Unification from Functional Renormalization

From

Microscopic Laws

(Interactions, classical action)

to

Fluctuations!

Macroscopic Observation

(Free energy functional, effective action)

- block spins

Kadanoff, Wilson

- exact renormalization group equations

Wilson, Kogut
Wegner, Houghton
Weinberg
Polchinski
Hasenfratz^2

- Lattice finite size scaling

Lüscher,...

- coarse grained free energy/average action

Effective potential includes all fluctuations

Average potential U_k

$\equiv$ scale dependent effective potential
$\equiv$ coarse grained free energy

Only fluctuations with momenta $q^2 > k^2$ included

k: infrared cutoff for fluctuations, "average scale"

Λ: characteristic scale for microphysics

$$U_\Lambda \approx S \rightarrow U_0 \equiv U$$

Unification from Functional Renormalization

- fluctuations in d=0,1,2,3,...
- linear and non-linear sigma models
- vortices and perturbation theory
- bosonic and fermionic models
- relativistic and non-relativistic physics
- classical and quantum statistics
- non-universal and universal aspects
- homogenous systems and local disorder
- equilibrium and out of equilibrium

unification

abstract laws

quantum gravity

grand unification

standard model

electro-magnetism

gravity

Landau theory	universal critical physics	functional renormalization

complexity

unification: functional integral / flow equation

- simplicity of average action
- explicit presence of scale
- differentiating is easier than integrating…

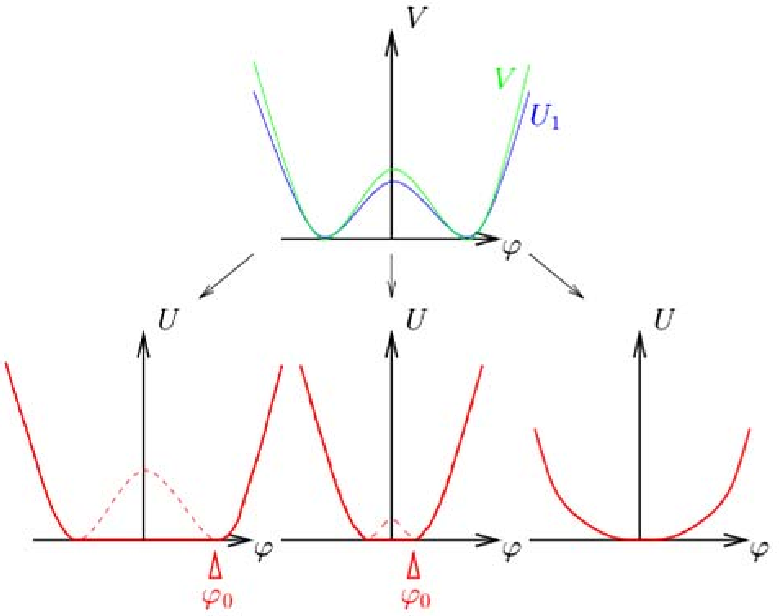
unified description of scalar models for all d and N

Scalar field theory

$\varphi_a(x)$: magnetization, density, chemical concentration, Higgs field, meson field, inflaton, cosmon

$O(N)$-symmetry:

$$S = \int d^dx \left\{ \frac{1}{2}\partial_\mu\varphi_a\partial_\mu\varphi_a + V(\rho) \right\}; \quad \rho = \frac{1}{2}\varphi_a\varphi_a$$



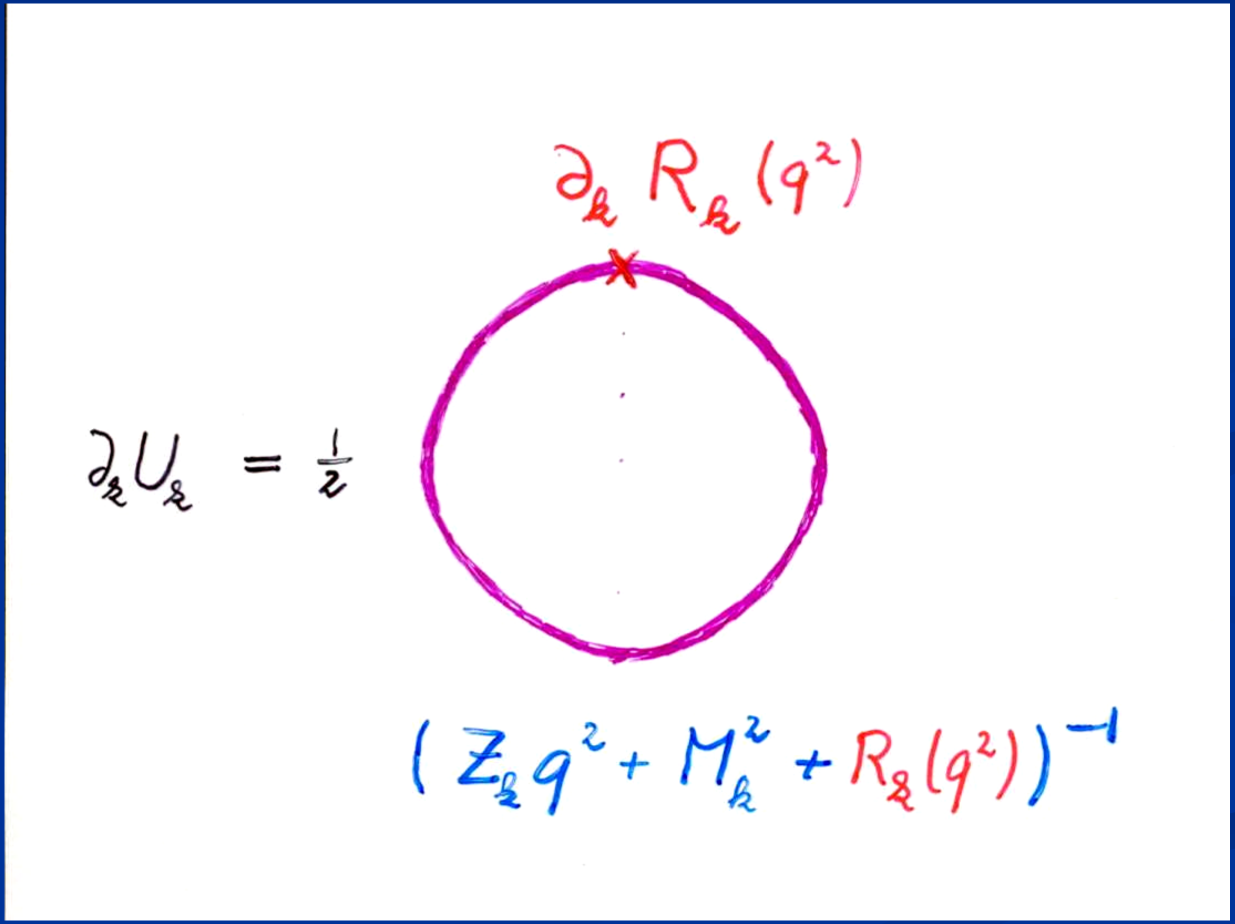
Flow equation for average potential

$$\partial_k U_k(\varphi) = \frac{1}{2}\sum_i \int \frac{d^d q}{(2\pi)^d} \frac{\partial_k R_k(q^2)}{Z_k q^2 + R_k(q^2) + \bar{M}^2_{k,i}(\varphi)}$$

$$\bar{M}^2_{k,ab} = \frac{\partial^2 U_k}{\partial\varphi_a \partial\varphi_b}$$: Mass matrix

$\bar{M}^2_{k,i}$: Eigenvalues of mass matrix

Simple one loop structure – nevertheless (almost) exact



Infrared cutoff

R_k : IR-cutoff

e.g $$R_k = \frac{Z_k q^2}{e^{q^2/k^2} - 1}$$

or $$R_k = Z_k(k^2 - q^2)\Theta(k^2 - q^2) \qquad \text{(Litim)}$$

$$\lim_{k\to 0} R_k = 0$$

$$\lim_{k\to\infty} R_k \to \infty$$

Wave function renormalization and anomalous dimension

Z_k: wave function renormalization

$$k\partial_k Z_k = -\eta_k Z_K$$

η_k: anomalous dimension

$$t = \ln(k/\Lambda)$$

$$\partial_t \ln Z = -\eta$$

for $Z_k(\varphi, q^2)$: flow equation is **exact !**

Scaling form of evolution equation

$$u = \frac{U_k}{k^d}$$
$$\tilde{\rho} = Z_k k^{2-d} \rho$$
$$u' = \frac{\partial u}{\partial \tilde{\rho}} \quad \text{etc.}$$

$$\partial_t u|_{\tilde{\rho}} = -du + (d-2+\eta)\tilde{\rho}u' + 2v_d\{l_0^d(u' + 2\tilde{\rho}u''; \eta) + (N-1)\, l_0^d(u'; \eta)\}$$

$$v_d^{-1} = 2^{d+1}\pi^{d/2}\Gamma\left(\frac{d}{2}\right)$$

linear cutoff:

$$l_0^d(w;\eta) = \frac{2}{d}\left(1 - \frac{\eta}{d+2}\right)\frac{1}{1+w}$$

On r.h.s. :
neither the scale k
nor the wave function
renormalization Z
appear explicitly.

Scaling solution:
no dependence on t;
corresponds
to second order
phase transition.

Tetradis …

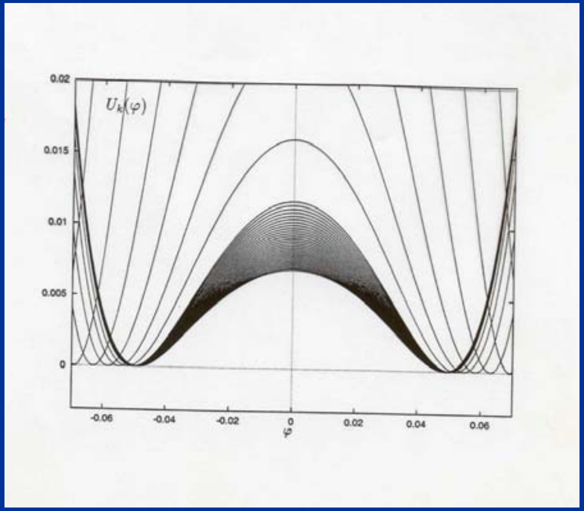
unified approach

- choose N
- choose d
- choose initial form of potential
- run !

(quantitative results : systematic derivative expansion in second order in derivatives)

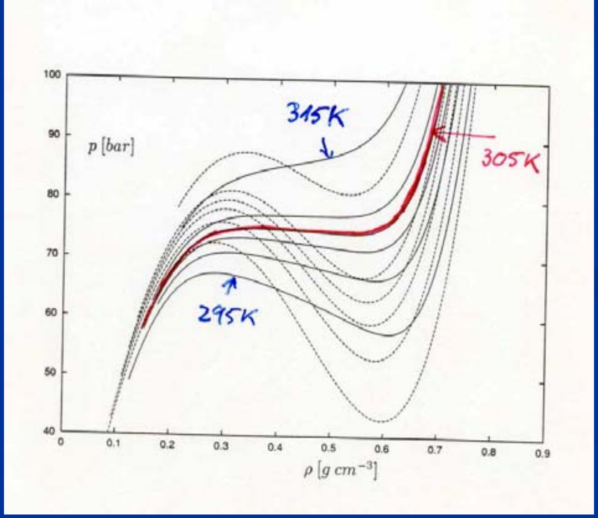
Flow of effective potential

Ising model



S.Seide …

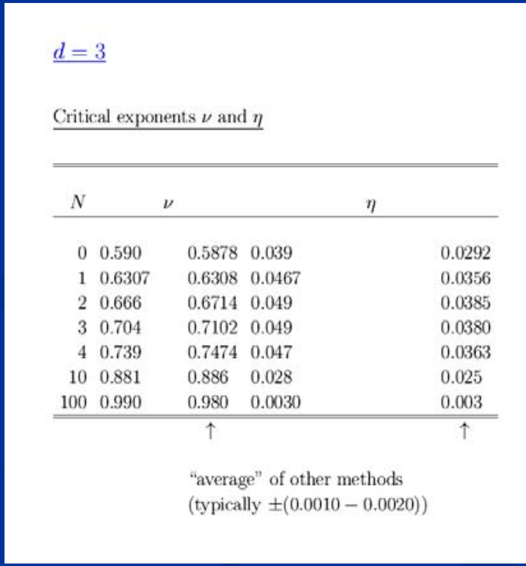
CO_2



Experiment :

T_* =304.15 K
p_* =73.8.bar
ρ_* = 0.442 g cm-2

Critical exponents



$d = 3$

Critical exponents ν and η

N	ν		η	
0	0.590	0.5878	0.039	0.0292
1	0.6307	0.6308	0.0467	0.0356
2	0.666	0.6714	0.049	0.0385
3	0.704	0.7102	0.049	0.0380
4	0.739	0.7474	0.047	0.0363
10	0.881	0.886	0.028	0.025
100	0.990	0.980	0.0030	0.003
		↑		↑

"average" of other methods
(typically $\pm(0.0010 - 0.0020)$)

Critical exponents , d=3

N	ν ERGE	ν world	η ERGE	η world
0	0.590	0.5878	0.039	0.0292
1	0.6307	0.6308	0.0467	0.0356
2	0.666	0.6714	0.049	0.0385
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"average" of other methods
(typically $\pm(0.0010 - 0.0020)$)

critical exponents , BMW approximation

N	η	η (other)	ν	ν (other)	ω (prelim.)	ω (other)
0	0.033(3)	0.028(3) [1]	0.588	0.588(1) [1]	0.80	
1	0.039(3)	0.0364(2) [2]	0.6298(4)	0.6301(2) [2]	0.78	0.79(1) [1]
		0.0368(2) [3]		0.6302(1) [3]		
		0.033(3) [1]		0.630(1) [1]		
2	0.041(3)	0.0381(2) [4]	0.6719(4)	0.6717(1) [4]	0.78	0.79(1) [1]
		0.035(3) [1]		0.670(2) [1]		
3	0.040(3)	0.0375(5) [5]	0.709	0.7112(5) [5]	0.73	
		0.036(3) [1]		0.707(4) [1]		
4	0.038(3)	0.035(5)[1]	0.738	0.741(6) [1]	0.74	0.77(2) [1]
		0.037(1) [6]		0.749(2) [6]		
5	0.035(3)	0.031(3) [8]	0.768	0.764(4) [8]	0.73	0.77(2) [1]
		0.034(1) [7]		0.779(3) [7]		
10	0.022(2)	0.024 [9]	0.860	0.859 [9]	0.81	
20	0.012(1)	0.014 [9]	0.929	0.930 [9]	0.94	
100	0.0023(2)	0.0027 [10]		0.989 [10]	0.99	

[1] R. Guida and J. Zinn-Justin '98. [2] M. Campostrini, A. Pelissetto, P. Rossi, E. Vicari '02.
[3] Y. Deng and H. W. J. Blote '03. [4] M. Campostrini, M. Hasenbusch, A. Pelissetto, E. Vicari '06.
[5] M. Campostrini, M. Hasenbusch, A. Pelissetto, P. Rossi, E. Vicari '02. [6] M. Hasenbusch '01.
[7] M. Hasenbusch, A. Pelissetto, E. Vicari '05. [8] A. Butti and F. Parisen Toldin '05.
[9] S. A. Antonenko and A. I. Sokolov '95. [10] M. Moshe and J. Zinn-Justin '03.

Blaizot, Benitez , … , Wschebor

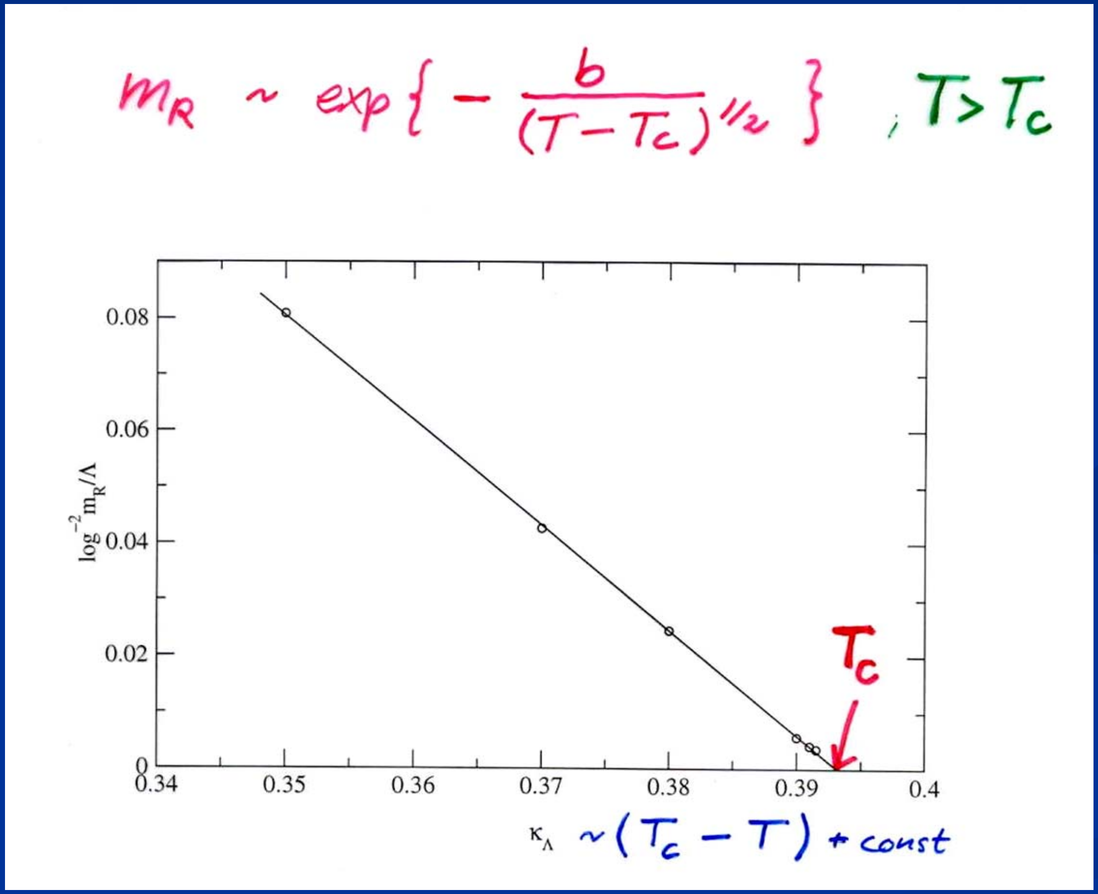
Solution of partial differential equation :

yields highly nontrivial non-perturbative results despite the one loop structure !

Example:

Kosterlitz-Thouless phase transition

Essential scaling : d=2,N=2



- Flow equation contains correctly the non-perturbative information !
- (essential scaling usually described by vortices)

Von Gersdorff …

Kosterlitz-Thouless phase transition (d=2,N=2)

Correct description of phase with

Goldstone boson

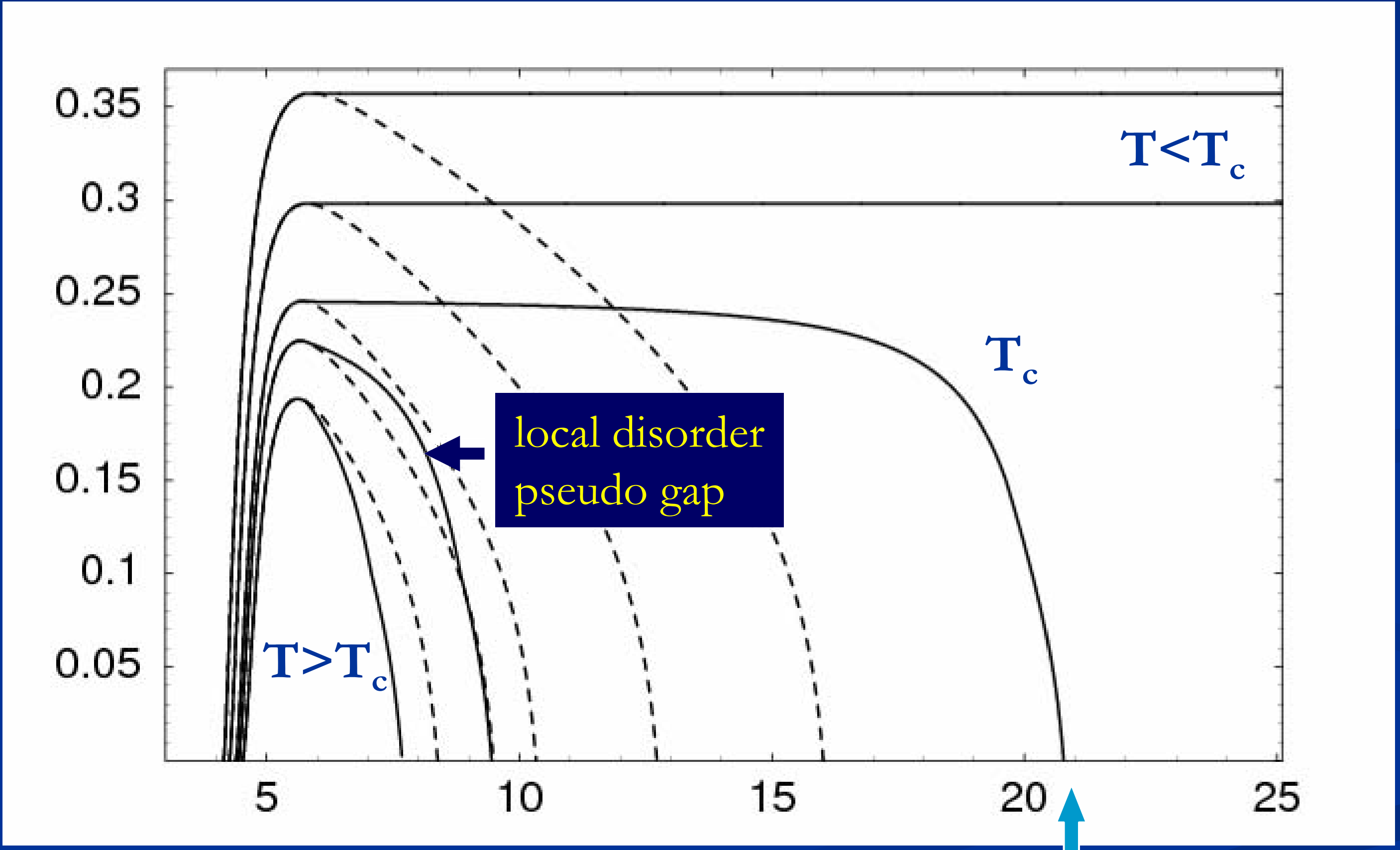
(infinite correlation length)

for $T<T_c$

Running renormalized d-wave superconducting order parameter ϰ in doped Hubbard (-type) model

ϰ

location of minimum of u

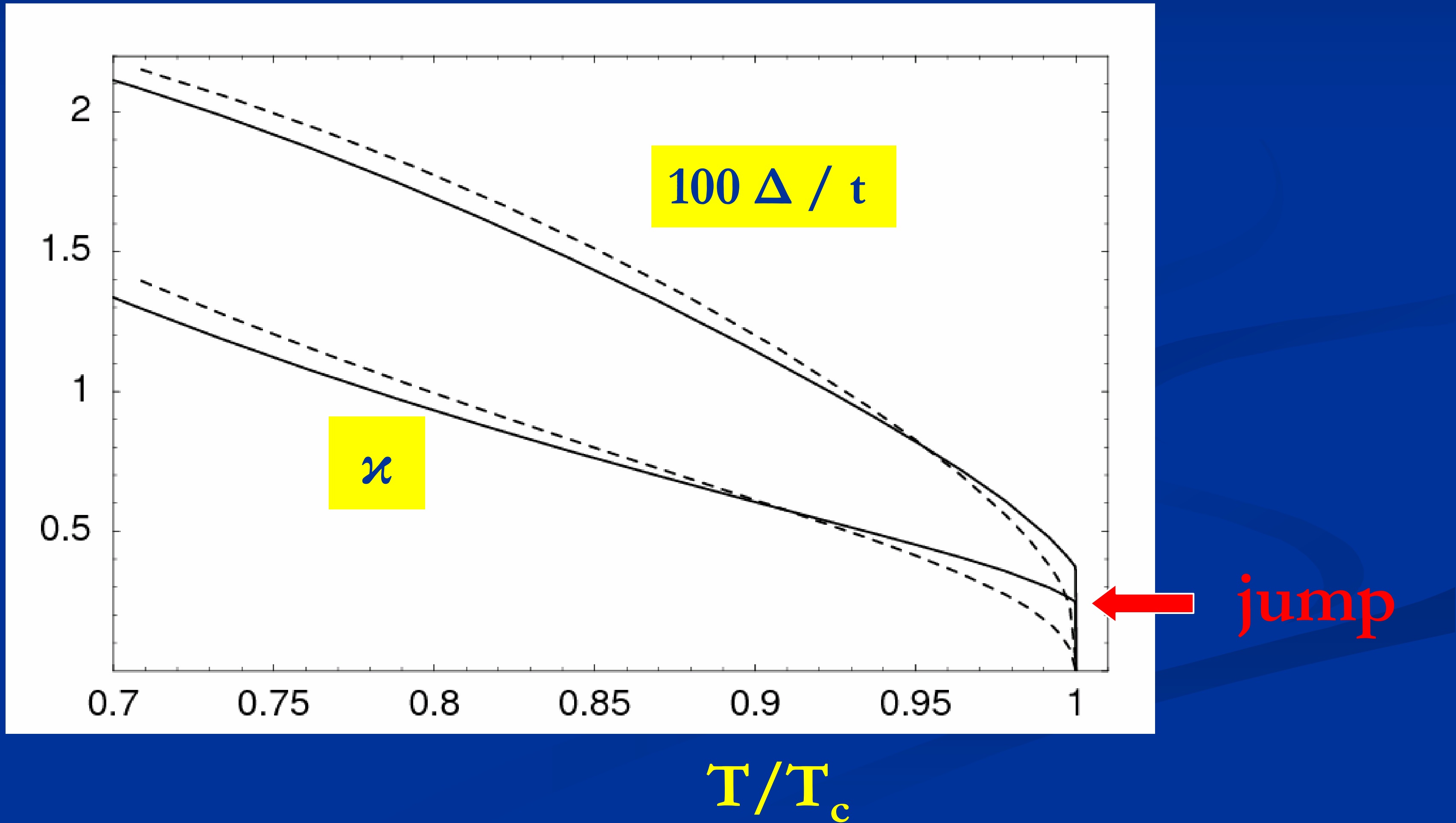


C.Krahl,…

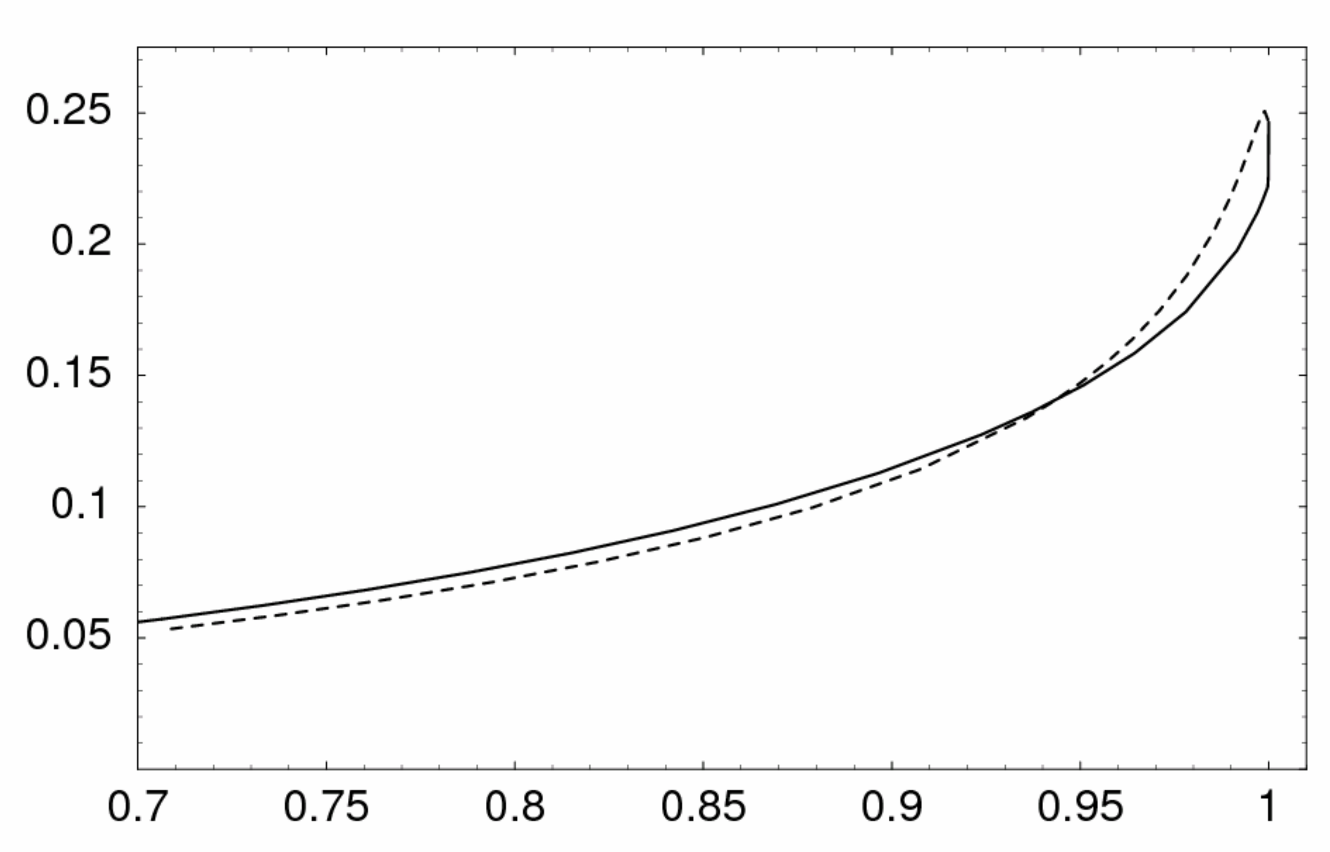
- ln (k/Λ)

macroscopic scale 1 cm

Renormalized order parameter $\varkappa$ and gap in electron propagator Δ in doped Hubbard model



Temperature dependent anomalous dimension η



η

T/T_c

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- bosonic and fermionic models
- relativistic and non-relativistic physics
- classical and quantum statistics
- ☺ non-universal and universal aspects
- homogenous systems and local disorder
- equilibrium and out of equilibrium

Exact renormalization group equation

Exact flow equation

for scale dependence of average action

$$\partial_k \Gamma_k[\varphi] = \frac{1}{2}\mathrm{Tr}\left\{\left(\Gamma_k^{(2)}[\varphi] + R_k\right)^{-1} \partial_k R_k\right\}$$

'92

$\left(\Gamma_k^{(2)}\right)_{ab}(q, q') = \frac{\delta^2 \Gamma_k}{\delta\varphi_a(-q)\delta\varphi_b(q')}$

$\mathrm{Tr} : \sum_a \int \frac{d^d q}{(2\pi)^d}$

(fermions : STr)

some history … (the parents)

- **exact RG equations :**

  Symanzik eq. , Wilson eq. , Wegner-Houghton eq. , Polchinski eq. ,

  mathematical physics

- **1PI :** RG for 1PI-four-point function and hierarchy

  Weinberg

  formal Legendre transform of Wilson eq.

  Nicoll, Chang

- **non-perturbative flow :**

  d=3 : sharp cutoff ,

  no wave function renormalization or momentum dependence

  Hasenfratz^2

qualitative changes that make non-perturbative physics accessible :

(1) basic object is simple

average action ~ classical action

~ generalized Landau theory

direct connection to thermodynamics

(coarse grained free energy)

qualitative changes that make non-perturbative physics accessible :

(2) Infrared scale k

instead of Ultraviolet cutoff Λ

short distance memory not lost

no modes are integrated out , but only part of the fluctuations is included

simple one-loop form of flow

simple comparison with perturbation theory

infrared cutoff k

cutoff on momentum resolution

or frequency resolution

e.g. distance from pure anti-ferromagnetic momentum or from Fermi surface

intuitive interpretation of k by association with physical IR-cutoff , i.e. finite size of system :

arbitrarily small momentum differences cannot be resolved !

qualitative changes that make non-perturbative physics accessible :

(3) only physics in small momentum range around k matters for the flow

ERGE regularization

simple implementation on lattice

artificial non-analyticities can be avoided

qualitative changes that make non-perturbative physics accessible :

(4) flexibility

- change of fields
- microscopic or composite variables
- simple description of collective degrees of freedom and bound states
- many possible choices of "cutoffs"

Proof of exact flow equation

$$\partial_k \Gamma|_\phi = -\partial_k W|_j - \partial_k \Delta_k S[\varphi]$$
$$= \frac{1}{2}\mathrm{Tr}\left\{\partial_k R_k(\langle\phi\phi\rangle - \langle\phi\rangle\langle\phi\rangle)\right\}$$
$$= \frac{1}{2}\mathrm{Tr}\left\{\partial_k R_k W_k^{(2)}\right\}$$

$$W_k^{(2)}(\Gamma_k^{(2)} + R_k) = \mathbb{1}$$
$$(\Delta_k S^{(2)} \equiv R_k)$$

$$\Longrightarrow$$

$$\partial_k \Gamma_k = \frac{1}{2}\mathrm{Tr}\left\{\partial_k R_k(\Gamma_k^{(2)} + R_k)^{-1}\right\}$$

sources j can multiply arbitrary operators

φ : associated fields

Truncations

Functional differential equation –

cannot be solved exactly

Approximative solution by truncation of

most general form of effective action

derivative expansion

Tetradis,...; Morris

$O(N)$-model:

$$\Gamma_k = \int d^d x \{U_k(\rho) + \frac{1}{2} Z_k(\rho) \partial_\mu \varphi_a \partial_\mu \varphi_a + \frac{1}{4} Y_k(\rho) \partial_\mu \rho \partial_\mu \rho + \cdots\}$$

$$(N = 1 : \quad Y_k \equiv 0)$$

field expansion
(flow eq. for 1PI vertices)

Weinberg; Ellwanger,...

$$\Gamma_k = \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=0}^{n} d^d x_j \, \Gamma_k^{(n)}(x_1, x_2, \ldots, x_n) \prod_{j=0}^{n} (\phi(x_j) - \phi_0)$$

Expansion in canonical dimension of couplings

Lowest order:

$$d = 4: \quad \rho_0, \bar{\lambda}, Z$$

$$d = 3: \quad \rho_0, \bar{\lambda}, \bar{\gamma}, Z$$

$$U = \frac{1}{2}\bar{\lambda}(\rho - \rho_0)^2 + \frac{1}{6}\bar{\gamma}(\rho - \rho_0)^3$$

works well for $O(N)$ models

Tetradis,...; Tsypin

polynomial expansion of potential converges

if expanded around ρ_0

Tetradis,...; Aoki et al.

convergence and errors

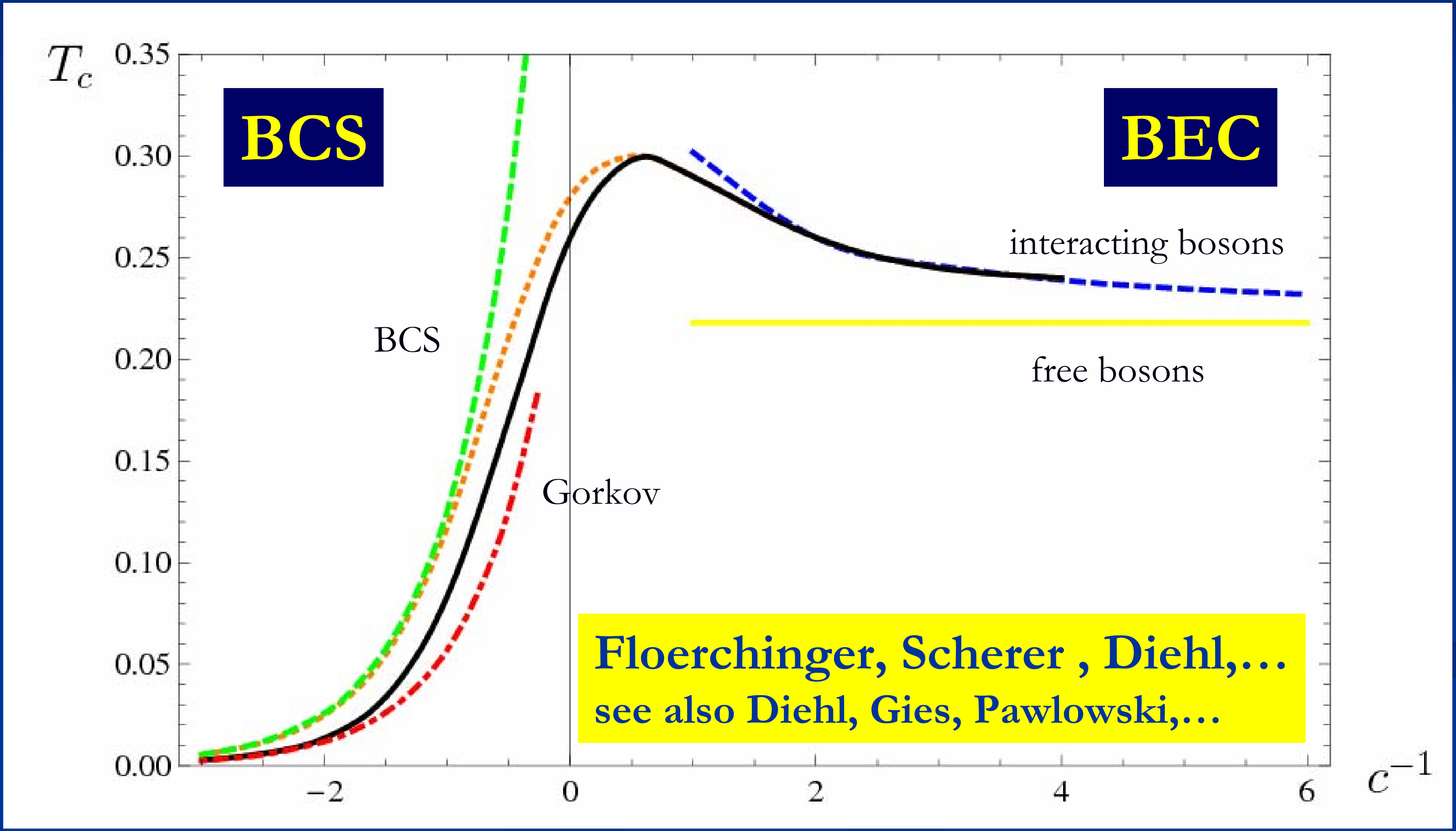
- apparent fast convergence : no series resummation
- rough error estimate by different cutoffs and truncations , Fierz ambiguity etc.
- in general : understanding of physics crucial
- no standardized procedure

including fermions :

no particular problem !

Universality in ultra-cold fermionic atom gases

BCS – BEC crossover



BEC – BCS crossover

Bound molecules of two atoms
on microscopic scale:

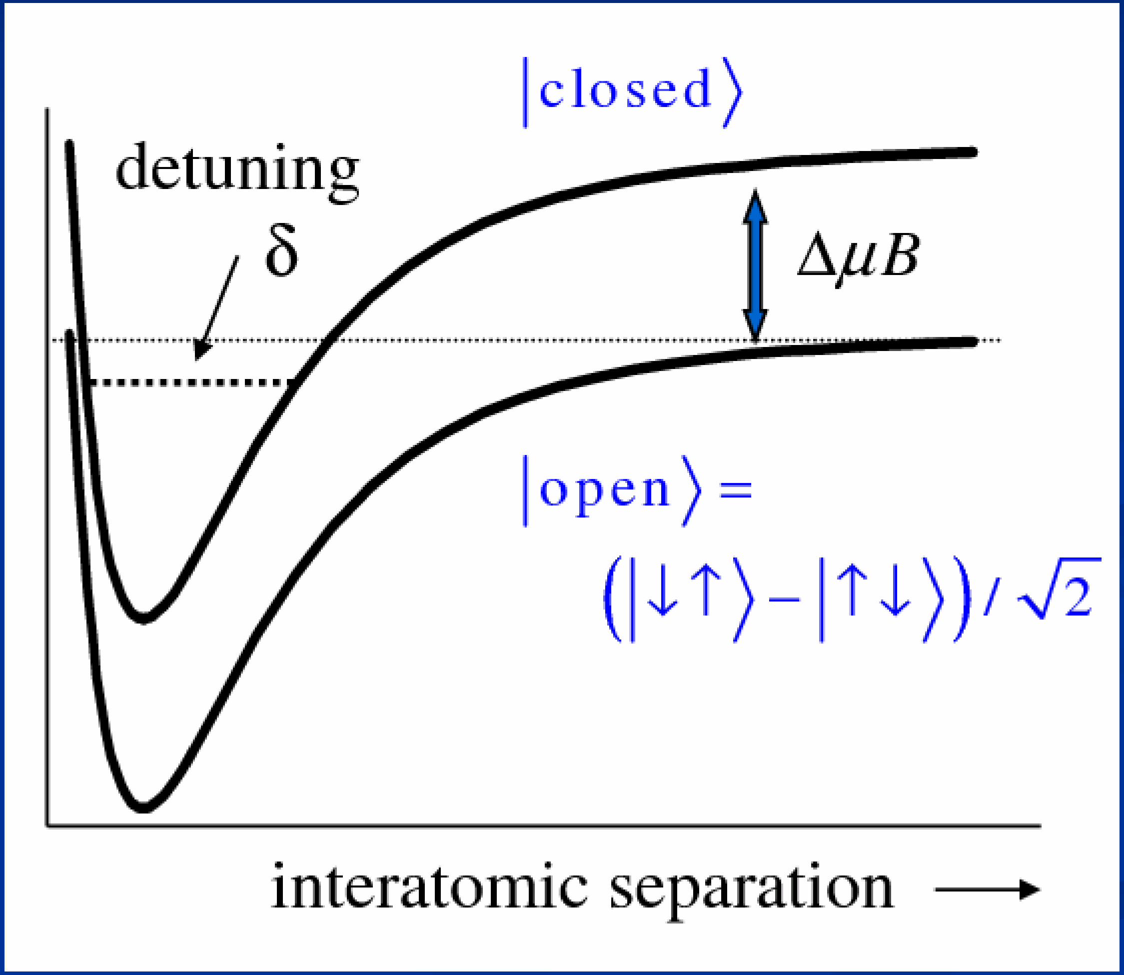
Bose-Einstein condensate (BEC) for low T

Fermions with attractive interactions
(molecules play no role) :

BCS – superfluidity at low T
by condensation of Cooper pairs

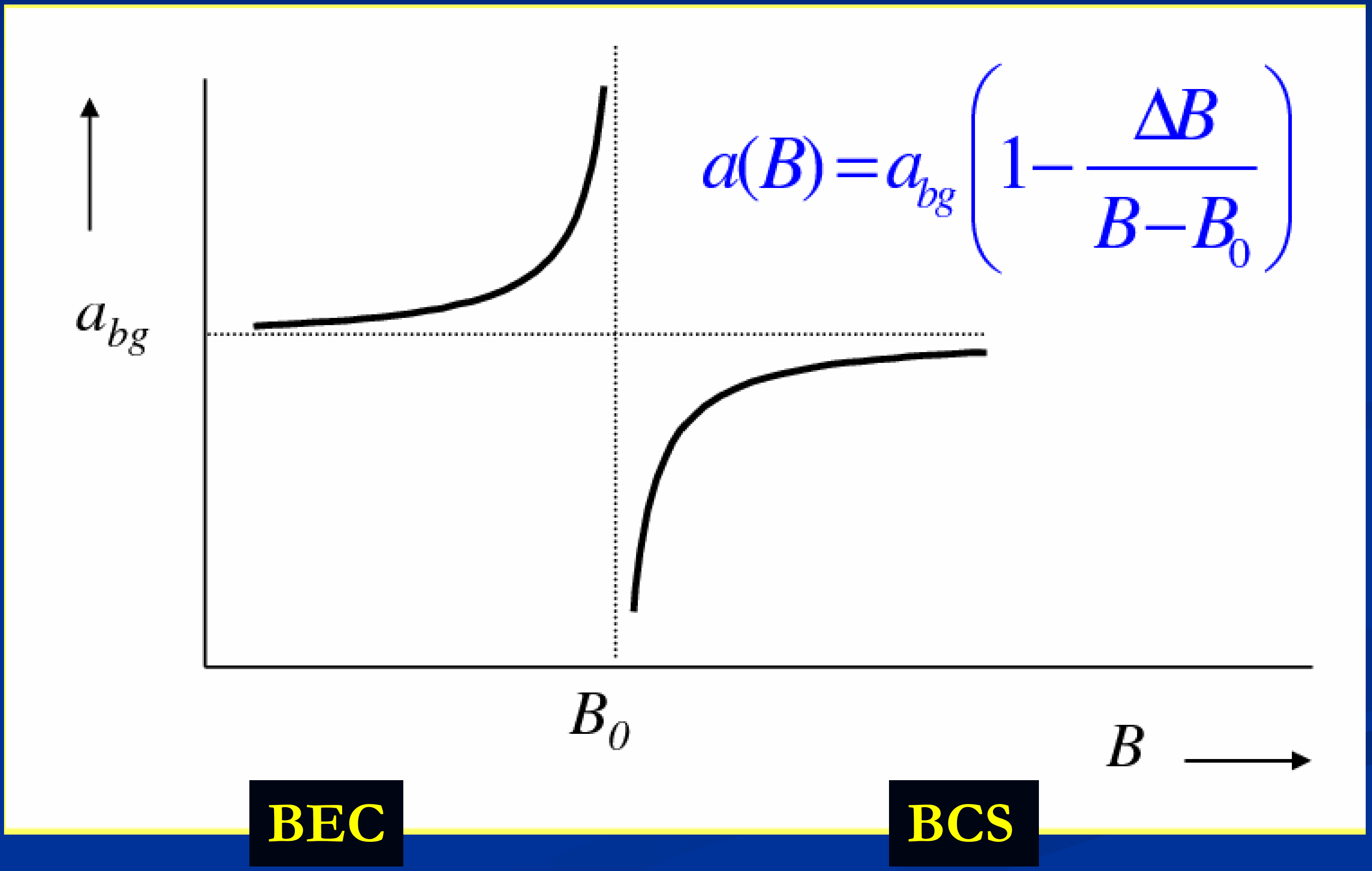
Crossover by Feshbach resonance
as a transition in terms of external magnetic field

Feshbach resonance

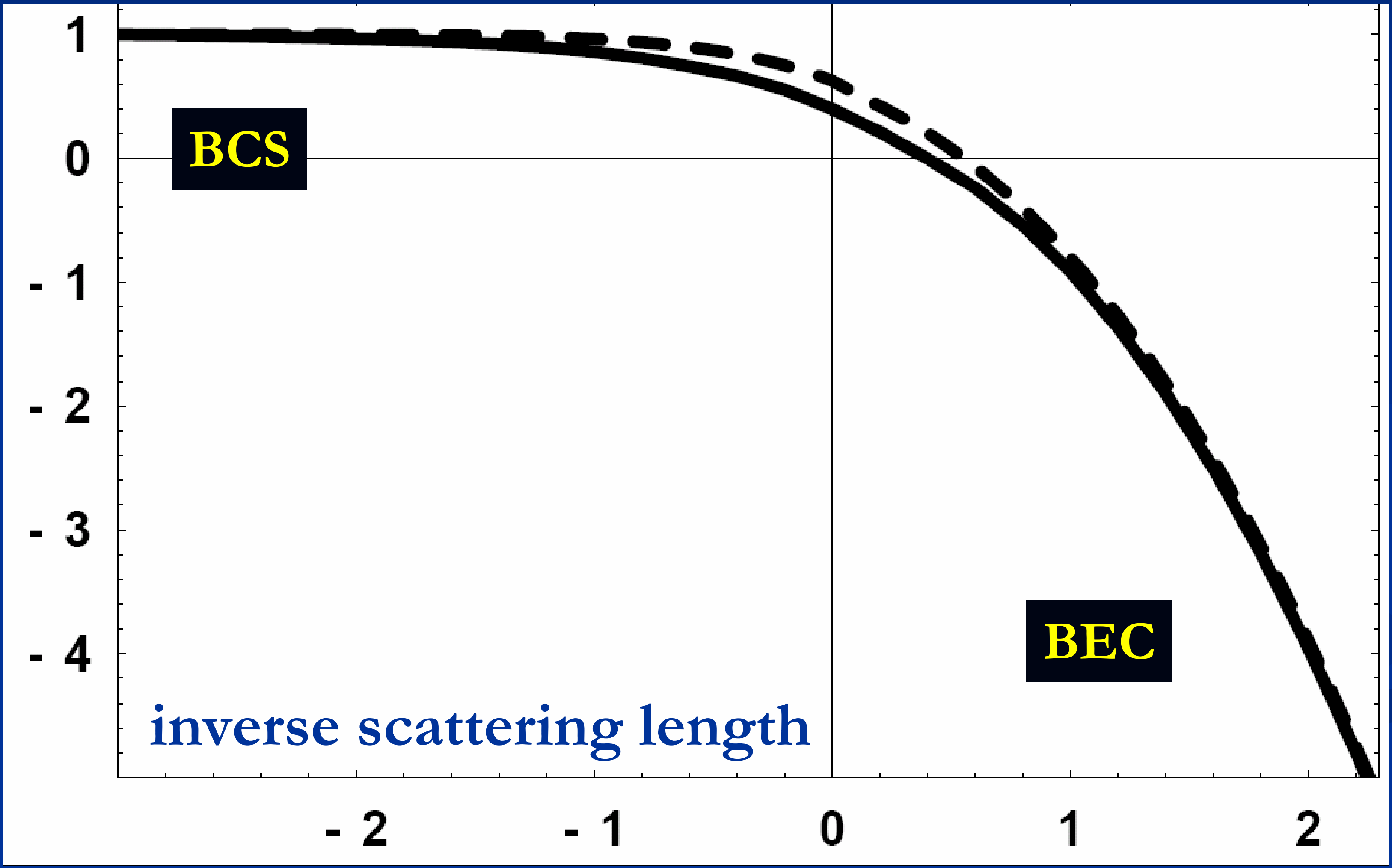


H.Stoof

scattering length

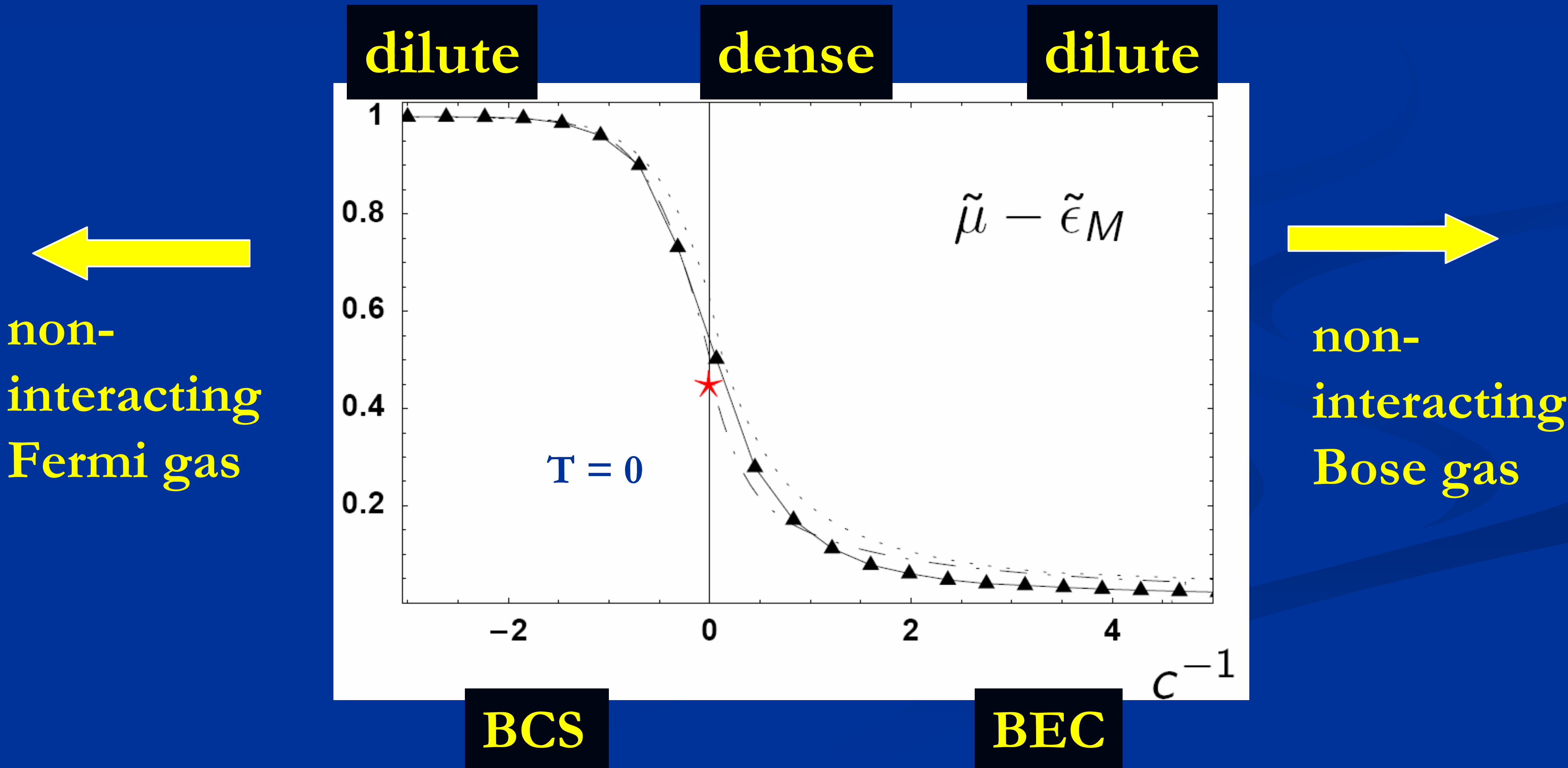


chemical potential



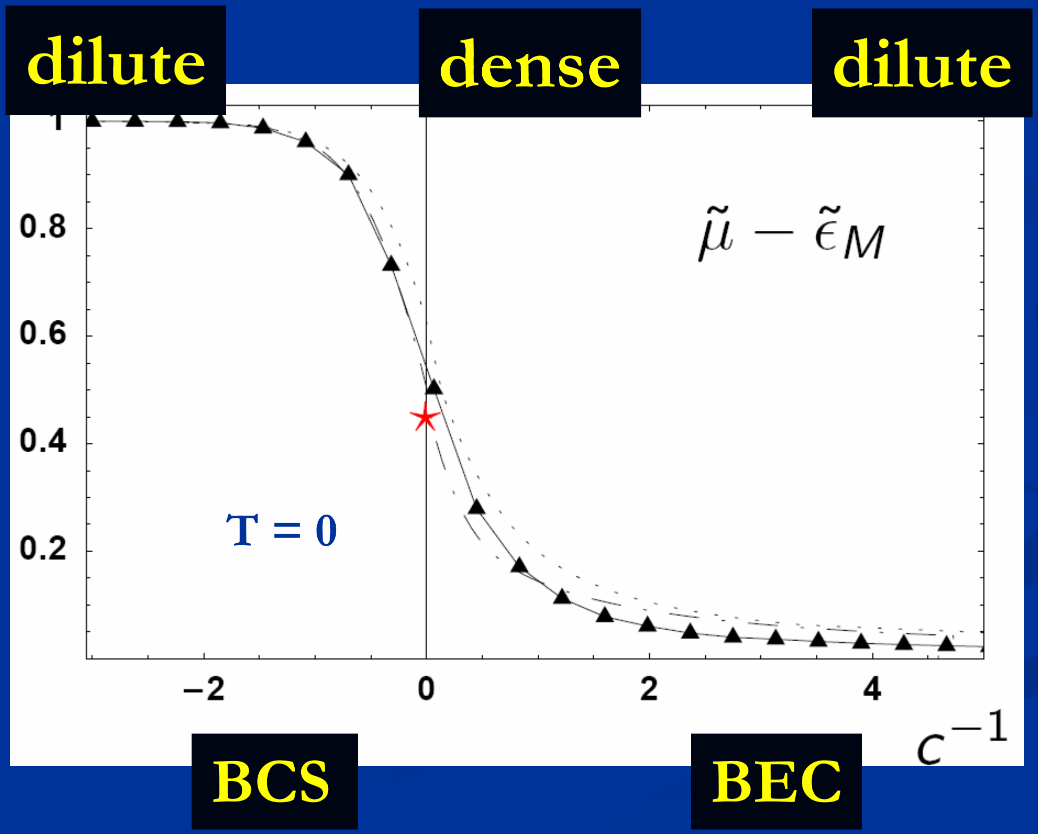
concentration

- $c = a\, k_F$, **a(B)** : scattering length
- needs computation of density $n = k_F^3/(3\pi^2)$

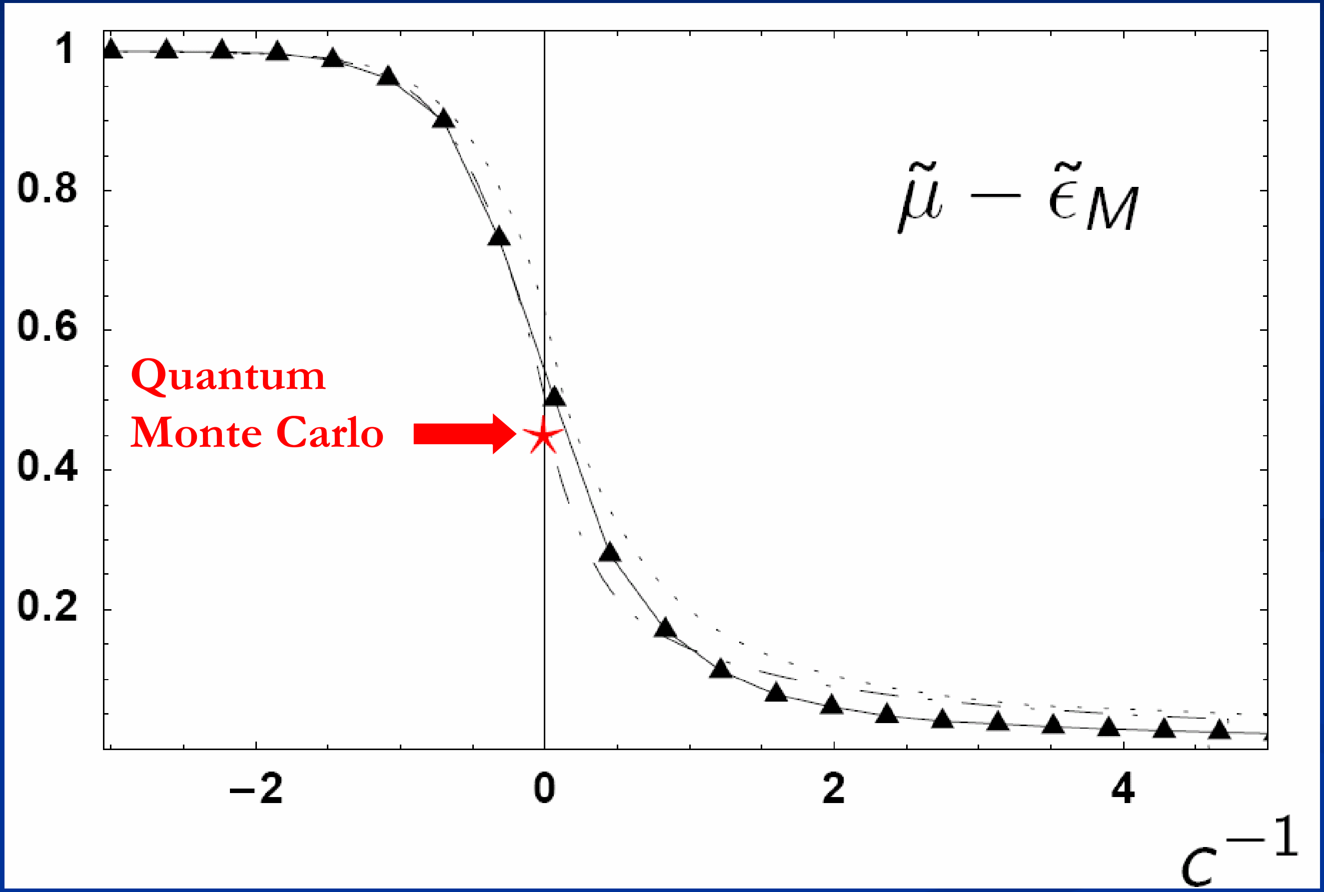


universality

same curve for Li and K atoms ?



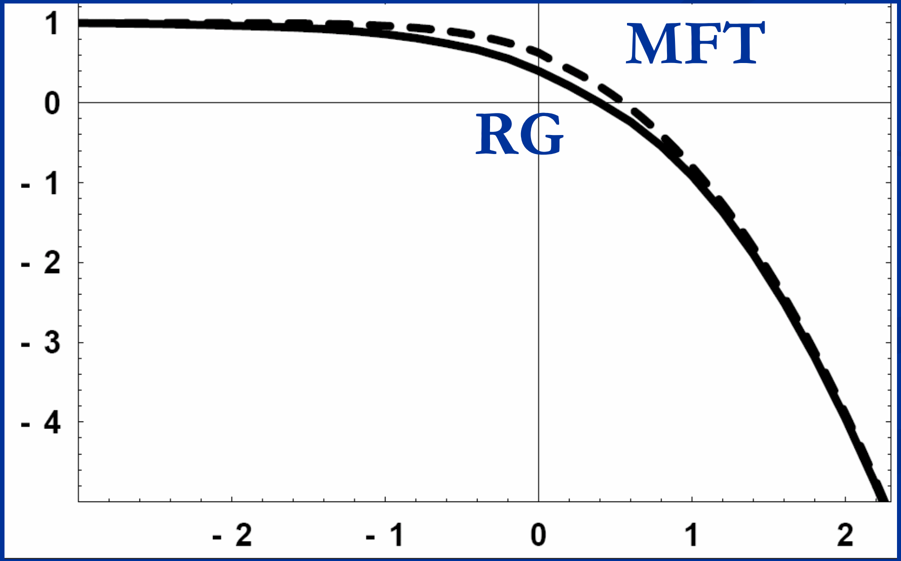
different methods



- Compare RGE (diamonds), SDE (dashed-dotted) and MFT (dashed) approximation schemes.

who cares about details ?

a theorists game …?



precision many body theory
- quantum field theory -

so far :

- particle physics : perturbative calculations

  magnetic moment of electron :

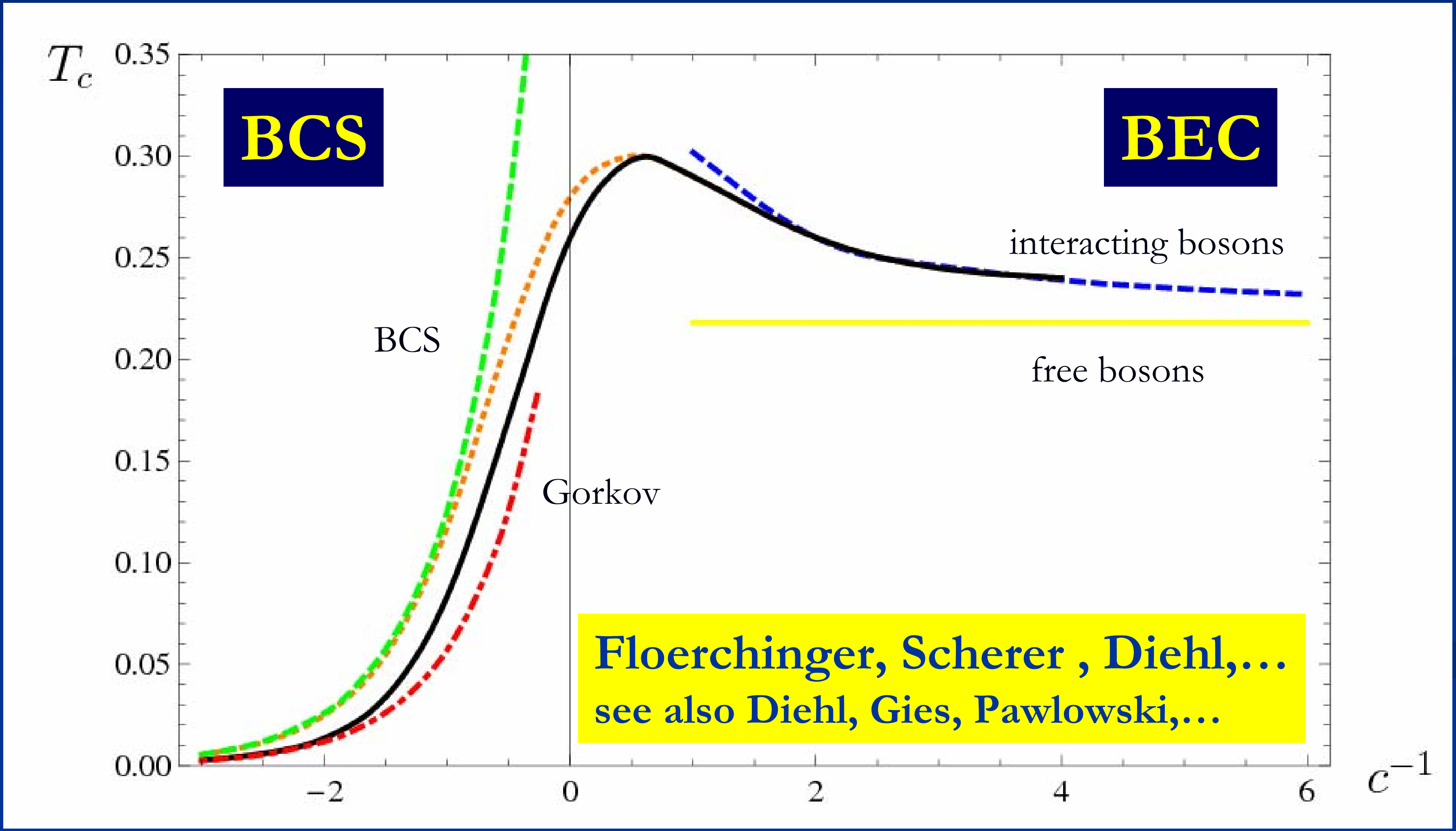
  g/2 = 1.001 159 652 180 85 (76) (Gabrielse et al.)

- statistical physics : universal critical exponents for second order phase transitions : $\nu = 0.6308\ (10)$

  renormalization group

- lattice simulations for bosonic systems in particle and statistical physics (e.g. QCD)

BCS – BEC crossover



QFT with fermions

needed:

universal theoretical tools for complex fermionic systems

wide applications :

electrons in solids ,

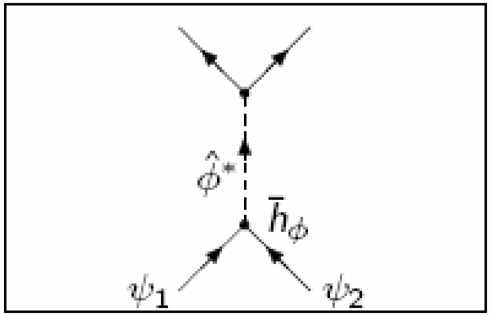
nuclear matter in neutron stars , ….

QFT for non-relativistic fermions

- functional integral, action

$$S = \int_x \{\psi^\dagger(\partial_\tau - \frac{\Delta}{2M} - \sigma)\psi$$
$$+\varphi^*(\partial_\tau - \frac{\Delta}{4M} + \bar{\nu}_\Lambda - 2\sigma)\varphi$$
$$-\bar{h}_\varphi(\varphi^*\psi_1\psi_2 - \varphi\psi_1^*\psi_2^*)\}$$

Molecule exchange



perturbation theory:
Feynman rules

τ : euclidean time on torus with circumference 1/T
σ : effective chemical potential

variables

- ψ : Grassmann variables
- φ : bosonic field with atom number two

What is φ ?

microscopic molecule,

macroscopic Cooper pair ?

All !

parameters

- detuning ν(B)

$$\bar{\nu}_\Lambda = \bar{\nu}_{\Lambda,0} + \bar{\mu}_B (B - B_0)$$

$$\frac{\partial \bar{\nu}_\Lambda}{\partial B} = \bar{\mu}_B$$

$$S = \int_x \{ \psi^\dagger (\partial_\tau - \frac{\Delta}{2M} - \sigma) \psi + \varphi^* (\partial_\tau - \frac{\Delta}{4M} + \bar{\nu}_\Lambda - 2\sigma) \varphi - \bar{h}_\varphi (\varphi^* \psi_1 \psi_2 - \varphi \psi_1^* \psi_2^*) \}$$

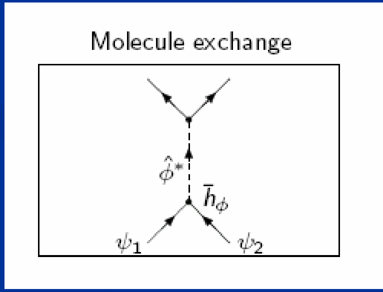
- Yukawa or Feshbach coupling h_φ

fermionic action

equivalent fermionic action , in general not local

$$S_F = \int_x \psi^\dagger(\partial_\tau - \frac{\Delta}{2M} - \sigma)\psi + S_{\text{int}}$$

$$S_{\text{int}} = -\frac{1}{2}\int_{Q_1,Q_2,Q_3} (\psi^\dagger(-Q_1)\psi(Q_2))(\psi^\dagger(Q_4)\psi(-Q_3)) \frac{\bar{h}_\varphi^2}{\bar{\nu}_\Lambda - 2\sigma + (\vec{q}_1 - \vec{q}_4)^2/4M + 2\pi i T(n_1 - n_4)}$$



scattering length a

$$\bar{\lambda} = -\frac{\bar{h}_\varphi^2}{\bar{\nu}_\Lambda}$$

a= M λ/4π

- broad resonance : pointlike limit
- large Feshbach coupling

$$\bar{h}_\varphi^2 \to \infty, \; \bar{\nu}_\Lambda \to \infty, \; \bar{\lambda} \text{ fixed}$$

$$S_{\rm int} = -\frac{1}{2}\int_{Q_1,Q_2,Q_3} (\psi^\dagger(-Q_1)\psi(Q_2))(\psi^\dagger(Q_4)\psi(-Q_3)) \frac{\bar{h}_\varphi^2}{\bar{\nu}_\Lambda - 2\sigma + (\vec{q}_1 - \vec{q}_4)^2/4M + 2\pi i T(n_1 - n_4)}$$

parameters

- Yukawa or Feshbach coupling h_φ
- scattering length a

Set of microscopic parameters:

$$\{\nu(B), \quad h_{\phi,0}\} \leftrightarrow \{a(B), \quad h_{\phi,0}\}.$$

- **broad resonance :** h_φ **drops out**

concentration c

$$c = ak_F = -\frac{Mk_F\bar{h}_\varphi^2}{4\pi\bar{\mu}_B(B - B_0)}$$

$$n = \frac{k_F^3}{3\pi^2}$$

universality

- Are these parameters enough for a quantitatively precise description ?
- Have Li and K the same crossover when described with these parameters ?
- Long distance physics looses memory of detailed microscopic properties of atoms and molecules !

universality for $c^{-1} = 0$: Ho,…(valid for broad resonance)

here: whole crossover range

analogy with particle physics

microscopic theory not known -

nevertheless "macroscopic theory" characterized by a finite number of

"renormalizable couplings"

m_e , α ; g_w , g_s , M_w , …

here : **c** , h_φ (only **c** for broad resonance)

analogy with universal critical exponents

only one relevant parameter :

$$T - T_c$$

units and dimensions

- $\hbar = 1$; $k_B = 1$
- momentum ~ length^{-1} ~ mass ~ eV
- energies : 2ME ~ (momentum)^{2}

  (M : atom mass)
- typical momentum unit : Fermi momentum
- typical energy and temperature unit : Fermi energy
- time ~ (momentum)$^{-2}$
- **canonical dimensions different from relativistic QFT !**

rescaled action

$$S = \int_{\hat{x}} \{ \hat{\psi}^\dagger (\hat{\partial}_\tau - \hat{\Delta} - \hat{\sigma}) \hat{\psi} + \hat{\varphi}^* (\hat{\partial}_\tau - \frac{1}{2}\hat{\Delta} + \hat{\nu} - 2\hat{\sigma}) \hat{\varphi} - \hat{h}_\varphi (\hat{\varphi}^* \hat{\psi}_1 \hat{\psi}_2 - \hat{\varphi} \hat{\psi}_1^* \hat{\psi}_2^*) \}$$

$$\hat{\psi} = \hat{k}^{-3/2}\psi \ , \ \hat{\varphi} = \hat{k}^{-3/2}\varphi,$$
$$\hat{x} = \hat{k}x \ , \ \hat{\tau} = \frac{\hat{k}^2}{2M}\tau,$$
$$\hat{\sigma} = \frac{2M\sigma}{\hat{k}^2} \ , \ \hat{h}_\varphi = \frac{2M\bar{h}_\varphi}{\sqrt{\hat{k}}}$$

- M drops out
- all quantities in units of k_F if $\hat{k} = k_F$

what is to be computed ?

Inclusion of fluctuation effects
via **functional integral**
leads to **effective action.**

This contains all relevant information for arbitrary T and n !

effective action

- integrate out all quantum and thermal fluctuations
- quantum effective action
- generates full propagators and vertices
- richer structure than classical action

$$\Gamma = \int_x \{\psi^\dagger(\partial_\tau - A_\psi\Delta - \sigma)\psi$$
$$+\varphi^*(\partial_\tau - A_\varphi\Delta)\varphi + u(\varphi)$$
$$-h_\varphi(\varphi^*\psi_1\psi_2 - \varphi\psi_1^*\psi_2^*) + \dots\}$$

effective action

$$\Gamma[\psi,\phi] = \int_0^{1/T} d\tau \int d^3x \Big\{ \psi^\dagger (\partial_\tau - A_\psi \triangle - \sigma)\psi +$$

$$\phi^* (\partial_\tau - A_\phi \triangle)\phi + U(\phi^*\phi) - \frac{h_\phi}{2}\Big(\phi^*\psi^T\epsilon\psi - \phi\psi^\dagger\epsilon\psi^*\Big) + \ldots\Big\}.$$

- includes all quantum and thermal fluctuations
- formulated here in terms of renormalized fields
- involves renormalized couplings

effective potential

minimum determines order parameter

$$u = m_\varphi^2 \rho + \frac{\lambda_\varphi}{2}\rho^2 \quad , \quad SYM$$
$$u = \frac{\lambda_\varphi}{2}(\rho - \rho_0)^2 \quad , \quad SSB$$

$$\rho = \varphi^* \varphi$$

condensate fraction

$$\Omega_c = 2\, \varrho_0 / n$$

$$\Gamma = \int_x \{\psi^\dagger(\partial_\tau - A_\psi \Delta - \sigma)\psi$$
$$+\varphi^*(\partial_\tau - A_\varphi \Delta)\varphi + u(\varphi)$$
$$-h_\varphi(\varphi^* \psi_1 \psi_2 - \varphi \psi_1^* \psi_2^*) + \dots \}$$

effective potential

- value of φ at potential minimum :
  order parameter , determines condensate fraction
- second derivative of U with respect to φ yields correlation length
- derivative with respect to σ yields density
- fourth derivative of U with respect to φ yields molecular scattering length

Quartic truncation for bosonic potential (displayed in symmetric phase):

$$U(\phi^*\phi) = (\nu(B) + \Delta m_\phi^2)\phi^*\phi + \frac{\lambda_\phi}{2}(\phi^*\phi)^2 + \ldots$$

renormalized fields and couplings

$$\psi = Z_\psi^{1/2}\hat{\psi}\ ,\ \varphi = Z_\varphi^{1/2}\hat{\varphi}$$

$$h_\varphi = Z_\varphi^{-1/2} Z_\psi^{-1} \hat{h}_\varphi$$

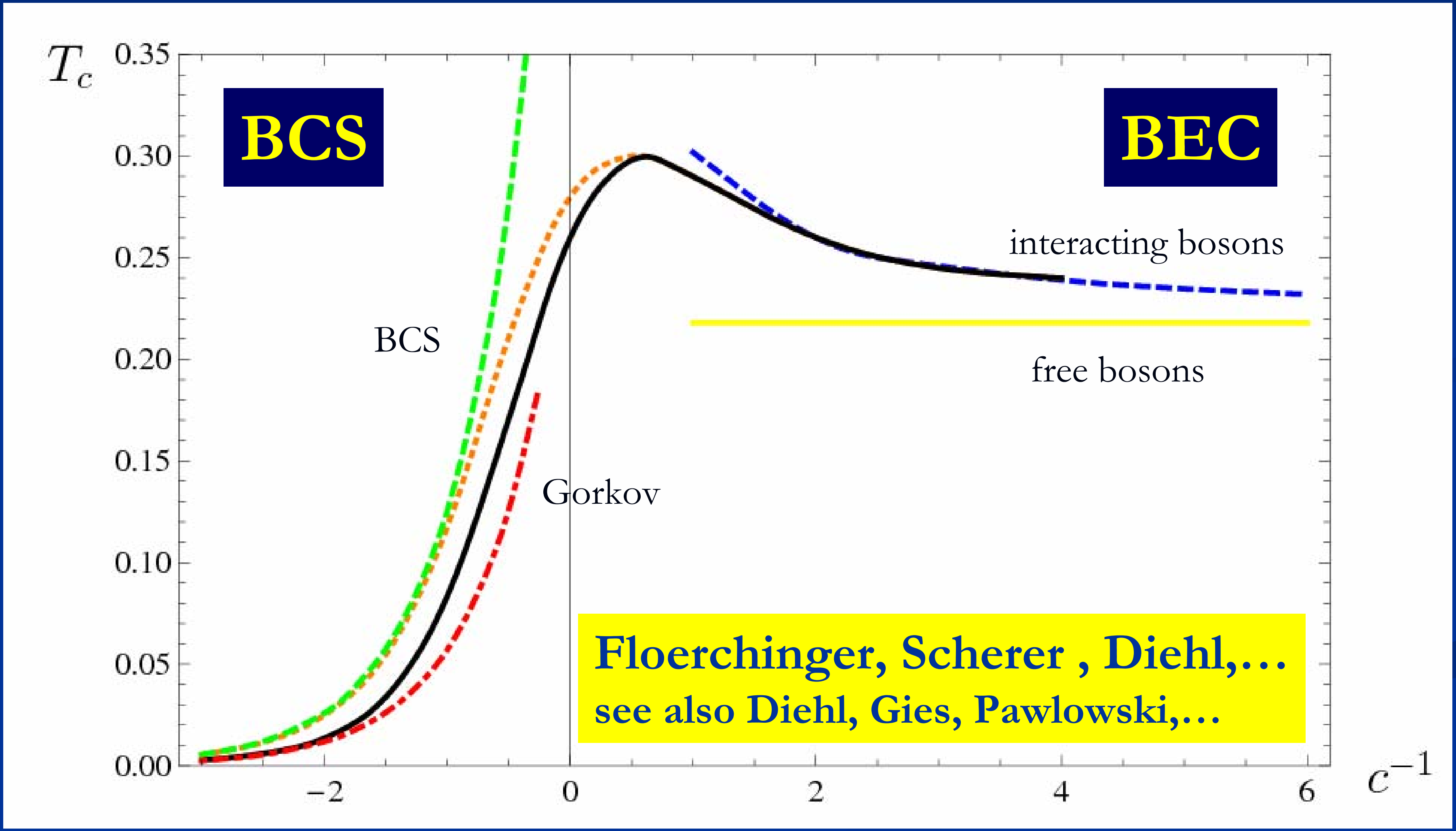
$$\Gamma = \int_x \{\psi^\dagger(\partial_\tau - A_\psi\Delta - \sigma)\psi$$
$$+\varphi^*(\partial_\tau - A_\varphi\Delta)\varphi + u(\varphi)$$
$$-h_\varphi(\varphi^*\psi_1\psi_2 - \varphi\psi_1^*\psi_2^*) + \dots\}$$

challenge for ultra-cold atoms :

Non-relativistic fermion systems with precision similar to particle physics !

(QCD with quarks)

BCS – BEC crossover



Unification from Functional Renormalization

- fluctuations in d=0,1,2,3,4,...
- ☺ linear and non-linear sigma models
- vortices and perturbation theory
- ☺ bosonic and fermionic models
- relativistic and non-relativistic physics
- ☺ classical and quantum statistics
- ☺ non-universal and universal aspects
- homogenous systems and local disorder
- equilibrium and out of equilibrium

wide applications

particle physics

- gauge theories, QCD

  Reuter,…, Marchesini et al, Ellwanger et al, Litim, Pawlowski, Gies ,Freire, Morris et al., Braun , many others

- electroweak interactions, gauge hierarchy problem

  Jaeckel, Gies,…

- electroweak phase transition

  Reuter, Tetradis,…Bergerhoff,

wide applications

gravity

- asymptotic safety

  Reuter, Lauscher, Schwindt et al, Percacci et al, Litim, Fischer, Saueressig

wide applications

condensed matter

- unified description for classical bosons

  CW , Tetradis , Aoki , Morikawa , Souma, Sumi , Terao , Morris , Graeter , v.Gersdorff , Litim , Berges , Mouhanna , Delamotte , Canet , Bervilliers , Blaizot , Benitez , Chatie , Mendes-Galain , Wschebor

- Hubbard model

  Baier , Bick,…, Metzner et al, Salmhofer et al, Honerkamp et al, Krahl , Kopietz et al, Katanin , Pepin , Tsai , Strack , Husemann , Lauscher

wide applications

condensed matter

- quantum criticality

  Floerchinger , Dupuis , Sengupta , Jakubczyk ,

- sine- Gordon model

  Nagy , Polonyi

- disordered systems

  Tissier , Tarjus , Delamotte , Canet

wide applications

condensed matter

- equation of state for CO_2 Seide,…
- liquid He^4 Gollisch,… and He^3 Kindermann,…
- frustrated magnets Delamotte, Mouhanna, Tissier
- nucleation and first order phase transitions
  Tetradis, Strumia,…, Berges,…

wide applications

condensed matter

- crossover phenomena

  Bornholdt , Tetradis ,…
- superconductivity (scalar QED_3)

  Bergerhoff , Lola , Litim , Freire,…
- non equilibrium systems

  Delamotte , Tissier , Canet , Pietroni , Meden , Schoeller , Gasenzer , Pawlowski , Berges , Pletyukov , Reininghaus

wide applications

nuclear physics

- effective NJL- type models

  Ellwanger , Jungnickel , Berges , Tetradis,…, Pirner , Schaefer , Wambach , Kunihiro , Schwenk

- di-neutron condensates

  Birse, Krippa,

- equation of state for nuclear matter

  Berges, Jungnickel …, Birse, Krippa

- nuclear interactions

  Schwenk

wide applications

ultracold atoms

- Feshbach resonances

  Diehl, Krippa, Birse , Gies, Pawlowski , Floerchinger , Scherer , Krahl ,

- BEC

  Blaizot, Wschebor, Dupuis, Sengupta, Floerchinger end