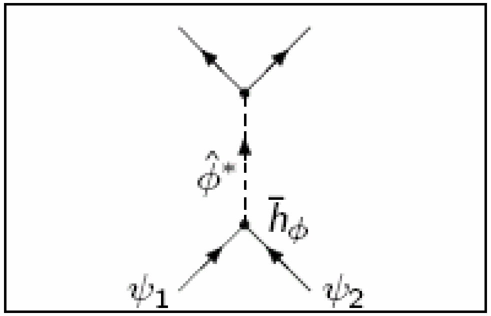
nuclear matter in neutron stars , ….

QFT for non-relativistic fermions

- functional integral, action

$$S = \int_x \{\psi^\dagger(\partial_\tau - \frac{\Delta}{2M} - \sigma)\psi$$
$$+\varphi^*(\partial_\tau - \frac{\Delta}{4M} + \bar{\nu}_\Lambda - 2\sigma)\varphi$$
$$-\bar{h}_\varphi(\varphi^*\psi_1\psi_2 - \varphi\psi_1^*\psi_2^*)\}$$

Molecule exchange



perturbation theory:
Feynman rules

τ : euclidean time on torus with circumference 1/T
σ : effective chemical potential

variables

- ψ : Grassmann variables
- φ : bosonic field with atom number two

What is φ ?

microscopic molecule,

macroscopic Cooper pair ?

All !

parameters

- detuning ν(B)

$$\bar{\nu}_\Lambda = \bar{\nu}_{\Lambda,0} + \bar{\mu}_B(B - B_0)$$

$$\frac{\partial \bar{\nu}_\Lambda}{\partial B} = \bar{\mu}_B$$

$$\begin{aligned} S = \int_x \{ & \psi^\dagger(\partial_\tau - \frac{\Delta}{2M} - \sigma)\psi \\ & + \varphi^*(\partial_\tau - \frac{\Delta}{4M} + \bar{\nu}_\Lambda - 2\sigma)\varphi \\ & - \bar{h}_\varphi(\varphi^*\psi_1\psi_2 - \varphi\psi_1^*\psi_2^*)\} \end{aligned}$$

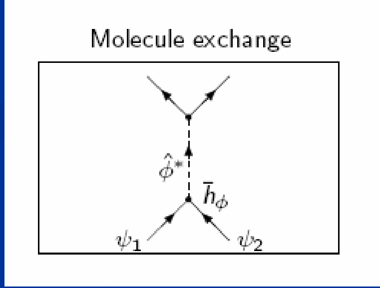
- Yukawa or Feshbach coupling h_φ

fermionic action

equivalent fermionic action , in general not local

$$S_F = \int_x \psi^\dagger \left(\partial_\tau - \frac{\Delta}{2M} - \sigma\right)\psi + S_{\text{int}}$$

$$S_{\text{int}} = -\frac{1}{2}\int_{Q_1,Q_2,Q_3} (\psi^\dagger(-Q_1)\psi(Q_2))(\psi^\dagger(Q_4)\psi(-Q_3)) \frac{\bar{h}_\varphi^2}{\bar{\nu}_\Lambda - 2\sigma + (\vec{q}_1 - \vec{q}_4)^2/4M + 2\pi i T(n_1 - n_4)}$$



scattering length a

$$\bar{\lambda} = -\frac{\bar{h}_\varphi^2}{\bar{\nu}_\Lambda}$$

$$a = M\,\lambda/4\pi$$

- broad resonance : pointlike limit
- large Feshbach coupling

$$\bar{h}_\varphi^2 \to \infty,\ \bar{\nu}_\Lambda \to \infty,\ \bar{\lambda}\ \text{fixed}$$

$$S_{\text{int}} = -\frac{1}{2}\int_{Q_1,Q_2,Q_3} (\psi^\dagger(-Q_1)\psi(Q_2))(\psi^\dagger(Q_4)\psi(-Q_3)) \frac{\bar{h}_\varphi^2}{\bar{\nu}_\Lambda - 2\sigma + (\vec{q}_1 - \vec{q}_4)^2/4M + 2\pi i T(n_1 - n_4)}$$

parameters

- Yukawa or Feshbach coupling h_φ
- scattering length a

Set of microscopic parameters:

$$\{\nu(B), \quad h_{\phi,0}\} \leftrightarrow \{a(B), \quad h_{\phi,0}\}.$$

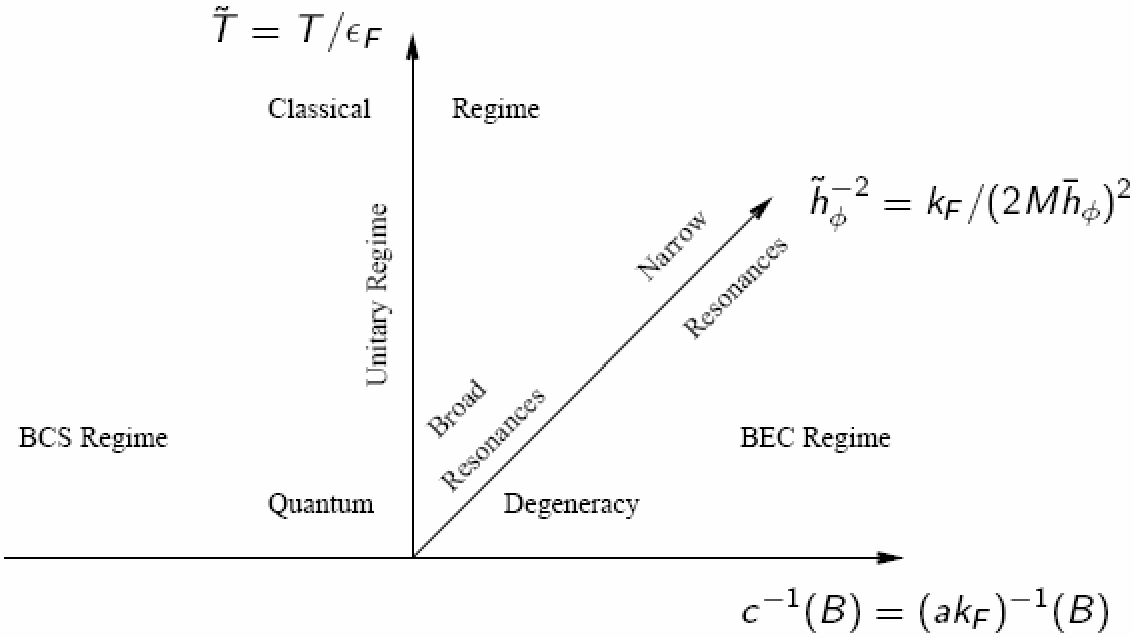
- **broad resonance : h_φ drops out**

concentration c

$$c = ak_F = -\frac{Mk_F\bar{h}_\varphi^2}{4\pi\bar{\mu}_B(B - B_0)}$$

$$n = \frac{k_F^3}{3\pi^2}$$

- Dimensionless axes: measure in units of Fermi momentum, $k_F = (3\pi^2 n)^{1/3}$ and Fermi energy, $\epsilon_F = k_F^2/(2M)$.
- Crossover induced by magnetic field (B) dependence of scattering length: Feshbach resonance.
- Narrow resonances: Nonlocal interactions, exact solution possible (S. Diehl, C. Wetterich, Phys. Rev. A **73** 033615 (2006)).
- Focus on the broad resonance limit $\tilde{h}_\phi \to \infty$: pointlike interactions.



universality

- Are these parameters enough for a quantitatively precise description ?
- Have Li and K the same crossover when described with these parameters ?
- Long distance physics looses memory of detailed microscopic properties of atoms and molecules !

universality for $c^{-1} = 0$: Ho,…(valid for broad resonance)
here: whole crossover range

analogy with particle physics

microscopic theory not known -

nevertheless "macroscopic theory" characterized by a finite number of

"renormalizable couplings"

m_e , α ; g_w , g_s , M_w , …

here : **c** , h_φ (only **c** for broad resonance)

analogy with universal critical exponents

only one relevant parameter :

$$T - T_c$$

units and dimensions

- $c = 1$; $\hbar = 1$; $k = 1$
- momentum ~ length^{-1} ~ mass ~ eV
- energies : $2ME$ ~ (momentum)^2

  (M : atom mass)
- typical momentum unit : Fermi momentum
- typical energy and temperature unit : Fermi energy
- time ~ (momentum)$^{-2}$
- **canonical dimensions different from relativistic QFT !**

rescaled action

$$S = \int_{\hat{x}} \{\hat{\psi}^\dagger(\hat{\partial}_\tau - \hat{\Delta} - \hat{\sigma})\hat{\psi} + \hat{\varphi}^*(\hat{\partial}_\tau - \frac{1}{2}\hat{\Delta} + \hat{\nu} - 2\hat{\sigma})\hat{\varphi} - \hat{h}_\varphi(\hat{\varphi}^*\hat{\psi}_1\hat{\psi}_2 - \hat{\varphi}\hat{\psi}_1^*\hat{\psi}_2^*)\}$$

$$\hat{\psi} = \hat{k}^{-3/2}\psi \ , \ \hat{\varphi} = \hat{k}^{-3/2}\varphi,$$
$$\hat{x} = \hat{k}x \ , \ \hat{\tau} = \frac{\hat{k}^2}{2M}\tau,$$
$$\hat{\sigma} = \frac{2M\sigma}{\hat{k}^2} \ , \ \hat{h}_\varphi = \frac{2M\bar{h}_\varphi}{\sqrt{\hat{k}}}$$

- M drops out
- all quantities in units of k_F if $\hat{k} = k_F$

what is to be computed ?

Inclusion of fluctuation effects
via **functional integral**
leads to **effective action.**

This contains all relevant information for arbitrary T and n !

effective action

- integrate out all quantum and thermal fluctuations
- quantum effective action
- generates full propagators and vertices
- richer structure than classical action

$$\Gamma = \int_x \{\psi^\dagger(\partial_\tau - A_\psi \Delta - \sigma)\psi$$
$$+\varphi^*(\partial_\tau - A_\varphi \Delta)\varphi + u(\varphi)$$
$$-h_\varphi(\varphi^*\psi_1\psi_2 - \varphi\psi_1^*\psi_2^*) + \dots\}$$

effective action

$$\Gamma[\psi, \phi] = \int_0^{1/T} d\tau \int d^3x \Big\{ \psi^\dagger (\partial_\tau - A_\psi \triangle - \sigma) \psi +$$

$$\phi^* (\partial_\tau - A_\phi \triangle) \phi + U(\phi^* \phi) - \frac{h_\phi}{2} \Big(\phi^* \psi^T \epsilon \psi - \phi \psi^\dagger \epsilon \psi^* \Big) + \ldots \Big\}.$$

- includes all quantum and thermal fluctuations
- formulated here in terms of renormalized fields
- involves renormalized couplings

effective potential

minimum determines order parameter

$$u = m_\varphi^2 \rho + \frac{\lambda_\varphi}{2}\rho^2 \quad , \quad SYM$$
$$u = \frac{\lambda_\varphi}{2}(\rho - \rho_0)^2 \quad , \quad SSB$$

$$\rho = \varphi^* \varphi$$

condensate fraction

$$\Omega_c = 2\,\rho_0 / n$$

$$\begin{aligned}\Gamma = \int_x \{ & \psi^\dagger(\partial_\tau - A_\psi \Delta - \sigma)\psi \\ & + \varphi^*(\partial_\tau - A_\varphi \Delta)\varphi + u(\varphi) \\ & - h_\varphi(\varphi^* \psi_1 \psi_2 - \varphi \psi_1^* \psi_2^*) + \dots \}\end{aligned}$$

effective potential

- value of φ at potential minimum :
  order parameter , determines condensate fraction
- second derivative of U with respect to φ yields correlation length
- derivative with respect to σ yields density
- fourth derivative of U with respect to φ yields molecular scattering length

Quartic truncation for bosonic potential (displayed in symmetric phase):

$$U(\phi^*\phi) = (\nu(B) + \Delta m_\phi^2)\phi^*\phi + \frac{\lambda_\phi}{2}(\phi^*\phi)^2 + \ldots$$

renormalized fields and couplings

$$\psi = Z_\psi^{1/2}\hat{\psi}\ ,\ \varphi = Z_\varphi^{1/2}\hat{\varphi}$$

$$h_\varphi = Z_\varphi^{-1/2} Z_\psi^{-1} \hat{h}_\varphi$$

$$\Gamma = \int_x \{\psi^\dagger(\partial_\tau - A_\psi\Delta - \sigma)\psi$$
$$+\varphi^*(\partial_\tau - A_\varphi\Delta)\varphi + u(\varphi)$$
$$-h_\varphi(\varphi^*\psi_1\psi_2 - \varphi\psi_1^*\psi_2^*) + \ldots\}$$

challenge for ultra-cold atoms :

Non-relativistic fermion systems with precision similar to particle physics !

(QCD with quarks)

results

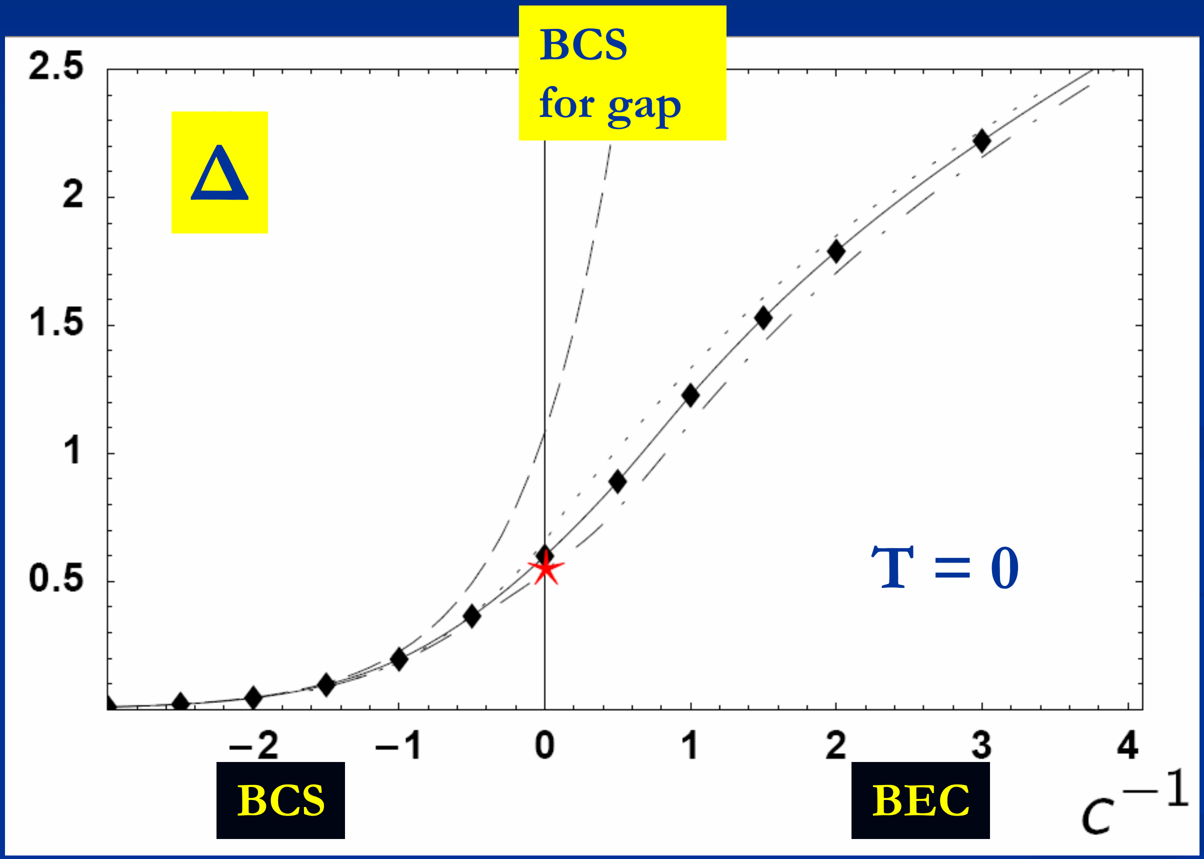
from

functional renormalization group

physics at different length scales

- microscopic theories : where the laws are formulated
- effective theories : where observations are made
- effective theory may involve different degrees of freedom as compared to microscopic theory
- example: microscopic theory only for fermionic atoms , macroscopic theory involves bosonic collective degrees of freedom (φ)

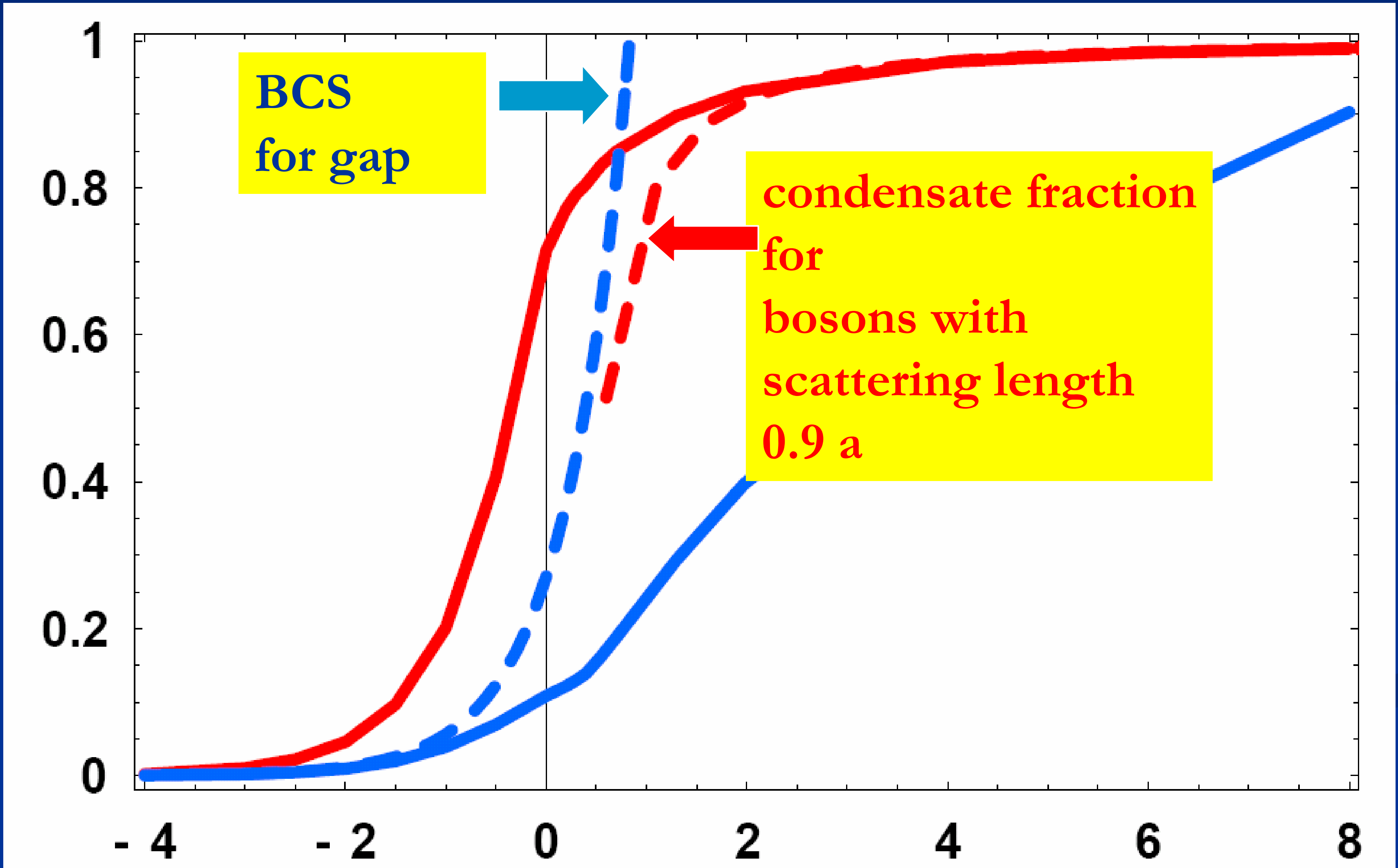
gap parameter



BCS regime: recover BCS gap result $\Delta/\Delta^{BCS}(c^{-1}) \approx 0.9$ for $c^{-1} < -2$.

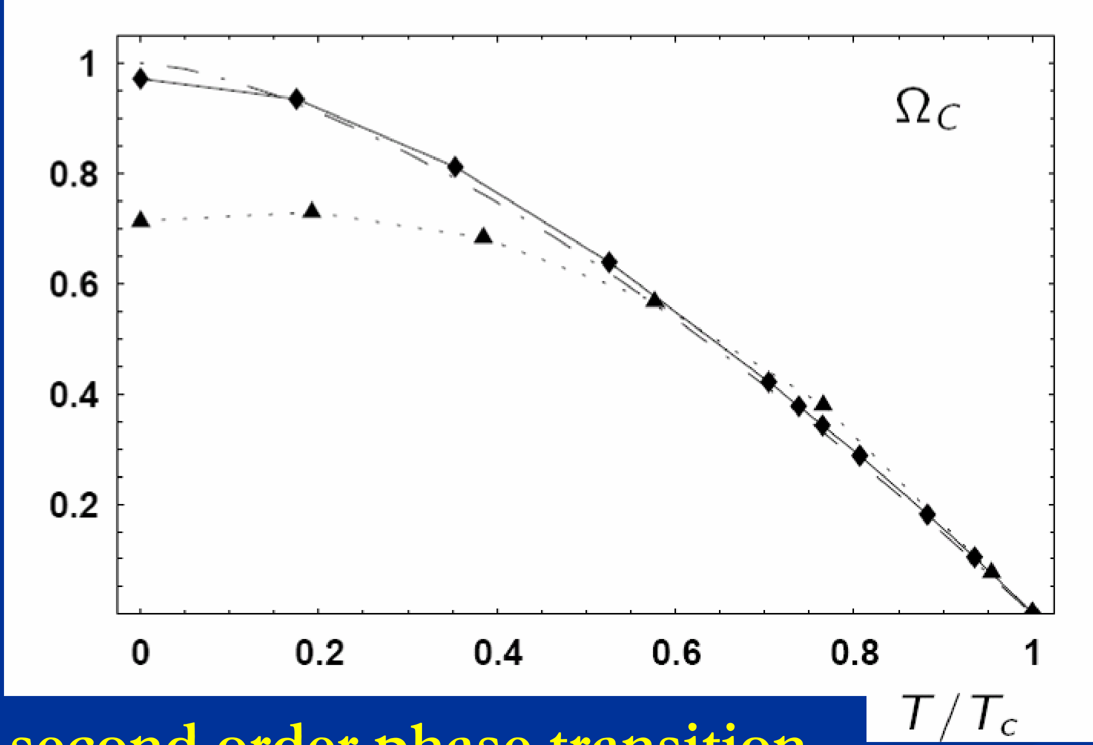
MFT (dashed): No boson interactions.
SDE (dashed-dotted): Overestimates interactions, $a_M = 2$.

limits



1
0.8
0.6
0.4
0.2
0
- 4
- 2
0
2
4
6
8
BCS
for gap
condensate fraction
for
bosons with
scattering length
0.9 a

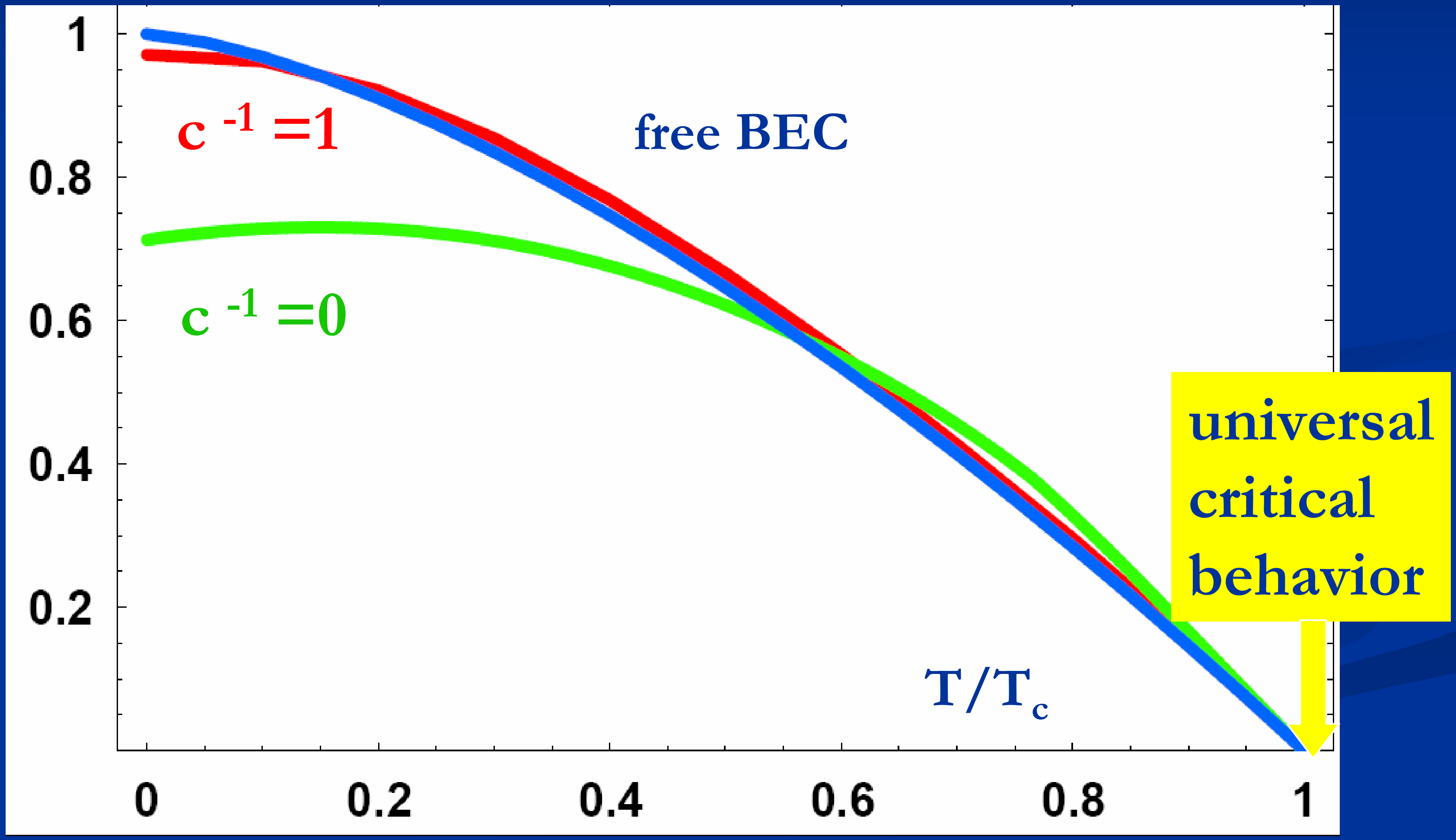
temperature dependence of condensate



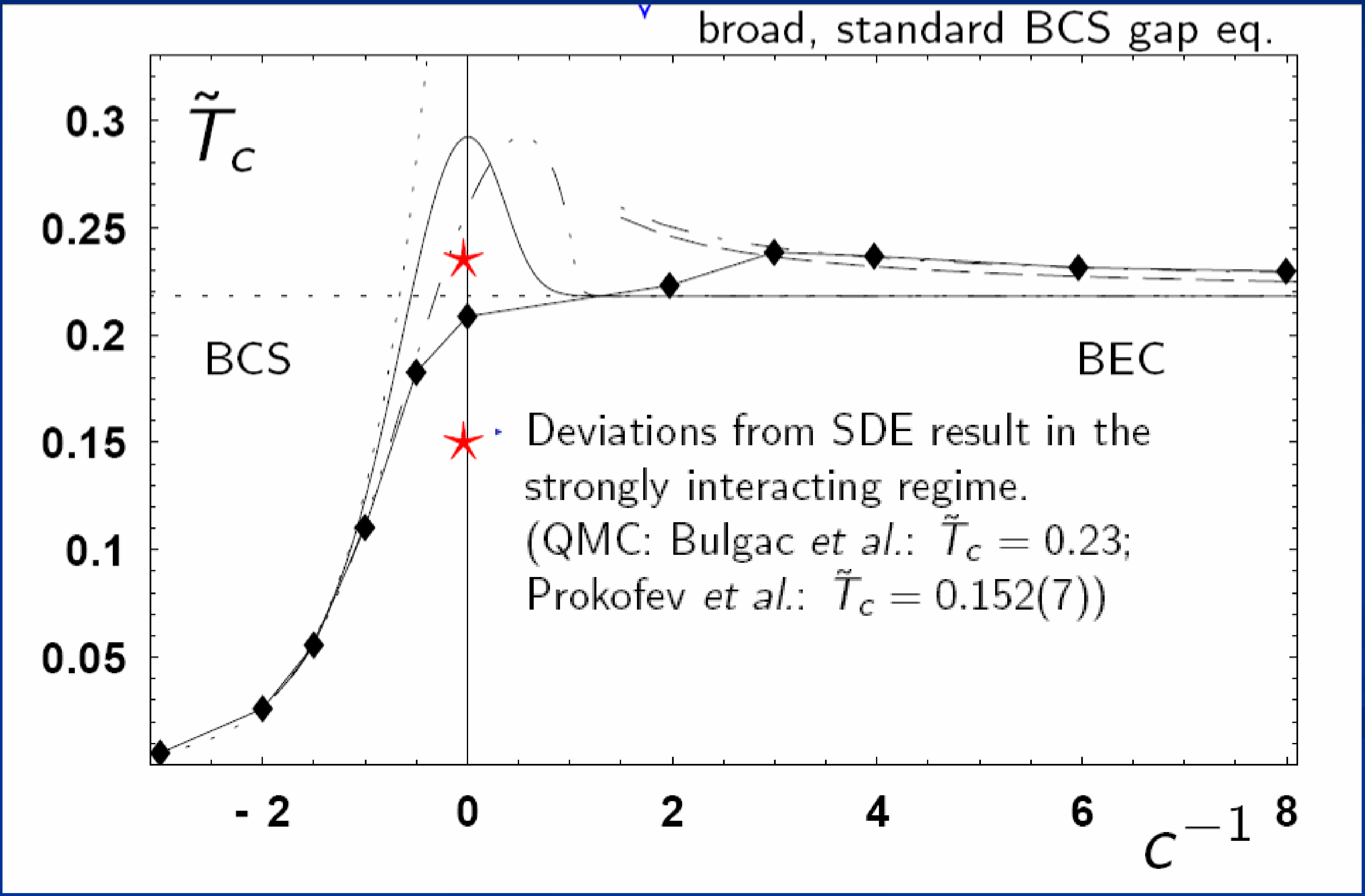
second order phase transition

- Compare free BE condensate fraction to result for $c^{-1} = 0$ (resonance, triangles) and $c^{-1} = 1$ (BEC regime, diamonds).
- Low temperature: Condensate fraction strongly depends on c^{-1}.
- Close to criticality:
  - Second order phase transition.
  - Similar approach to T_c: dominance of boson fluctuations, system attracted to universal critical point.

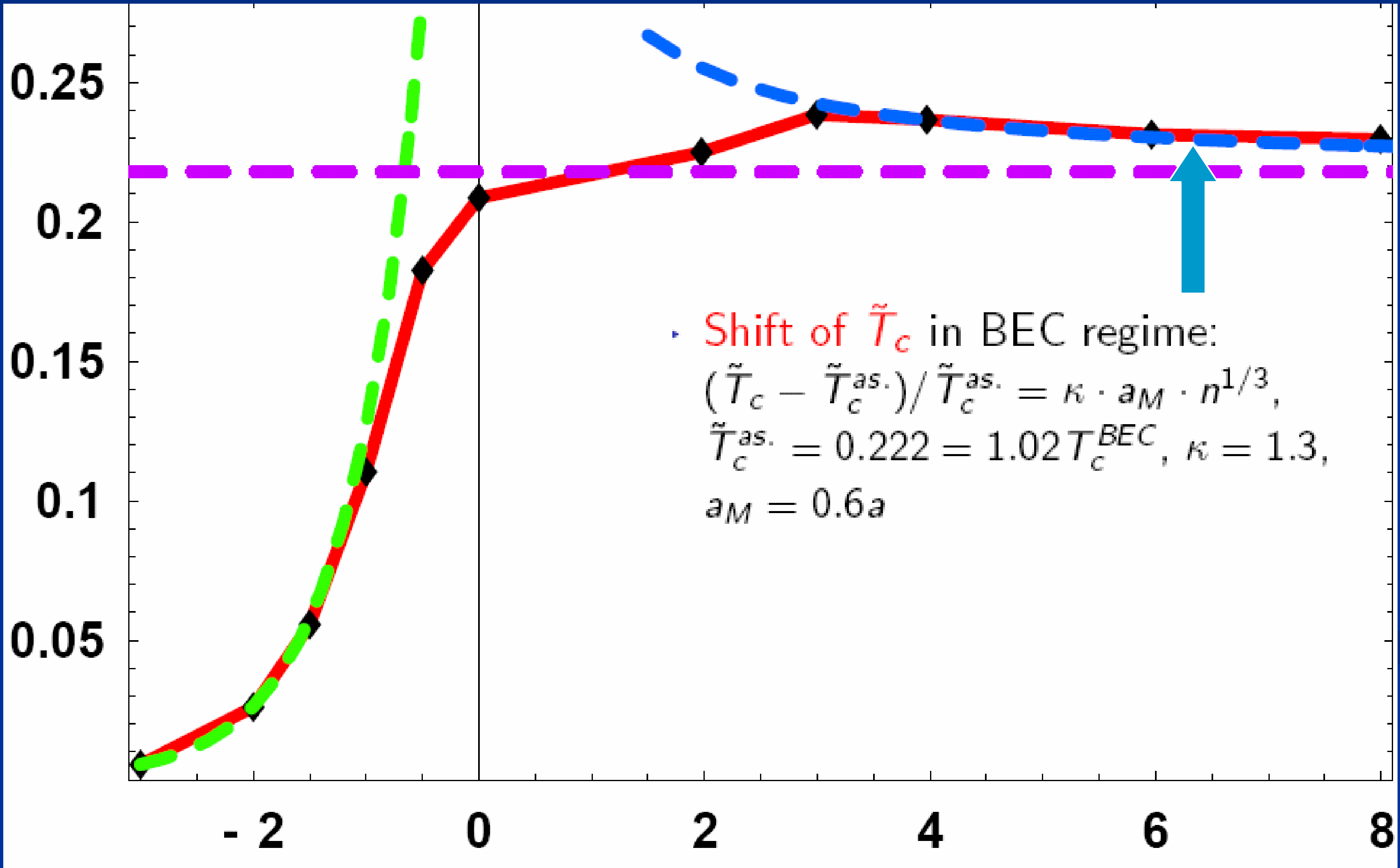
condensate fraction : second order phase transition



crossover phase diagram



shift of BEC critical temperature

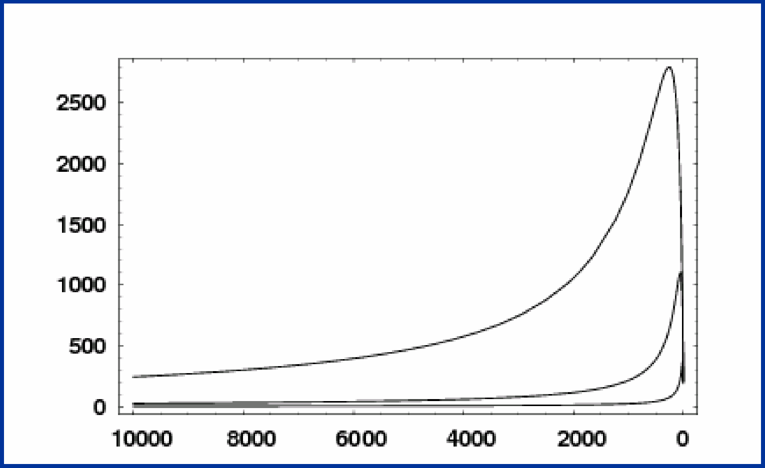


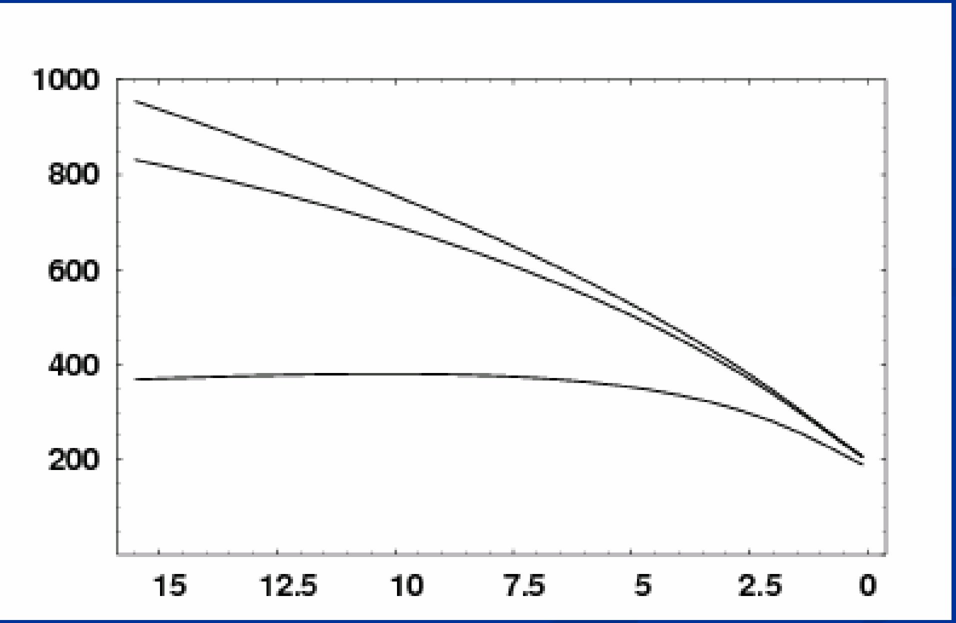
running couplings : crucial for universality

for large Yukawa couplings h_φ :

- only one relevant parameter c
- all other couplings are strongly attracted to partial fixed points
- macroscopic quantities can be predicted in terms of c and T/ε_F (in suitable range for c^{-1})

Flow of Yukawa coupling





k^2

k^2

T=0.5 , c=1

universality for broad resonances

for large Yukawa couplings h_φ :

- only one relevant parameter c
- all other couplings are strongly attracted to partial fixed points
- macroscopic quantities can be predicted

  in terms of c and T/ε_F

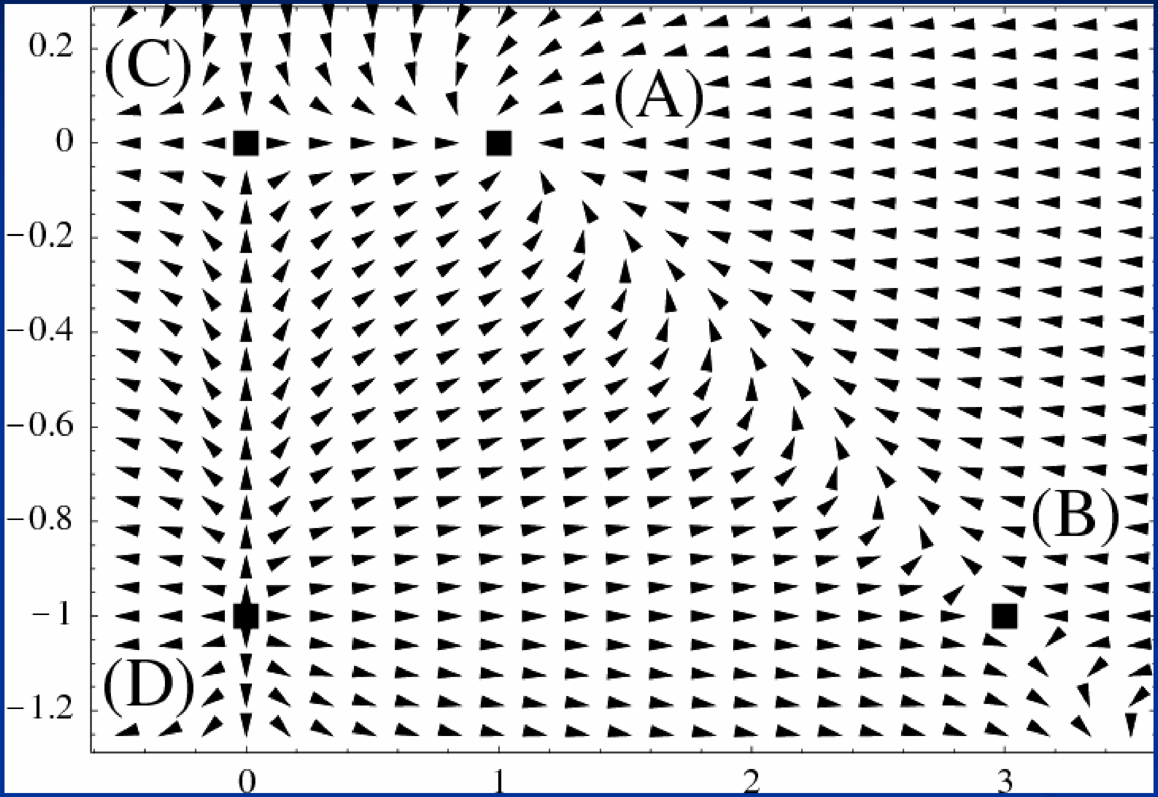
  (in suitable range for c^{-1} ; density sets scale)

universality for narrow resonances

- Yukawa coupling becomes additional parameter (marginal coupling)
- also background scattering important

Flow of Yukawa and four fermion coupling

$\lambda_\psi/8\pi$



$h^2/32\pi$

(A) broad Feshbach resonance

(C) narrow Feshbach resonance

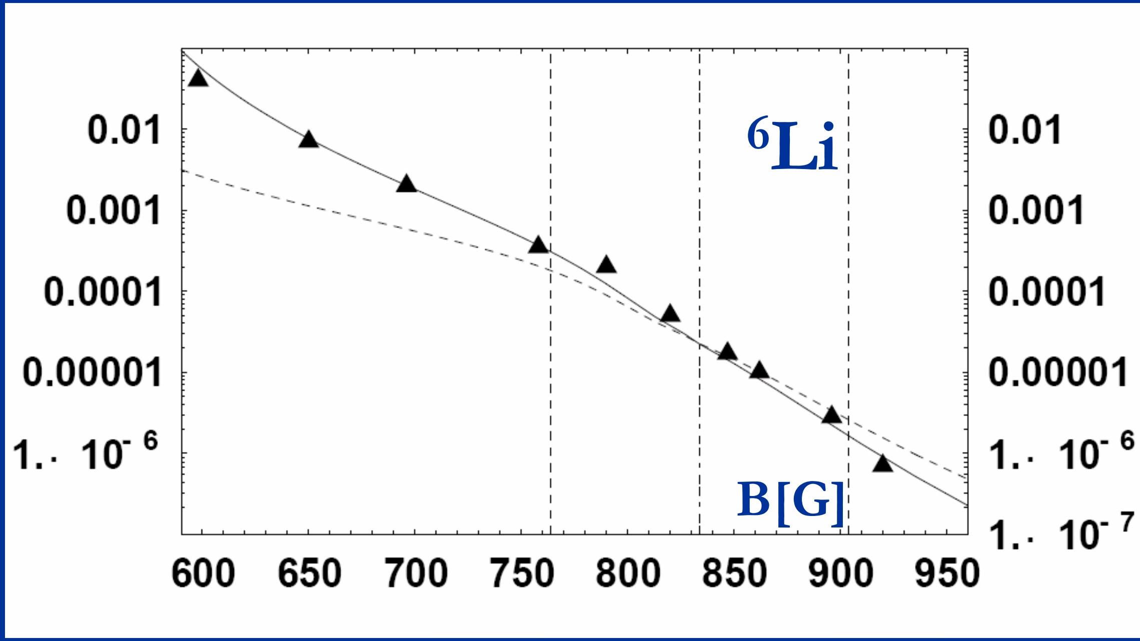
Universality is due to fixed points !

not all quantities are universal !

bare molecule fraction

(fraction of microscopic closed channel molecules)

- not all quantities are universal
- bare molecule fraction involves wave function renormalization that depends on value of Yukawa coupling



Experimental points by Partridge et al.

conclusions

the challenge of precision :

- substantial theoretical progress needed
- "phenomenology" has to identify quantities that are accessible to precision both for experiment and theory
- dedicated experimental effort needed

challenges for experiment

- study the simplest system
- identify quantities that can be measured with precision of a few percent and have clear theoretical interpretation
- precise thermometer that does not destroy probe
- same for density

end