Emergence of new laws with Functional Renormalization

From

Microscopic Laws

(Interactions, classical action)

to

Fluctuations!

Macroscopic Observation

(Free energy functional, effective action)

different laws at different scales

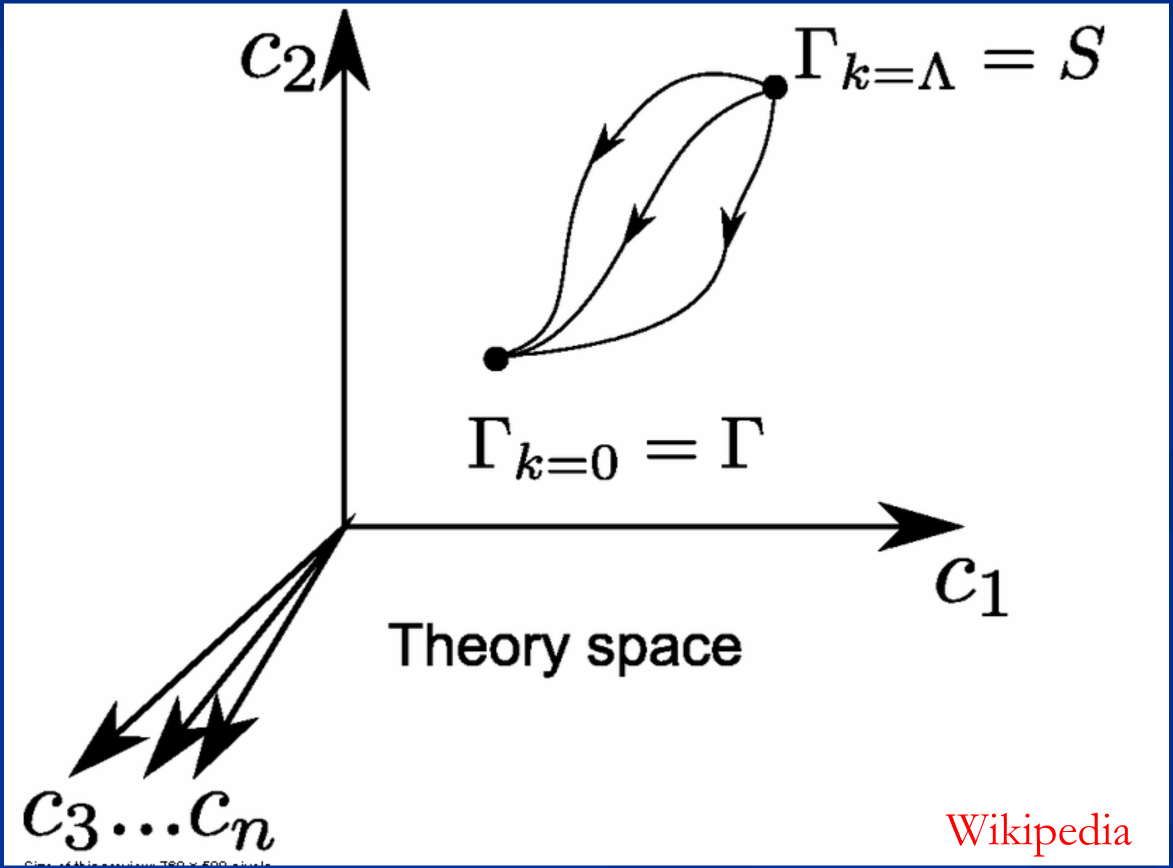
- fluctuations wash out many details of microscopic laws
- new structures as bound states or collective phenomena emerge
- elementary particles – earth – Universe :

key problem in Physics !

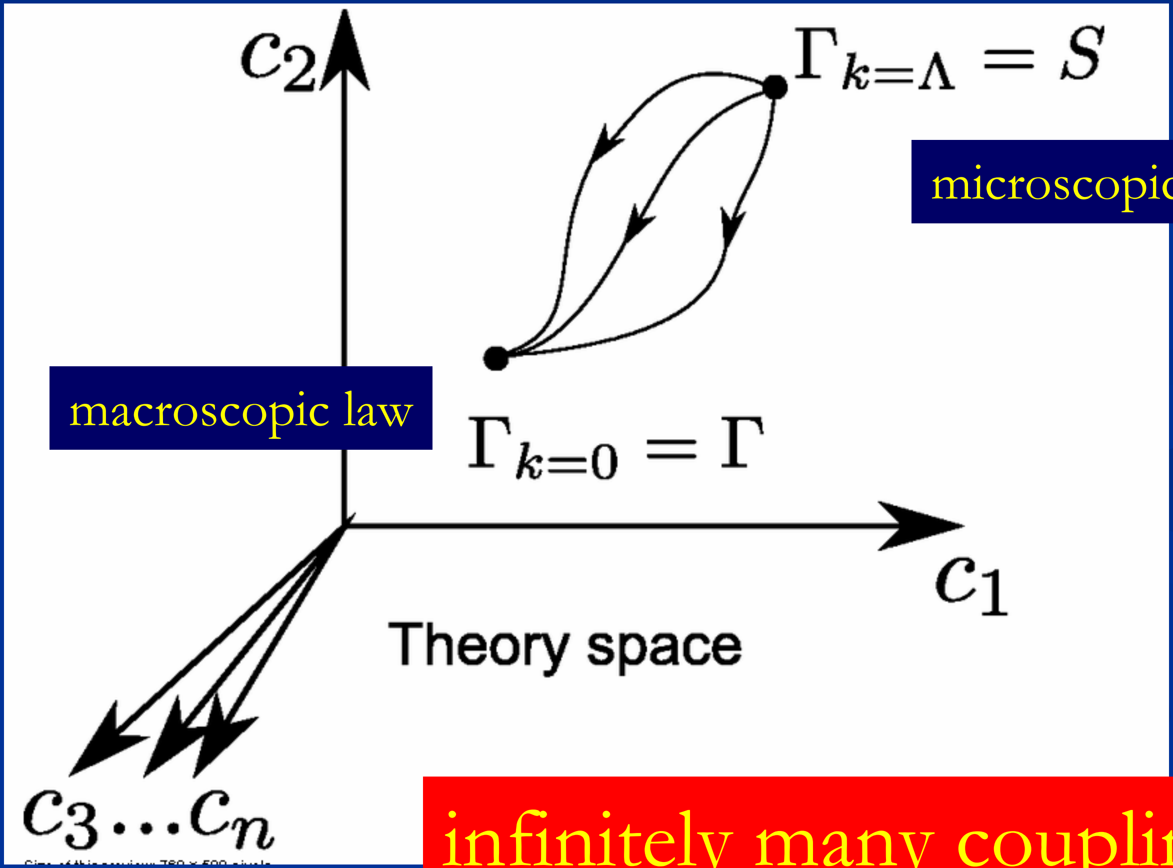
scale dependent laws

- scale dependent (running or flowing) couplings
- flowing functions
- flowing functionals

flowing action



flowing action



microscopic law

macroscopic law

infinitely many couplings

effective theories

planets

fundamental microscopic law for matter in solar system:

- Schroedinger equation for many electrons and nucleons,
- in gravitational potential of sun
- with electromagnetic and gravitational interactions (strong and weak interactions neglected)

effective theory for planets

at long distances , large time scales :

point-like planets , only mass of planets plays a role

- effective theory : Newtonian mechanics for point particles
- loss of memory
- new simple laws
- only a few parameters : masses of planets
- determined by microscopic parameters + history

QCD :
Short and long distance degrees of freedom are different !

Short distances : quarks and gluons
Long distances : baryons and mesons

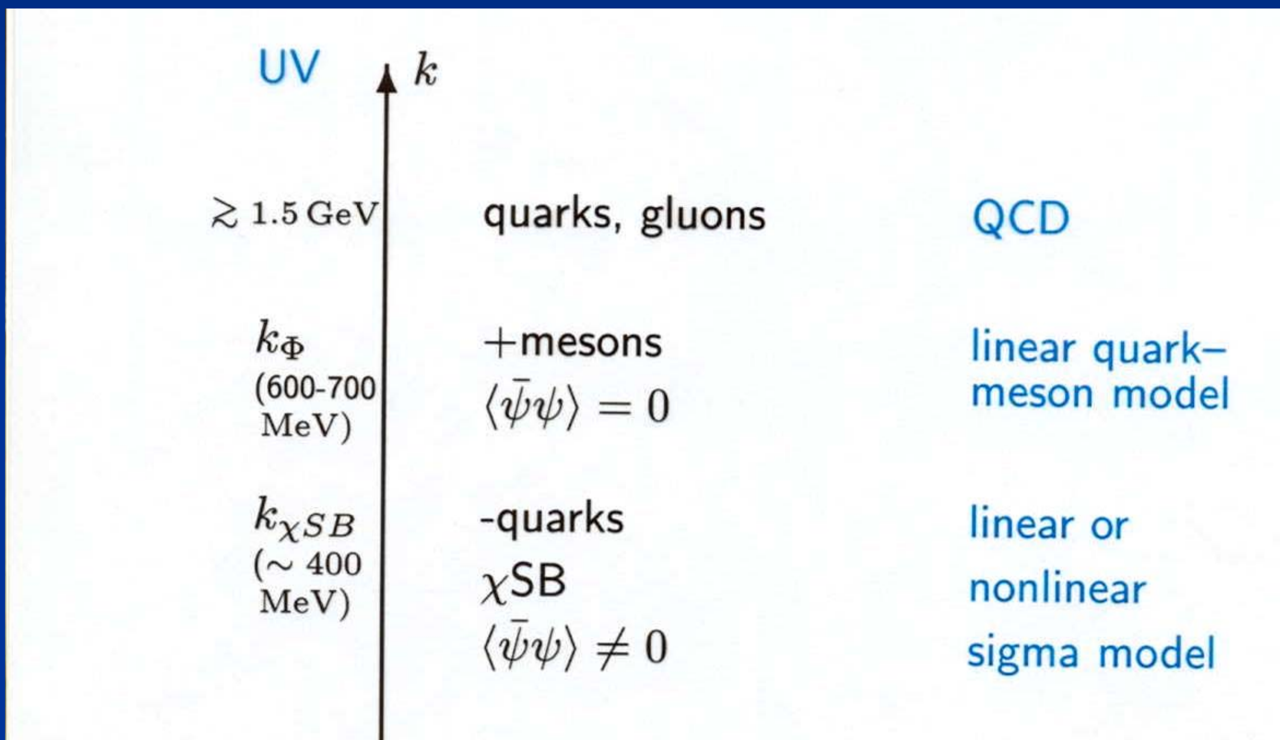
How to make the transition?

confinement/chiral symmetry breaking

functional renormalization

- transition from microscopic to effective theory is made continuous
- effective laws depend on scale k
- flow in space of theories
- flow from simplicity to complexity –
  if theory is simple for large k
- or opposite , if theory gets simple for small k

Scales in strong interactions



simple

complicated

simple

flow of functions

Effective potential includes all fluctuations

Average potential U_k

$\equiv$ scale dependent effective potential
$\equiv$ coarse grained free energy

Only fluctuations with momenta $q^2 > k^2$ included

k: infrared cutoff for fluctuations, "average scale"

Λ: characteristic scale for microphysics

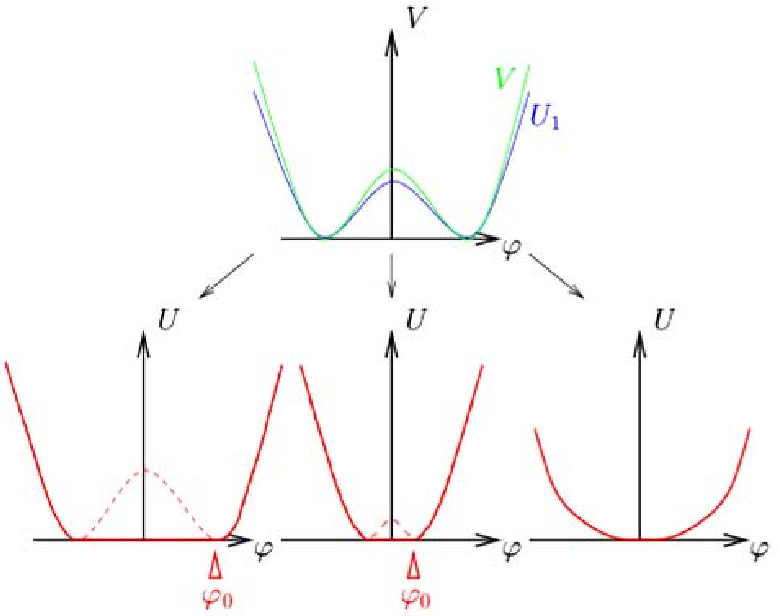
$$U_\Lambda \approx S \to U_0 \equiv U$$

Scalar field theory

$\varphi_a(x)$: magnetization, density, chemical concentration, Higgs field, meson field, inflaton, cosmon

$O(N)$-symmetry:

$$S = \int d^d x \left\{ \frac{1}{2} \partial_\mu \varphi_a \partial_\mu \varphi_a + V(\rho) \right\}; \quad \rho = \frac{1}{2} \varphi_a \varphi_a$$



Flow equation for average potential

$$\partial_k U_k(\varphi) = \frac{1}{2}\sum_i \int \frac{d^d q}{(2\pi)^d} \frac{\partial_k R_k(q^2)}{Z_k q^2 + R_k(q^2) + \bar{M}^2_{k,i}(\varphi)}$$

$\bar{M}^2_{k,ab} = \frac{\partial^2 U_k}{\partial\varphi_a \partial\varphi_b}$: Mass matrix

$\bar{M}^2_{k,i}$: Eigenvalues of mass matrix

R_k : IR-cutoff

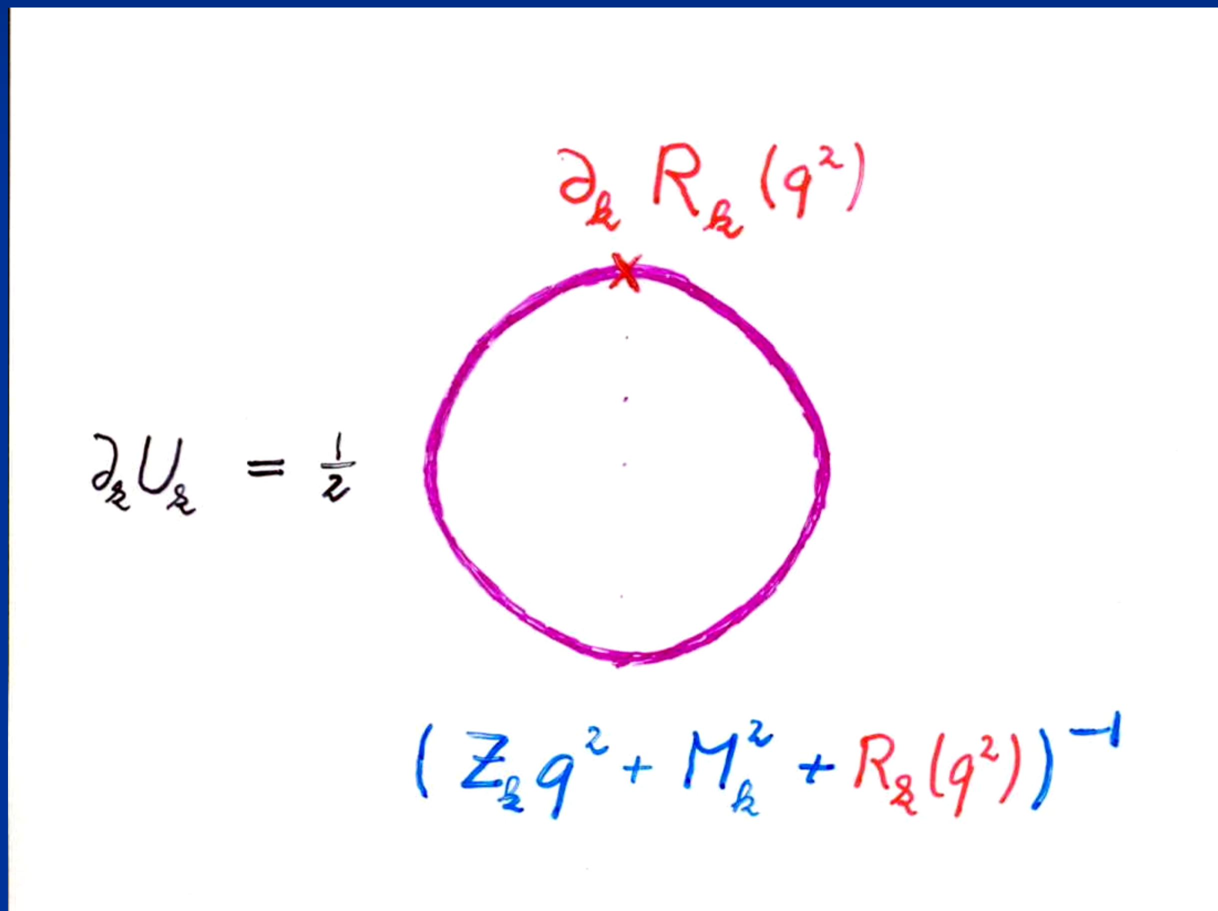
e.g $R_k = \frac{Z_k q^2}{e^{q^2/k^2} - 1}$

or $R_k = Z_k(k^2 - q^2)\Theta(k^2 - q^2)$ (Litim)

$\lim_{k\to 0} R_k = 0$

$\lim_{k\to\infty} R_k \to \infty$

Simple one loop structure – nevertheless (almost) exact



Infrared cutoff

R_k : IR-cutoff

e.g $\quad R_k = \dfrac{Z_k q^2}{e^{q^2/k^2} - 1}$

or $\quad R_k = Z_k(k^2 - q^2)\Theta(k^2 - q^2) \qquad \text{(Litim)}$

$\lim_{k\to 0} R_k = 0$

$\lim_{k\to\infty} R_k \to \infty$

Wave function renormalization and anomalous dimension

Z_k: wave function renormalization

$$k\partial_k Z_k = -\eta_k Z_K$$

η_k: anomalous dimension

$$t = \ln(k/\Lambda)$$

$$\partial_t \ln Z = -\eta$$

for Z_k (φ,q^2) : flow equation is **exact !**

Scaling form of evolution equation

$$u = \frac{U_k}{k^d}$$
$$\tilde{\rho} = Z_k k^{2-d} \rho$$
$$u' = \frac{\partial u}{\partial \tilde{\rho}} \quad \text{etc.}$$

$$\begin{aligned} \partial_t u|_{\tilde{\rho}} = & -du + (d - 2 + \eta)\tilde{\rho}u' \\ & +2v_d\{l_0^d(u' + 2\tilde{\rho}u''; \eta) \\ & +(N-1)\, l_0^d(u'; \eta)\} \end{aligned}$$

$$v_d^{-1} = 2^{d+1}\pi^{d/2}\Gamma\left(\frac{d}{2}\right)$$

linear cutoff:

$$l_0^d(w;\eta) = \frac{2}{d}\left(1 - \frac{\eta}{d+2}\right)\frac{1}{1+w}$$

On r.h.s. :
neither the scale k
nor the wave function
renormalization Z
appear explicitly.

Scaling solution:
no dependence on t;
corresponds
to second order
phase transition.

Tetradis …

unified approach

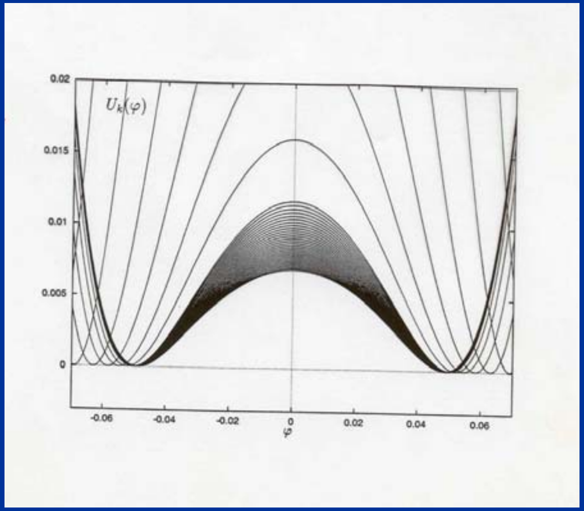
- choose N
- choose d
- choose initial form of potential
- run !

(quantitative results : systematic derivative expansion in second order in derivatives)

unified description of
scalar models for all d and N

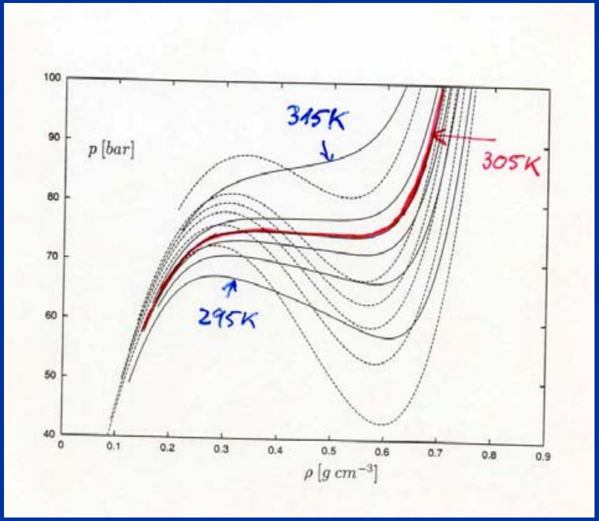
Flow of effective potential

Ising model



S.Seide …

CO_2



Experiment :

T_* =304.15 K
p_* =73.8.bar
ρ_* = 0.442 g cm-2

Critical exponents

$d = 3$

Critical exponents ν and η

N	ν		η	
0	0.590	0.5878	0.039	0.0292
1	0.6307	0.6308	0.0467	0.0356
2	0.666	0.6714	0.049	0.0385
3	0.704	0.7102	0.049	0.0380
4	0.739	0.7474	0.047	0.0363
10	0.881	0.886	0.028	0.025
100	0.990	0.980	0.0030	0.003

↑ ↑

"average" of other methods
(typically $\pm(0.0010 - 0.0020)$)

critical exponents , BMW approximation

N	η	η (other)	ν	ν (other)	ω (prelim.)	ω (other)
0	0.033(3)	0.028(3) [1]	0.588	0.588(1) [1]	0.80	
1	0.039(3)	0.0364(2) [2] 0.0368(2) [3] 0.033(3) [1]	0.6298(4)	0.6301(2) [2] 0.6302(1) [3] 0.630(1) [1]	0.78	0.79(1) [1]
2	0.041(3)	0.0381(2) [4] 0.035(3) [1]	0.6719(4)	0.6717(1) [4] 0.670(2) [1]	0.78	0.79(1) [1]
3	0.040(3)	0.0375(5) [5] 0.036(3) [1]	0.709	0.7112(5) [5] 0.707(4) [1]	0.73	
4	0.038(3)	0.035(5)[1] 0.037(1) [6]	0.738	0.741(6) [1] 0.749(2) [6]	0.74	0.77(2) [1]
5	0.035(3)	0.031(3) [8] 0.034(1) [7]	0.768	0.764(4) [8] 0.779(3) [7]	0.73	0.77(2) [1]
10	0.022(2)	0.024 [9]	0.860	0.859 [9]	0.81	
20	0.012(1)	0.014 [9]	0.929	0.930 [9]	0.94	
100	0.0023(2)	0.0027 [10]		0.989 [10]	0.99	

[1] R. Guida and J. Zinn-Justin '98. [2] M. Campostrini, A. Pelissetto, P. Rossi, E. Vicari '02.
[3] Y. Deng and H. W. J. Blote '03. [4] M. Campostrini, M. Hasenbusch, A. Pelissetto, E. Vicari '06.
[5] M. Campostrini, M. Hasenbusch, A. Pelissetto, P. Rossi, E. Vicari '02. [6] M. Hasenbusch '01.
[7] M. Hasenbusch, A. Pelissetto, E. Vicari '05. [8] A. Butti and F. Parisen Toldin '05.
[9] S. A. Antonenko and A. I. Sokolov '95. [10] M. Moshe and J. Zinn-Justin '03.

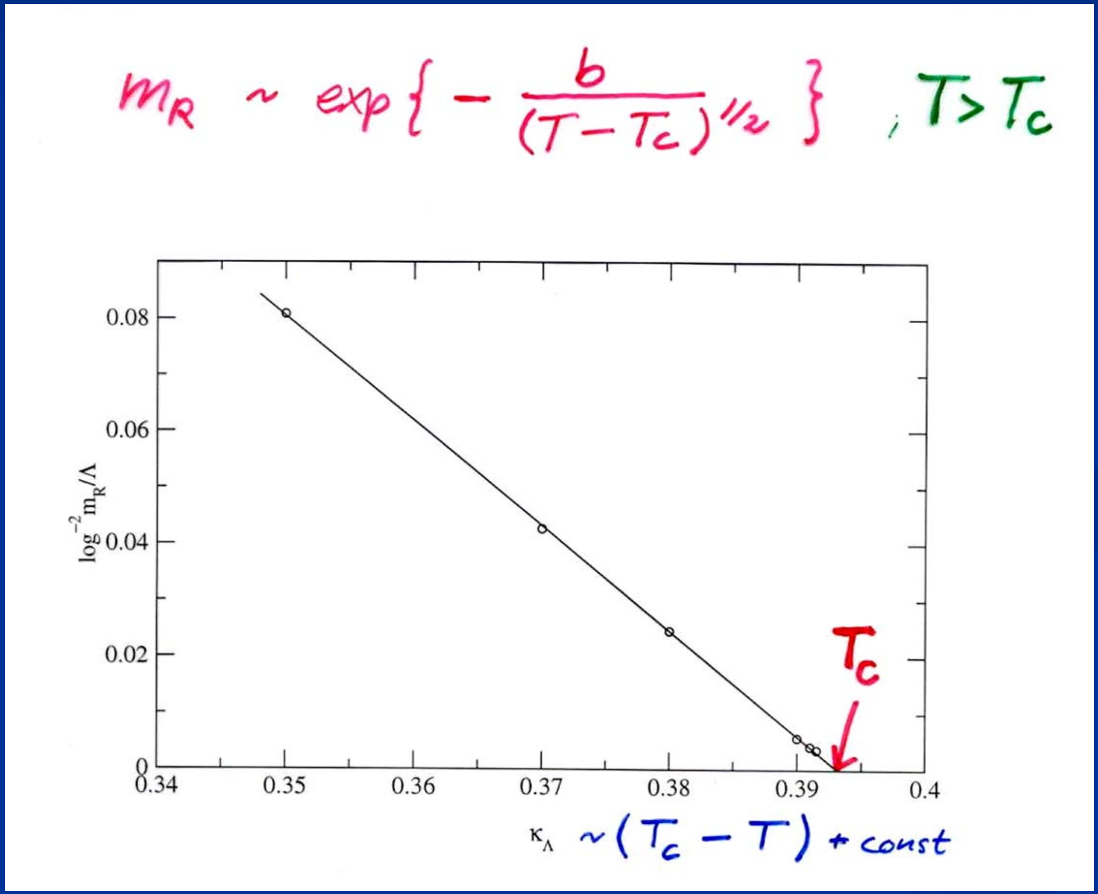
Blaizot, Benitez , … , Wschebor

Solution of partial differential equation :

yields highly nontrivial non-perturbative results despite the one loop structure !

Example:
Kosterlitz-Thouless phase transition

Essential scaling : d=2,N=2



- Flow equation contains correctly the non-perturbative information !
- (essential scaling usually described by vortices)

Von Gersdorff …

Kosterlitz-Thouless phase transition (d=2,N=2)

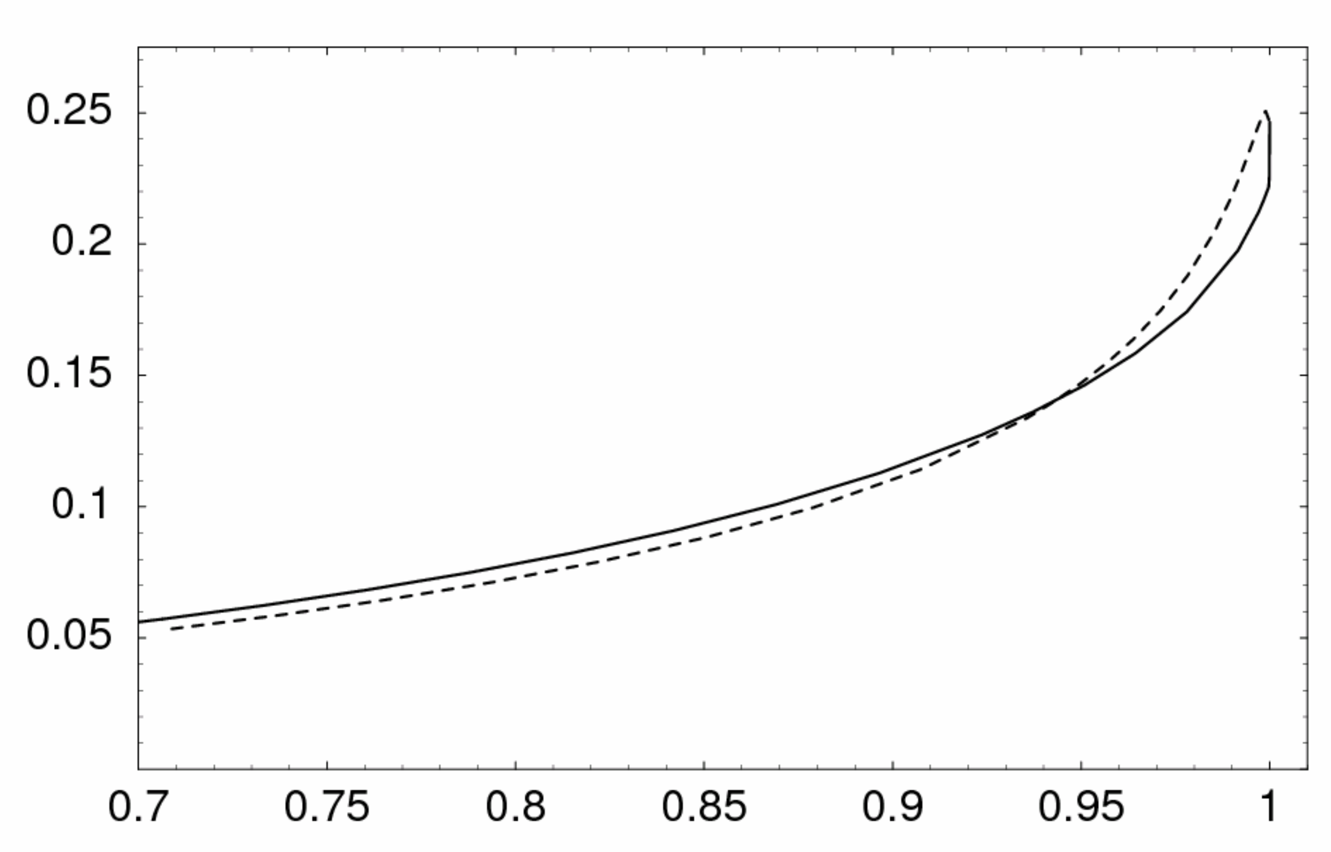
Correct description of phase with

Goldstone boson

(infinite correlation length)

for $T<T_c$

Temperature dependent anomalous dimension η



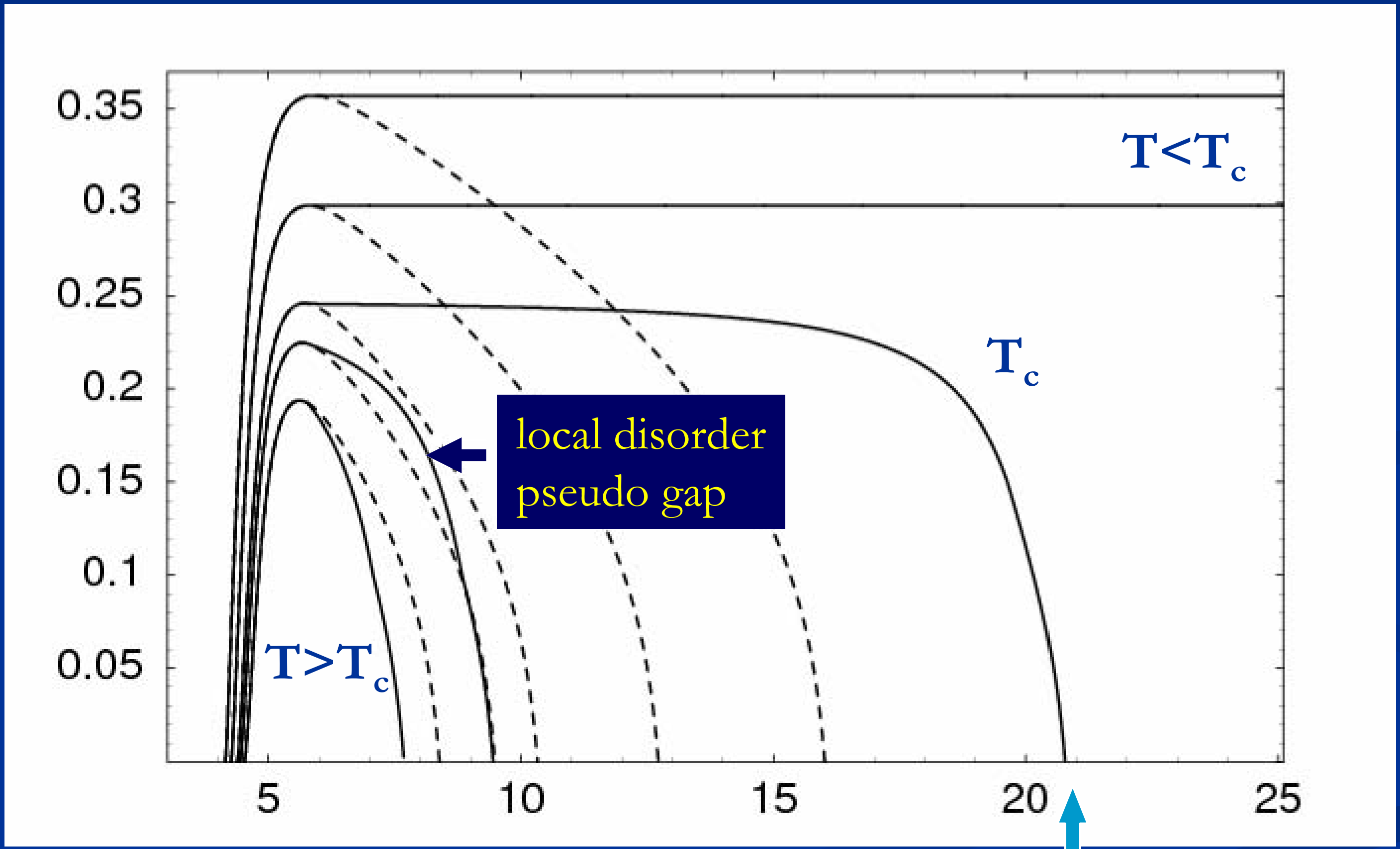
η

T/T_c

Running renormalized d-wave superconducting order parameter $\varkappa$ in doped Hubbard (-type) model

$\varkappa$

location of minimum of u

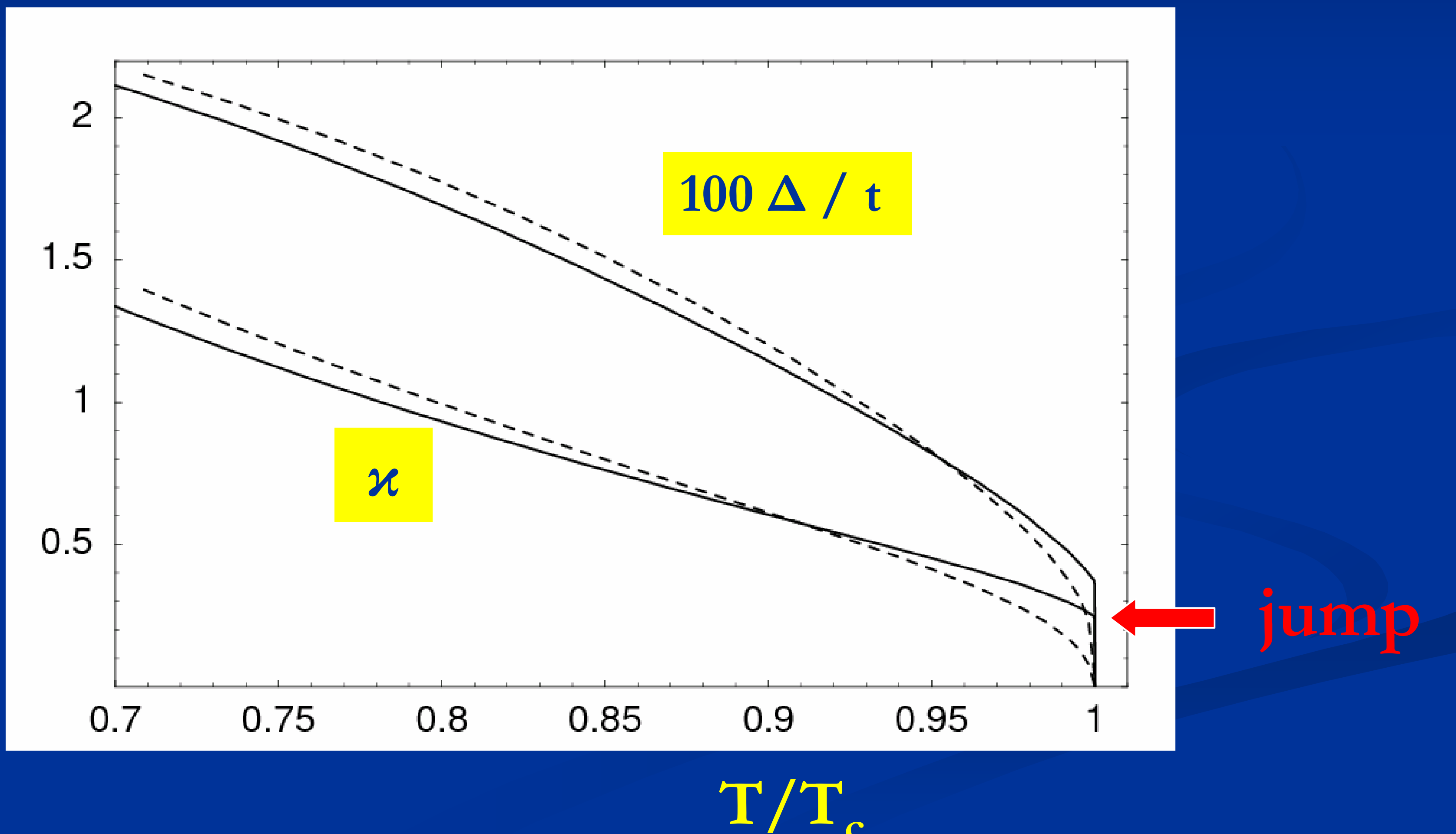


C.Krahl,…

- ln (k/Λ)

macroscopic scale 1 cm

Renormalized order parameter $\varkappa$ and gap in electron propagator Δ in doped Hubbard model



unification

abstract laws

quantum gravity

grand unification

standard model

electro-magnetism

gravity

Landau theory | universal critical physics | functional renormalization

complexity

flow of functionals

$$f(x) \longrightarrow f\,[\varphi(x)]$$

Exact renormalization group equation

Exact flow equation

for scale dependence of average action

$$\partial_k \Gamma_k[\varphi] = \frac{1}{2} \mathrm{Tr} \left\{ \left(\Gamma_k^{(2)}[\varphi] + R_k \right)^{-1} \partial_k R_k \right\}$$

'92

$\left(\Gamma_k^{(2)}\right)_{ab}(q, q') = \frac{\delta^2 \Gamma_k}{\delta\varphi_a(-q)\delta\varphi_b(q')}$

$\mathrm{Tr} : \sum_a \int \frac{d^d q}{(2\pi)^d}$

(fermions : STr)

some history … (the parents)

- **exact RG equations :**

  Symanzik eq. , Wilson eq. , Wegner-Houghton eq. , Polchinski eq. ,

  mathematical physics

- **1PI :** RG for 1PI-four-point function and hierarchy

  Weinberg

  formal Legendre transform of Wilson eq.

  Nicoll, Chang

- **non-perturbative flow :**

  d=3 : sharp cutoff ,

  no wave function renormalization or momentum dependence

  Hasenfratz^2

flow equations and composite degrees of freedom

Flowing quark interactions

quark – model (gluons integrated out)

rough truncation : (with U. Ellwanger)

$$\Gamma[\psi] = \Gamma_{2,k} + \Gamma_{4,k}$$

$$\Gamma_{2,k} = \int \frac{d^4q}{(2\pi)^4}\, \bar\psi_a^i(q)\, \not q\, \psi_i^a(q)$$

$$\Gamma_{4,k} = \frac{1}{2}\int \prod_{\ell=1}^{4}\left(\frac{d^4p_\ell}{(2\pi)^4}\right)(2\pi)^4\,\delta^4(p_1+p_2-p_3-p_4)\,\lambda_k(p_1,p_2,p_3,p_4)\,\mathcal{M}$$

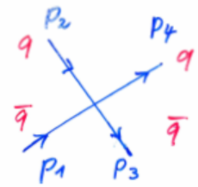
$$\mathcal{M} = [\bar\psi_a^i(-p_1)\,\psi_i^b(p_2)][\bar\psi_b^j(p_4)\,\psi_j^a(-p_3)] - [\bar\psi_a^i(-p_1)\gamma_5\,\psi_i^b(p_2)][\bar\psi_b^j(p_4)\gamma_5\,\psi_j^a(-p_3)]$$

$i = 1 \ldots N_c$ colour index

$a = 1 \ldots N_F$ flavour index

$\mathcal{M}$ is not the most general chirally invariant four point function !

Projection on colour singlet scalars



: $\lambda_k(p_1,p_2,p_3,p_4)$

initial conditions : λ_{k_0} , $k_0 \approx 1.5\,\mathrm{GeV}$

$$\lambda_{k_0} = \frac{2\pi\alpha_s}{t} + \frac{8\pi\lambda}{t^2} \quad \left(\equiv \tfrac{1}{2}V(t)\right)$$

$$s = (p_1+p_2)^2 = (p_3+p_4)^2$$

$$t = (p_1-p_3)^2 = (p_2-p_4)^2$$

$\lambda \sim 0.18\ \mathrm{GeV}^2$ (string tension)

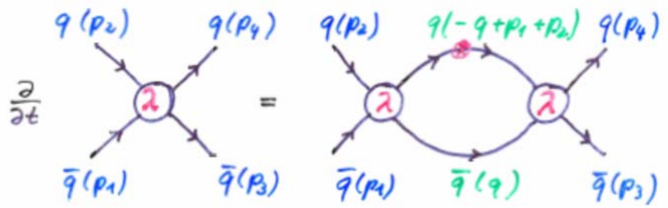
$\alpha_s \sim 0.3$

(λ_{k_0} should be given in next step by integrating out gluons)

U. Ellwanger,… Nucl.Phys.B423(1994)137

Flowing four-quark vertex

evolution equation



$$\frac{\partial}{\partial t}\lambda_k(p_1,p_2,p_3,p_4) = -\frac{8N_c}{k^2}\cdot$$

$$\cdot\int\frac{d^4q}{(2\pi)^4}\,\lambda_k(p_1,p_2,q,-q+p_1+p_2)\,\lambda_k(q,-q+p_1+p_2,p_3,p_4)$$

$$\cdot\left[\frac{q^\mu(q-p_1-p_2)_\mu}{q^2}\exp\left(-\frac{(q-p_1-p_2)^2}{k^2}\right)\cdot\right.$$

$$\left.\cdot\left(\exp\left(-\frac{q^2}{k_0^2}\right)-\exp\left(-\frac{q^2}{k^2}\right)\right)+q\rightarrow p_1+p_2-q\right]$$

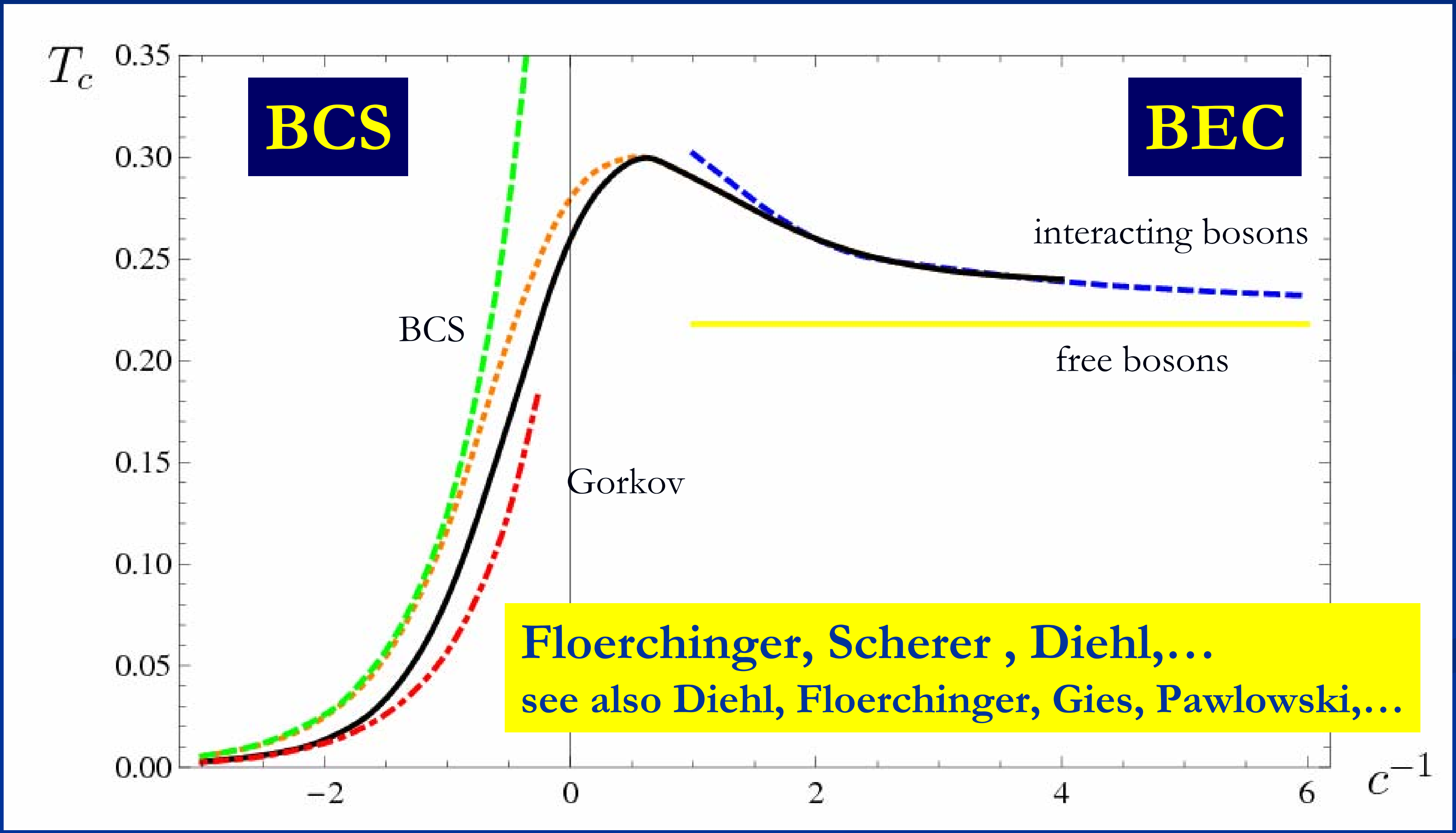
At $k \approx 0.65$ GeV

λ_k becomes „independent of t"

Then: „poles" in s-channel appear

emergence of mesons

BCS – BEC crossover



changing degrees of freedom

Anti-ferromagnetic order in the Hubbard model

transition from

microscopic theory for fermions to
macroscopic theory for bosons

T.Baier, E.Bick, …
C.Krahl, J.Mueller, S.Friederich

Hubbard model

Functional integral formulation

$$
\begin{aligned}
Z[\eta] = & \int_{\hat\psi(\beta)=-\hat\psi(0),\hat\psi^*(\beta)=-\hat\psi^*(0)} \mathcal{D}(\hat\psi^*(\tau),\hat\psi(\tau)) \\
& \exp\Big(-\int_0^\beta d\tau \Big(\sum_{\mathbf{x}} \hat\psi^\dagger_{\mathbf{x}}(\tau)\big(\frac{\partial}{\partial\tau}-\mu\big)\hat\psi_{\mathbf{x}}(\tau) \\
& \quad + \sum_{\mathbf{xy}} \hat\psi^\dagger_{x}(\tau)\mathcal{T}_{\mathbf{xy}}\hat\psi_{\mathbf{y}}(\tau) \\
& \quad + \frac{1}{2}U\sum_{\mathbf{x}} \big(\hat\psi^\dagger_{\mathbf{x}}(\tau)\hat\psi_{\mathbf{x}}(\tau)\big)^2 \\
& \quad - \sum_{\mathbf{x}} \big(\eta^\dagger_{\mathbf{x}}(\tau)\hat\psi_{\mathbf{x}}(\tau) + \eta^T_{\mathbf{x}}(\tau)\hat\psi^*_{\mathbf{x}}(\tau)\big)\Big)\Big)
\end{aligned}
$$

next neighbor interaction

$$
\mathcal{T}_{xy} = \begin{cases} -t & \text{, if } \boldsymbol{x} \text{ and } \boldsymbol{y} \text{ are nearest neighbors} \\ 0 & \text{, else} \end{cases}
$$

External parameters
T : temperature
μ : chemical potential
(doping)

U > 0 :
repulsive local interaction

Fermion bilinears

$$\tilde{\rho}(X) = \hat{\psi}^\dagger(X)\hat{\psi}(X),$$
$$\tilde{\vec{m}}(X) = \hat{\psi}^\dagger(X)\vec{\sigma}\hat{\psi}(X)$$

Introduce sources for bilinears

$$S_F = S_{F,\mathrm{kin}} + \frac{1}{2}U(\hat{\psi}^\dagger\hat{\psi})^2 - J_\rho\tilde{\rho} - \vec{J}_m\tilde{\vec{m}}$$

Functional variation with respect to sources J yields expectation values and correlation functions

$$Z = \int \mathcal{D}(\psi^*,\psi)\exp\left(-(S_F + S_\eta)\right)$$
$$S_\eta = -\eta^\dagger\psi - \eta^T\psi^*$$

Partial Bosonisation

- collective bosonic variables for fermion bilinears
- insert identity in functional integral
  (Hubbard-Stratonovich transformation)
- replace four fermion interaction by equivalent bosonic interaction (e.g. mass and Yukawa terms)
- problem : decomposition of fermion interaction into bilinears not unique (Grassmann variables)

$$(\hat{\psi}^\dagger(X)\hat{\psi}(X))^2 = \tilde{\rho}(X)^2 = -\frac{1}{3}\tilde{\vec{m}}(X)^2$$

Partially bosonised functional integral

$$Z[\eta, \eta^*, J_\rho, \vec{J}_m] = \int \mathcal{D}(\psi^*, \psi, \hat{\rho}, \hat{\vec{m}}) \exp\left(-(S + S_\eta + S_J)\right)$$

$$S = S_{F,\mathrm{kin}} + \frac{1}{2}U_\rho \hat{\rho}^2 + \frac{1}{2}U_m \hat{\vec{m}}^2 - U_\rho \hat{\rho}\tilde{\rho} - U_m \hat{\vec{m}}\tilde{\vec{m}},$$
$$S_J = -\ J_\rho \hat{\rho} - \vec{J}_m \hat{\vec{m}}$$

equivalent to
fermionic functional integral

if $U = -U_\rho + 3U_m$

Bosonic integration is Gaussian

or:

solve bosonic field equation as functional of fermion fields and reinsert into action

$$\hat{\rho} = \tilde{\rho} + \frac{J_\rho}{U_\rho}, \qquad \hat{\vec{m}} = \tilde{\vec{m}} + \frac{\vec{J}_m}{U_m}$$

more bosons …

- additional fields may be added formally :
- only mass term + source term : decoupled boson
- introduction of boson fields not linked to Hubbard-Stratonovich transformation

fermion – boson action

$$S = S_{F,\mathrm{kin}} + S_B + S_Y + S_J,$$

fermion kinetic term

$$S_{F,\mathrm{kin}} = \sum_Q \hat{\psi}^\dagger(Q)(i\omega_F - \mu - 2t(\cos q_1 + \cos q_2))\hat{\psi}(Q),$$

boson quadratic term ("classical propagator")

$$S_B = \frac{1}{2}\sum_Q \left(U_\rho \hat{\rho}(Q)\hat{\rho}(-Q) + U_m \hat{\vec{m}}(Q)\hat{\vec{m}}(-Q)\right),$$

Yukawa coupling

$$S_Y = -\sum_{QQ'Q''} \delta(Q - Q' + Q'') \times$$
$$(U_\rho \hat{\rho}(Q)\hat{\psi}^\dagger(Q')\hat{\psi}(Q'') + U_m \hat{\vec{m}}(Q)\hat{\psi}^\dagger(Q')\vec{\sigma}\hat{\psi}(Q'')),$$

source term

$$S_J = -\sum_Q \left(J_\rho(-Q)\hat{\rho}(Q) + \vec{J}_m(-Q)\hat{\vec{m}}(Q) \right)$$

is now linear in the bosonic fields

effective action treats fermions
and composite bosons on equal footing !

Mean Field Theory (MFT)

Evaluate Gaussian fermionic integral in background of bosonic field , e.g.

$$\begin{aligned}\hat{\rho}(Q) &\rightarrow \rho\delta(Q)\\ \hat{\vec{m}}(Q) &\rightarrow \vec{a}\delta(Q-\Pi)\end{aligned}$$

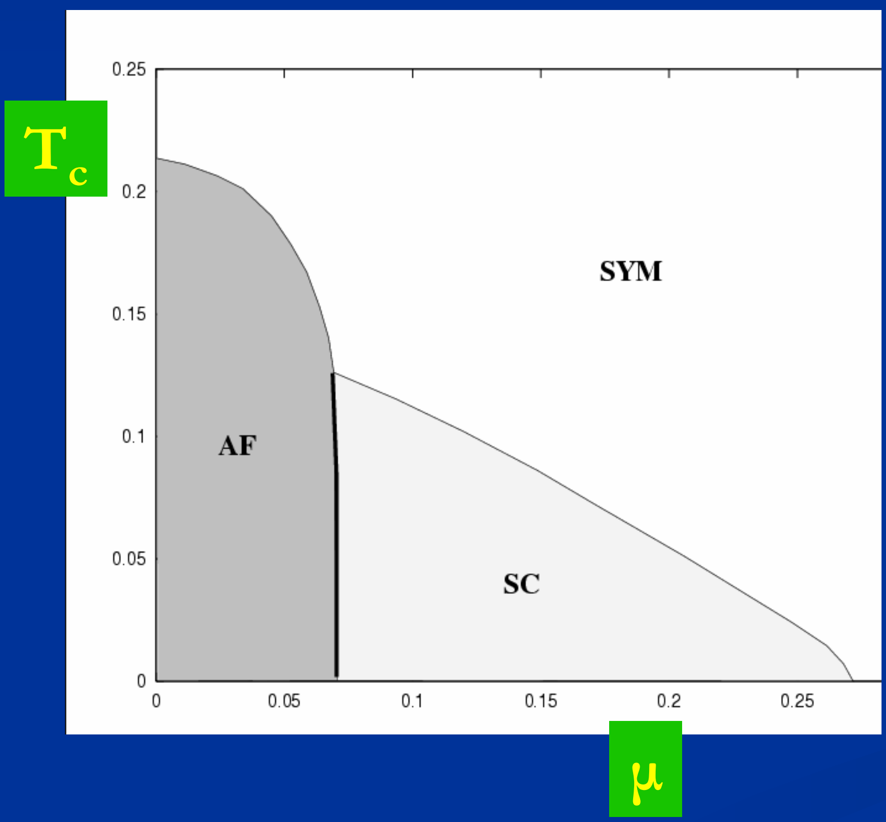
$$\begin{aligned}Z_{\mathrm{MF}} &= \int \mathcal{D}(\hat{\psi}^*, \hat{\psi}) \exp(-S_{\mathrm{MF}}),\\ S_{\mathrm{MF}} &= \sum_Q \hat{\psi}^\dagger(Q)(i\omega_F - \mu - 2t(\cos q_1 + \cos q_2))\hat{\psi}(Q)\\ &\quad -\sum_Q (U_\rho \rho \hat{\psi}^\dagger(Q)\hat{\psi}(Q) + U_m \vec{a}\hat{\psi}^\dagger(Q+\Pi)\vec{\sigma}\hat{\psi}(Q))\\ &\quad +\frac{V_2}{2T}(U_\rho\rho^2 + U_m\vec{a}^2) - J_\rho(0)\rho - \vec{J}_m(-\Pi)\vec{a}\end{aligned}$$

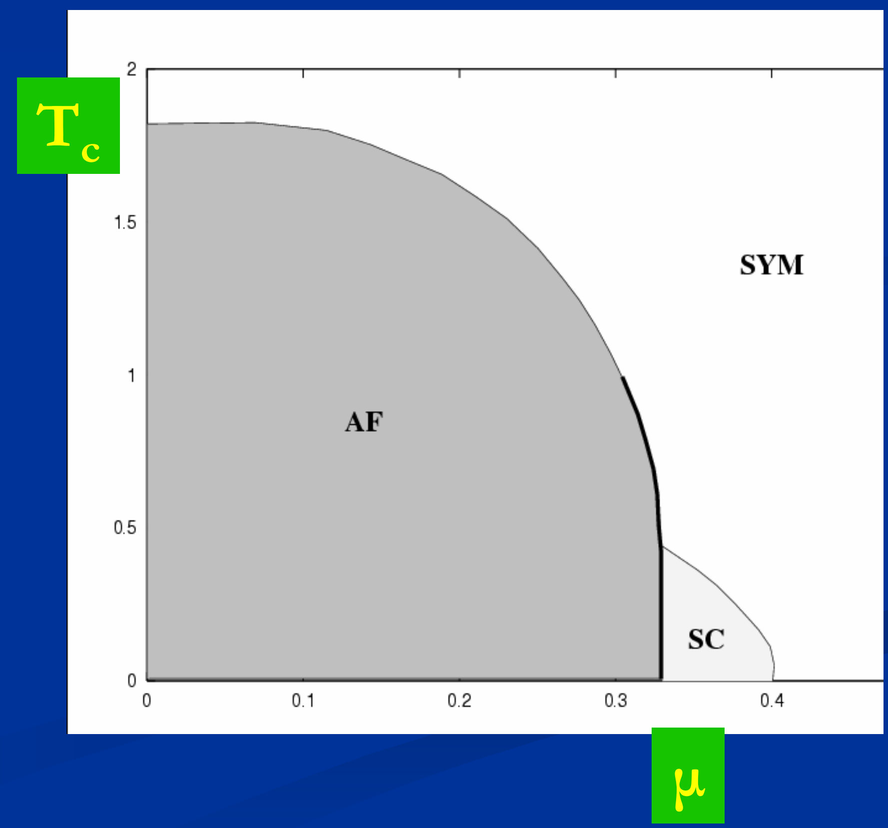
$$U = -U_\rho + 3U_m$$

$$\Gamma_{\mathrm{MF}} = -\ln Z_{\mathrm{MF}} + J_\rho(0)\rho + \vec{J}_m(-\Pi)\vec{a}$$

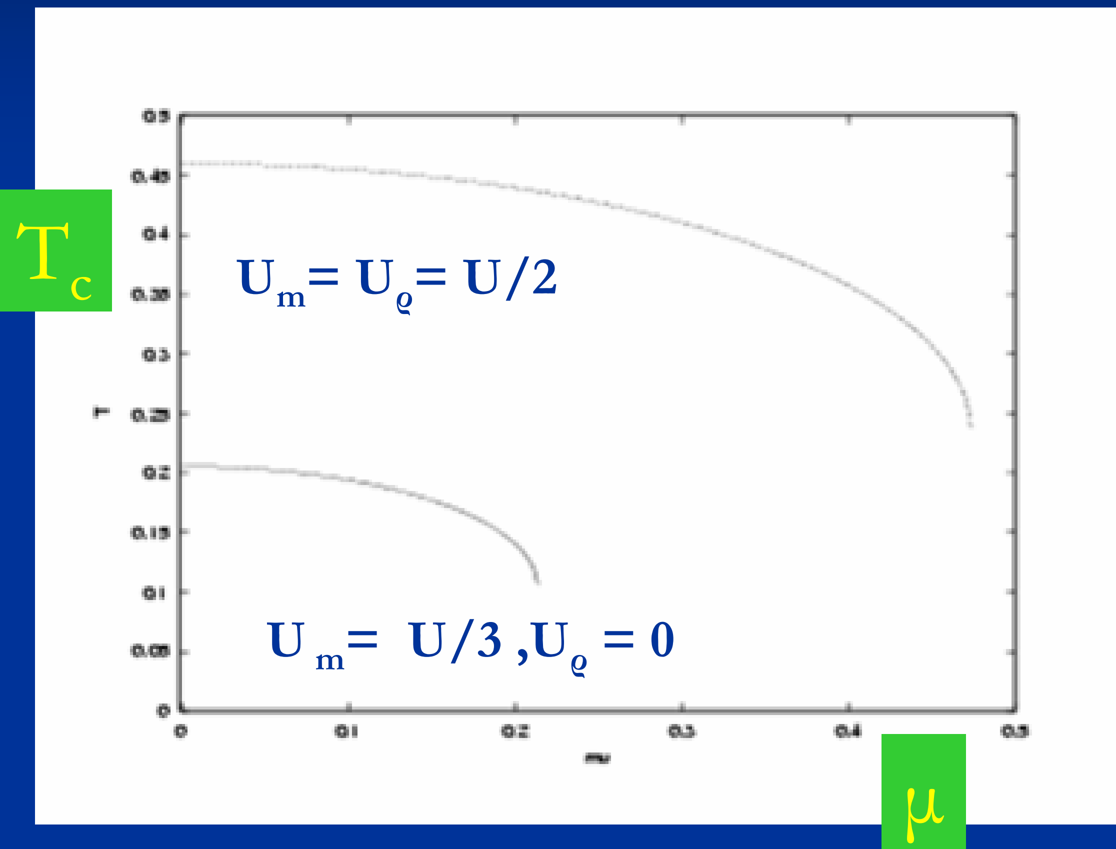
Mean field phase diagram

for two different choices of couplings – same U !





Mean field ambiguity



mean field phase diagram

Artefact of approximation …

cured by inclusion of bosonic fluctuations

J.Jaeckel,…

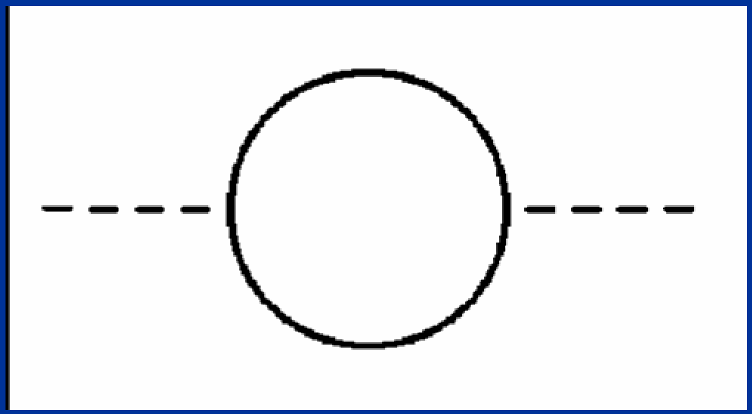
$$U = -U_\rho + 3U_m$$

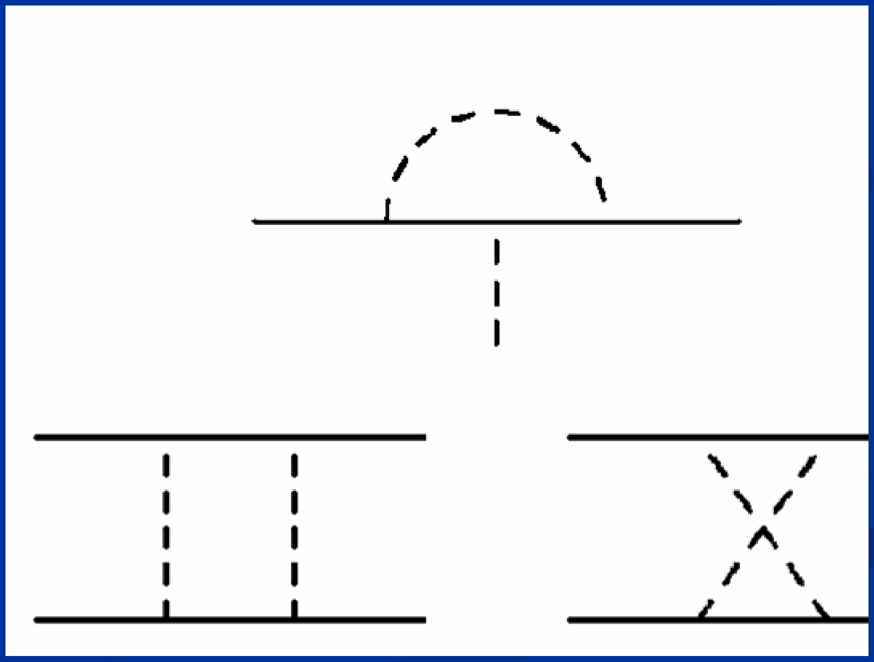
partial bosonisation and the mean field ambiguity

Bosonic fluctuations

fermion loops

boson loops

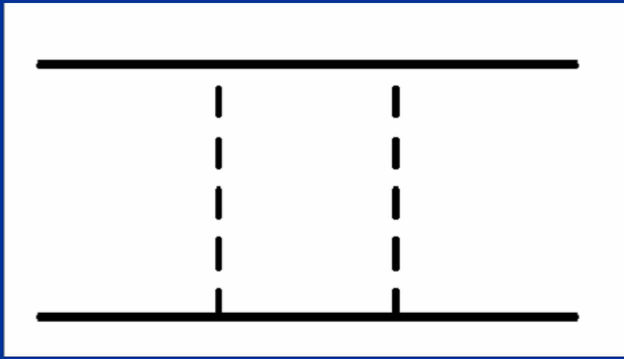




mean field theory

flowing bosonisation

- adapt bosonisation to every scale k such that



is translated to bosonic interaction

H.Gies , …

$$
\begin{aligned}
\Gamma_k[\psi,\psi^*,\phi] = & \sum_Q \psi^*(Q) P_{\psi,k} \psi(Q) \\
& + \frac{1}{2} \sum_Q \phi(-Q) P_{\phi,k}(Q) \phi(Q) \\
& - \sum_Q h_k(Q) \phi(Q) \tilde{\phi}(-Q) \\
& + \sum_Q \lambda_{\psi,k}(Q) \tilde{\phi}(Q) \tilde{\phi}(-Q)
\end{aligned}
$$

k-dependent field redefinition

$$\phi_k(Q) = \phi_{\bar{k}}(Q) + \Delta\alpha_k(Q)\tilde{\phi}(Q)$$

$$\partial_k \phi_k(Q) = -\,\partial_k \alpha_k(Q) \tilde{\phi}(Q)$$

absorbs four-fermion coupling

flowing bosonisation

Evolution with k-dependent field variables

$$\partial_k \Gamma_k[\psi,\psi^*,\phi_k] = \partial_k \Gamma_k[\psi,\psi^*,\phi_k]|_{\phi_k} + \sum_Q \left(\frac{\delta}{\delta\phi_k}\Gamma[\psi,\psi^*,\phi_k]\right)\partial_k\phi_k$$
$$= \partial_k \Gamma_k[\psi,\psi^*,\phi_k]|_{\phi_k} + \sum_Q \big(-\partial_k\alpha_k(Q) P_{\phi,k}(Q)\phi_k(Q)\tilde{\phi}(-Q) + h_k(Q)\partial_k\alpha_k(Q)\tilde{\phi}(Q)\tilde{\phi}(-Q)\big)$$

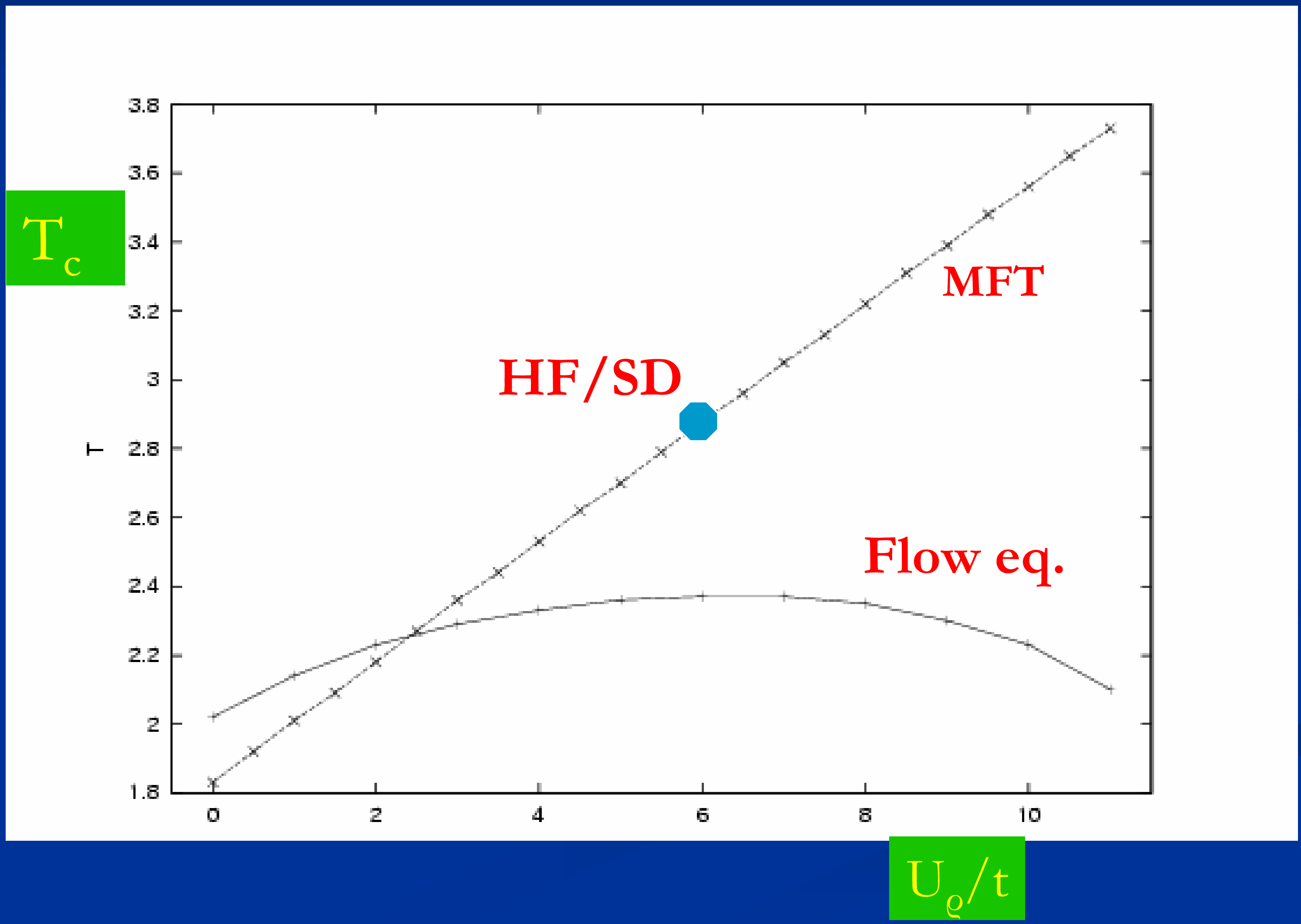
modified flow of couplings

$$\begin{aligned}\partial_k h_k(Q) &= \partial_k h_k(Q)|_{\phi_k} + \partial_k\alpha_k(Q) P_{\phi,k}(Q),\\ \partial_k \lambda_{\psi,k}(Q) &= \partial_k \lambda_{\psi,k}(Q)|_{\phi_k} + h_k(Q)\,\partial_k\alpha_k(Q).\end{aligned}$$

Choose α_k in order to absorb the four fermion coupling in corresponding channel ⇒

$$\partial_k h_k(Q) = \partial_k h_k(Q)|_{\phi_k} - \frac{P_{\phi,k}(Q)}{h_k(Q)}\partial_k \lambda_{\psi,k}(Q)|_{\phi_k}$$

Bosonisation cures mean field ambiguity



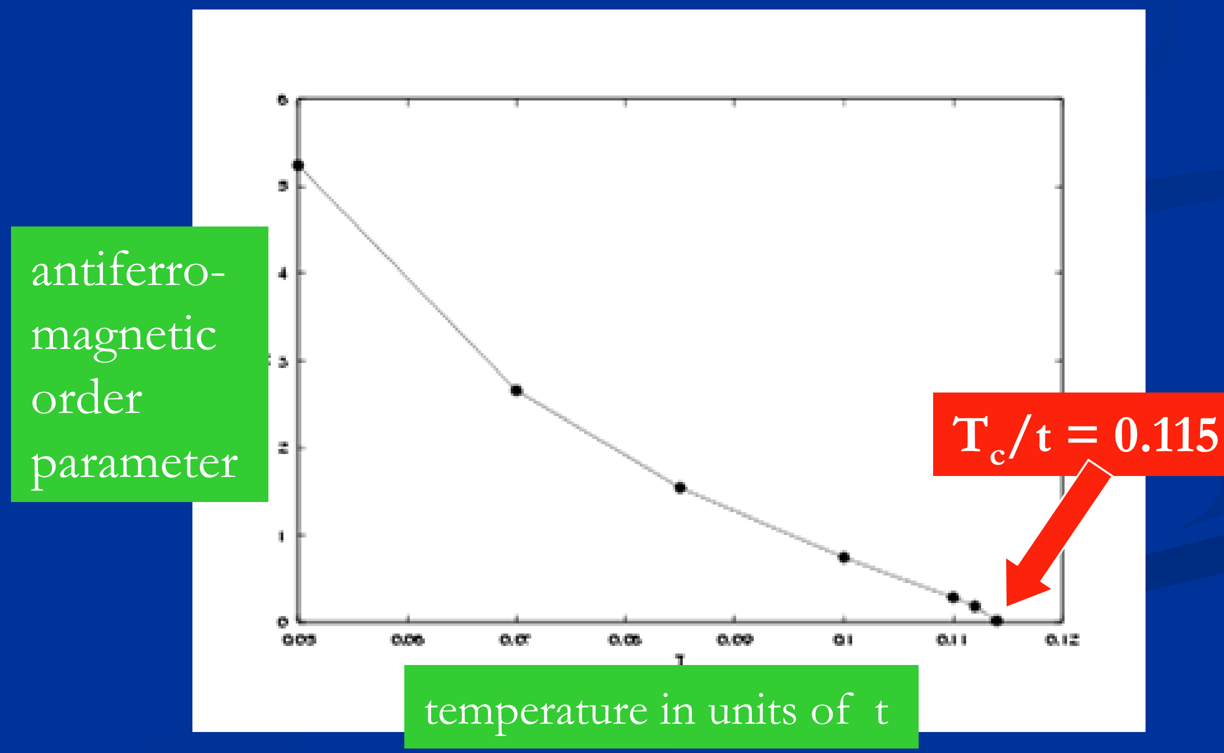
Flow equation for the Hubbard model

T.Baier , E.Bick , …,C.Krahl, J.Mueller, S.Friederich

Below the critical temperature :

Infinite-volume-correlation-length becomes larger than sample size

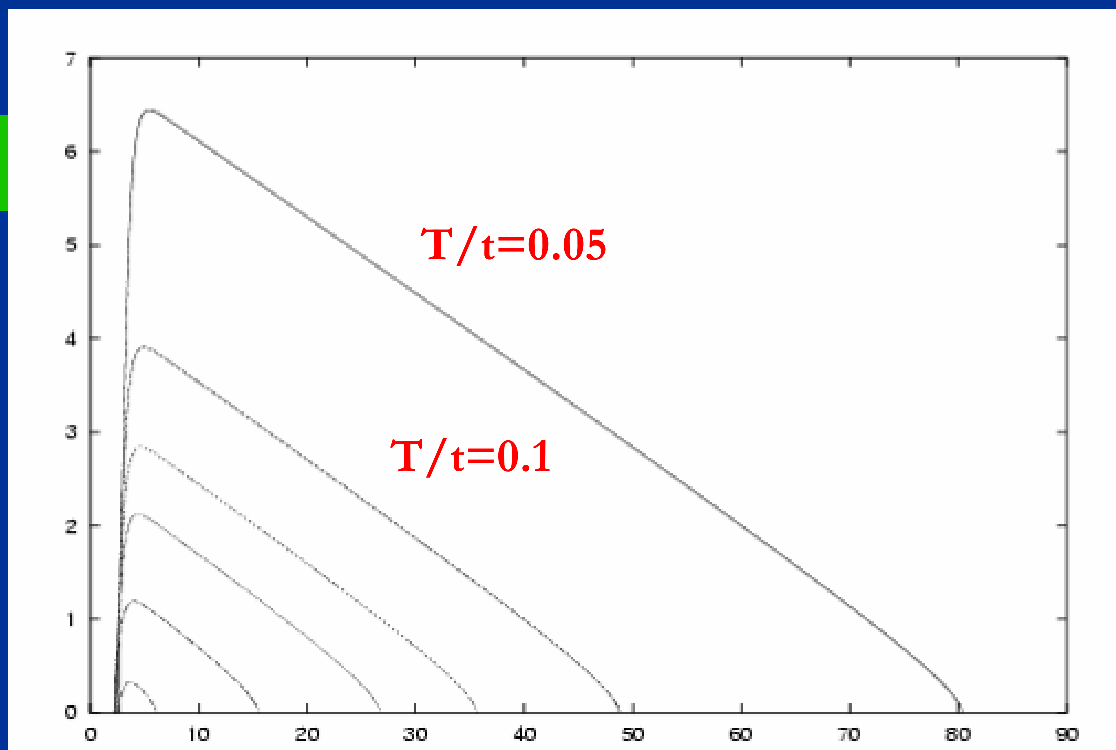
finite sample ≈ finite k : order remains effectively



Critical temperature

For $T<T_c$: ϰ remains positive for $k/t > 10^{-9}$
size of probe > 1 cm

ϰ



-ln(k/t)

local disorder
pseudo gap

SSB

$$\kappa_a = \frac{Z_a t^2}{T} \alpha_0$$

$T_c=0.115$

Mermin-Wagner theorem ?

No spontaneous symmetry breaking of continuous symmetry in d=2 !

not valid in practice !

Pseudo-critical temperature T_{pc}

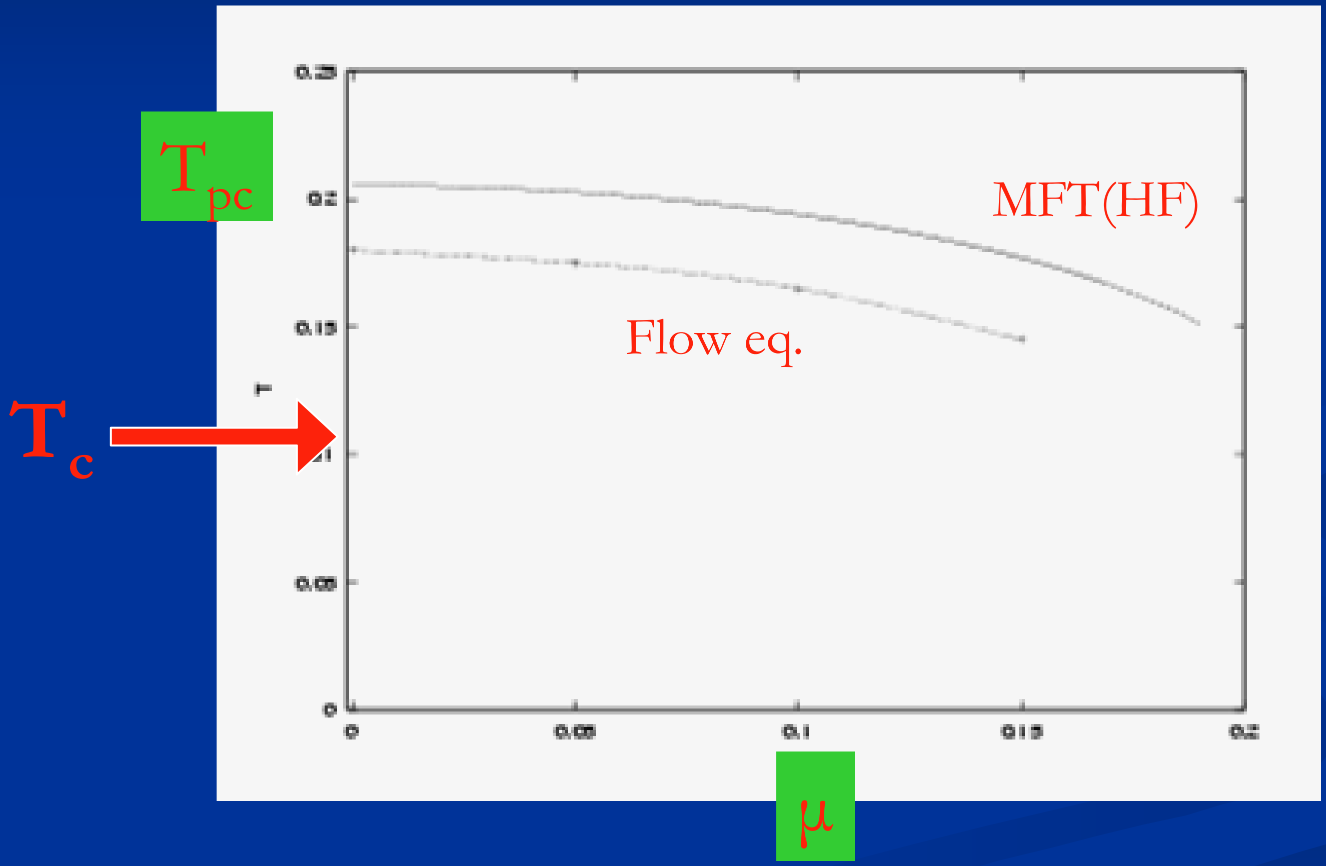
Limiting temperature at which bosonic mass term vanishes ($\varkappa$ becomes nonvanishing)

It corresponds to a diverging four-fermion coupling

This is the "critical temperature" computed in MFT !

Pseudo-gap behavior below this temperature

Pseudocritical temperature

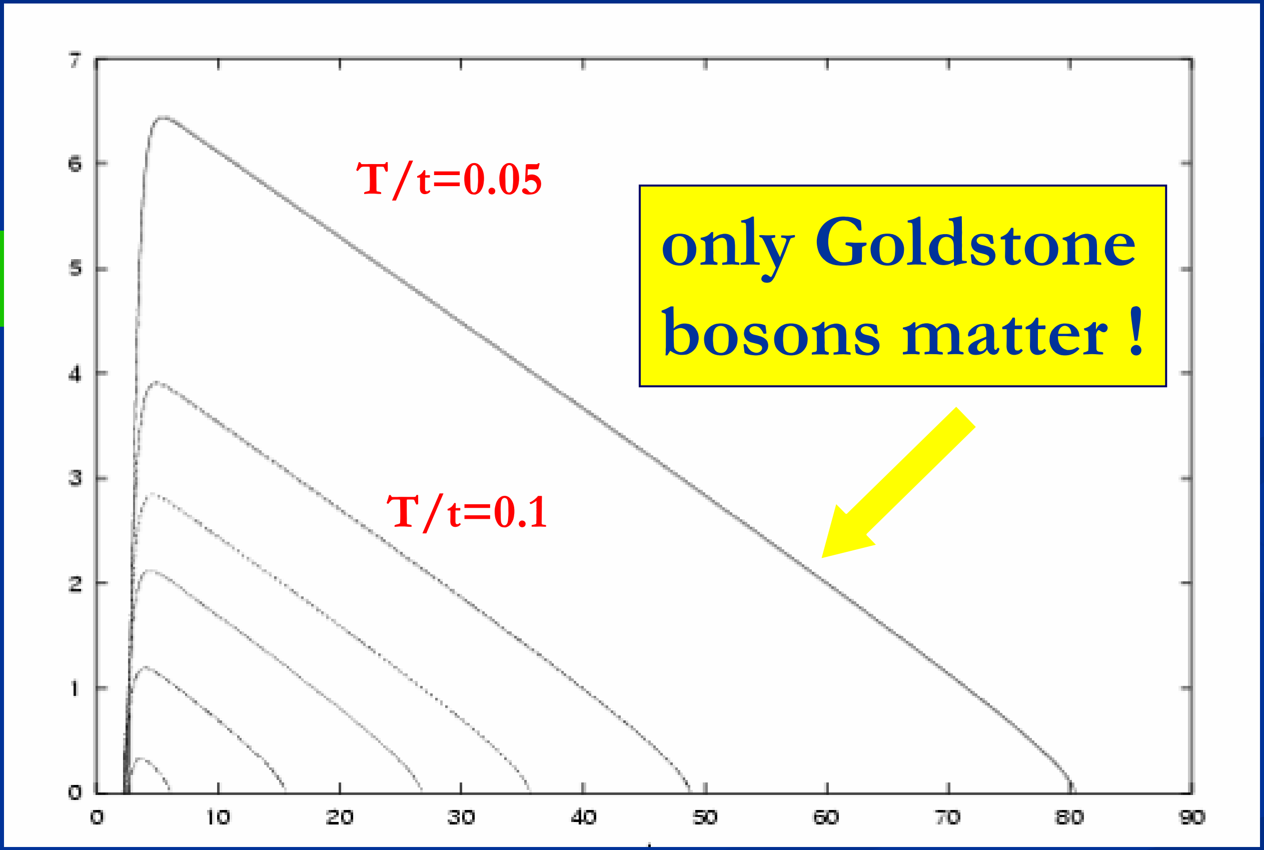


Below the pseudocritical temperature

the reign of the goldstone bosons

effective nonlinear O(3) – σ - model

Critical temperature



ϰ

local disorder
pseudo gap

-ln(k/t)

SSB

T_c=0.115

critical behavior

for interval $T_c < T < T_{pc}$
evolution as for classical Heisenberg model

cf. Chakravarty,Halperin,Nelson

$$k\partial_k \kappa = \frac{1}{4\pi} + \frac{1}{16\pi^2 \kappa} + 0(\kappa^{-2})$$

dimensionless coupling of non-linear sigma-model : $g^2 \sim \varkappa^{-1}$

two-loop beta function for g

effective theory

non-linear O(3)-sigma-model

asymptotic freedom

from fermionic microscopic law

to bosonic macroscopic law

transition to linear sigma-model

large coupling regime of non-linear sigma-model :

small renormalized order parameter $\varkappa$

transition to symmetric phase

again change of effective laws :
linear sigma-model is simple ,
strongly coupled non-linear sigma-model is complicated

critical correlation length

$$\xi t = c(T) \exp\left\{ 20.7 \beta(T) \frac{T_c}{T} \right\}$$

c,β : slowly varying functions

exponential growth of correlation length
compatible with observation !

at T_c : correlation length reaches sample size !

conclusion

- functional renormalization offers an efficient method for adding new relevant degrees of freedom or removing irrelevant degrees of freedom
- continuous description of the emergence of new laws

Unification from Functional Renormalization

- fluctuations in d=0,1,2,3,...
- linear and non-linear sigma models
- vortices and perturbation theory
- bosonic and fermionic models
- relativistic and non-relativistic physics
- classical and quantum statistics
- non-universal and universal aspects
- homogenous systems and local disorder
- equilibrium and out of equilibrium

end

unification: functional integral / flow equation

- simplicity of average action
- explicit presence of scale
- differentiating is easier than integrating…

qualitative changes that make non-perturbative physics accessible :

(1) basic object is simple

average action ~ classical action

~ generalized Landau theory

direct connection to thermodynamics

(coarse grained free energy)

qualitative changes that make non-perturbative physics accessible :

(2) Infrared scale k

instead of Ultraviolet cutoff Λ

short distance memory not lost

no modes are integrated out , but only part of the fluctuations is included

simple one-loop form of flow

simple comparison with perturbation theory

infrared cutoff k

cutoff on momentum resolution

or frequency resolution

e.g. distance from pure anti-ferromagnetic momentum or from Fermi surface

intuitive interpretation of k by association with physical IR-cutoff , i.e. finite size of system :

arbitrarily small momentum differences cannot be resolved !

qualitative changes that make non-perturbative physics accessible :

(3) only physics in small momentum range around k matters for the flow

ERGE regularization

simple implementation on lattice

artificial non-analyticities can be avoided

qualitative changes that make non-perturbative physics accessible :

(4) flexibility

change of fields

microscopic or composite variables

simple description of collective degrees of freedom and bound states

many possible choices of "cutoffs"