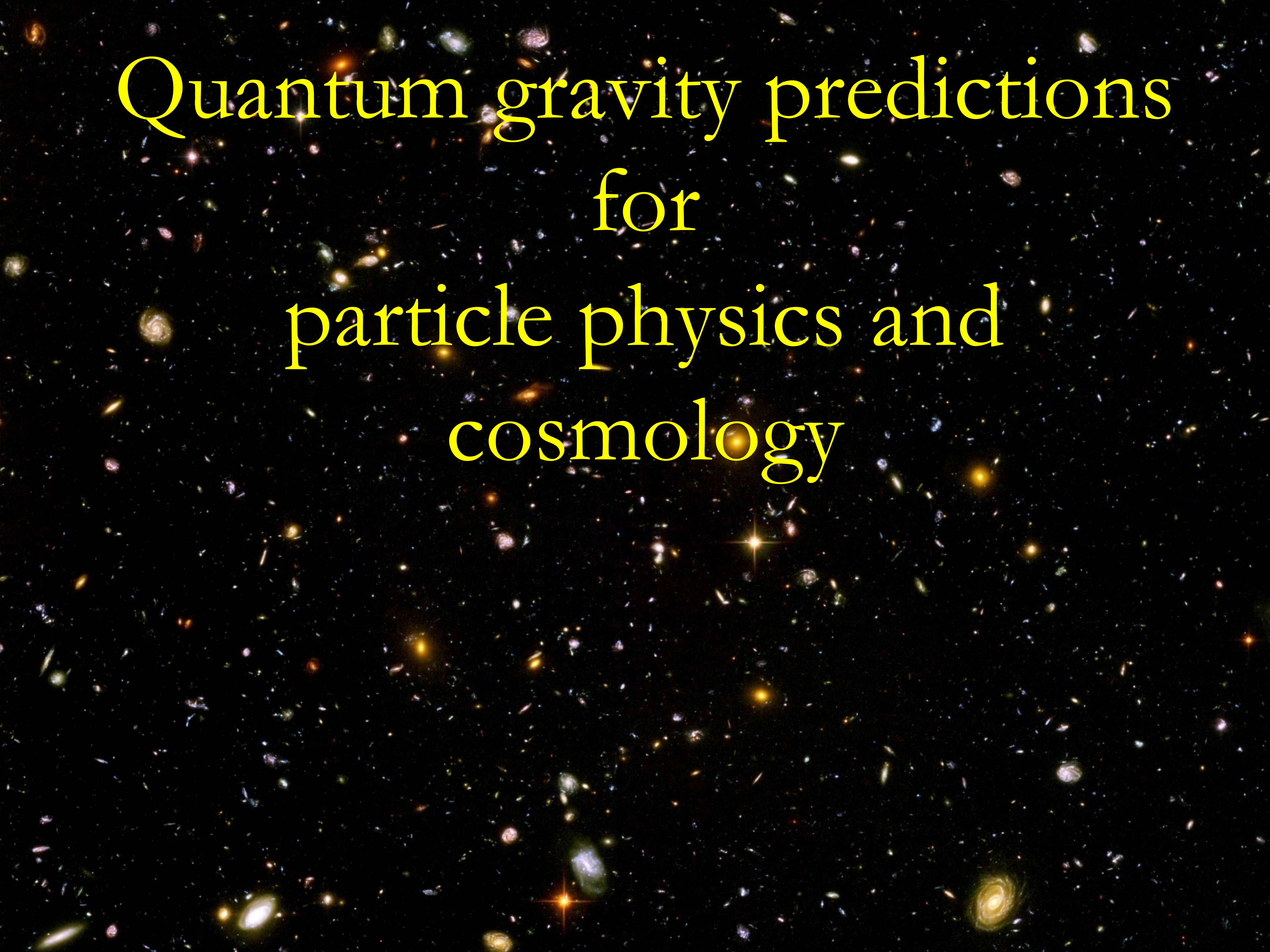
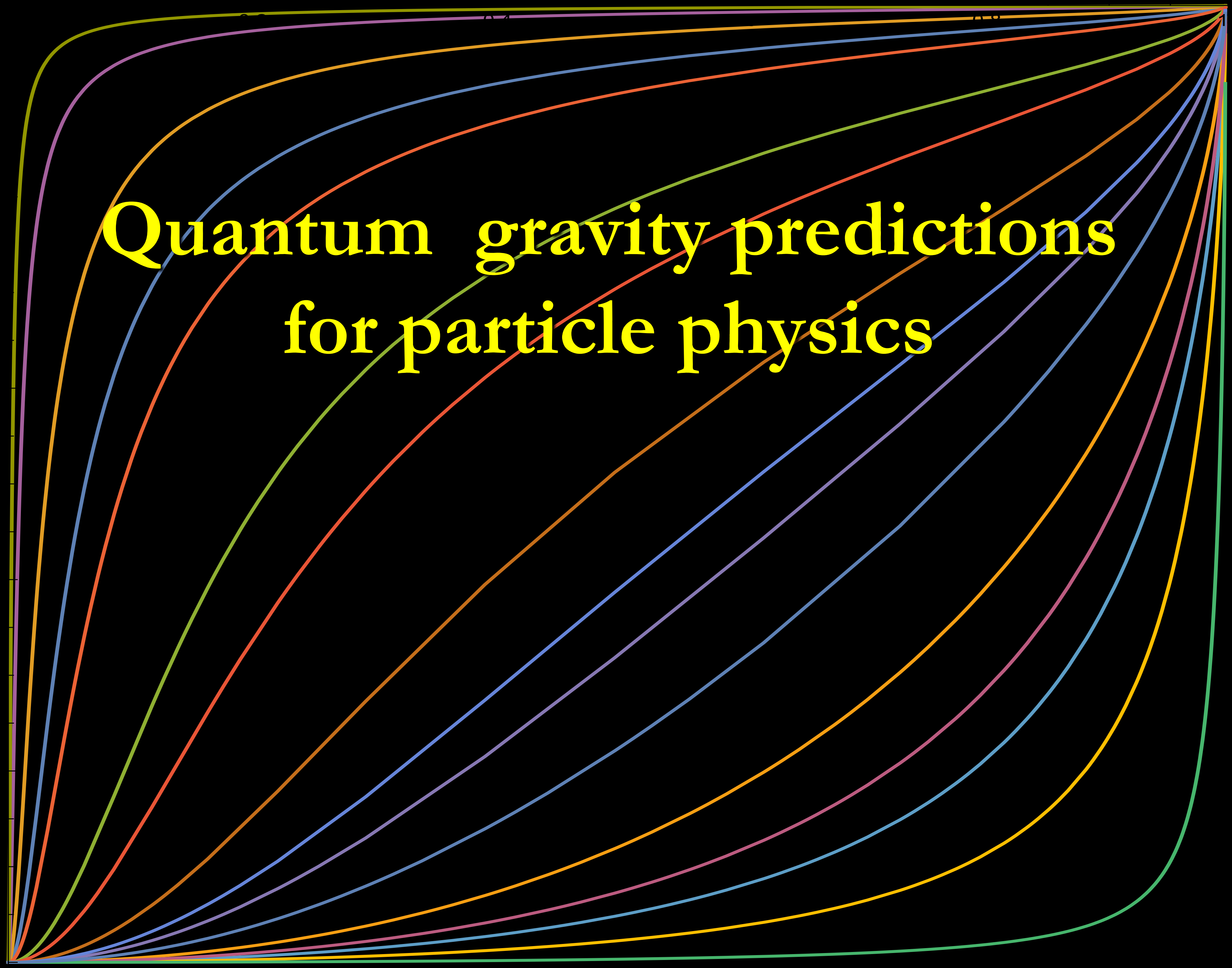
Quantum gravity predictions for particle physics and cosmology



Quantum gravity predictions for particle physics



Prediction of mass of Higgs boson

Asymptotic safety of gravity and the Higgs boson mass

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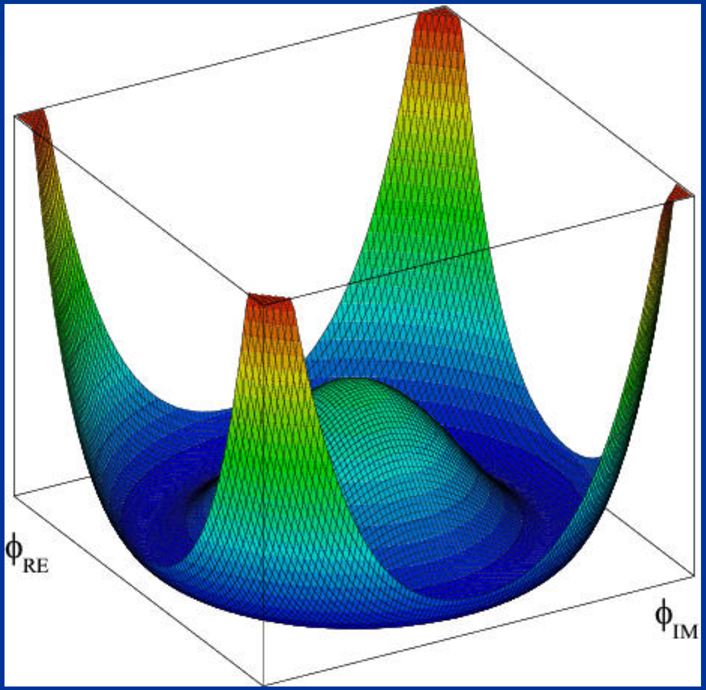
Abstract

There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalous dimension $A_\lambda > 0$ the running of the quartic scalar self interaction λ at scales beyond the Planck mass is determined by a fixed point at zero. This results in $m_H = m_{\rm min} = 126$ GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well.

s in $m_H = m_{\rm min} = 126$ GeV, with o

Why can quantum gravity make predictions for particle physics ?

Effective potential for Higgs scalar



$$V(\varphi) = -\mu^2 \varphi^\dagger \varphi + \frac{1}{2}\lambda(\varphi^\dagger \varphi)^2$$

$$= \frac{1}{2}\lambda(\varphi^\dagger \varphi - \varphi_0^2)^2 + \text{const.})$$

Fermi scale

$$\varphi_0 = 175 \text{ GeV}$$

Radial mode and Goldstone mode

expand around minimum of potential

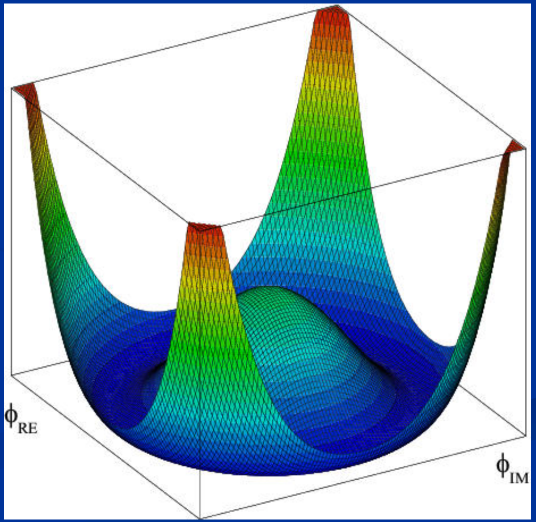
$$\varphi = \varphi_0 + \sigma + i\eta$$

$$\varphi_0, \sigma, \eta : \text{ real}$$

$$-\mathcal{L}_\varphi = \tfrac{1}{2}\partial^\mu\sigma\partial_\mu\sigma + \tfrac{1}{2}\partial^\mu\eta\partial_\mu\eta + \tfrac{1}{2}m^2\sigma^2 + \dots$$

mass term for radial mode

$$m^2 = 2\lambda\varphi_0^2$$



Quartic scalar coupling

prediction of mass of Higgs boson

=

prediction of value of quartic scalar coupling λ at Fermi scale

$$m^2 = 2\lambda\varphi_0^2$$

Why can quantum gravity make predictions for quartic scalar coupling ?

Mass scales

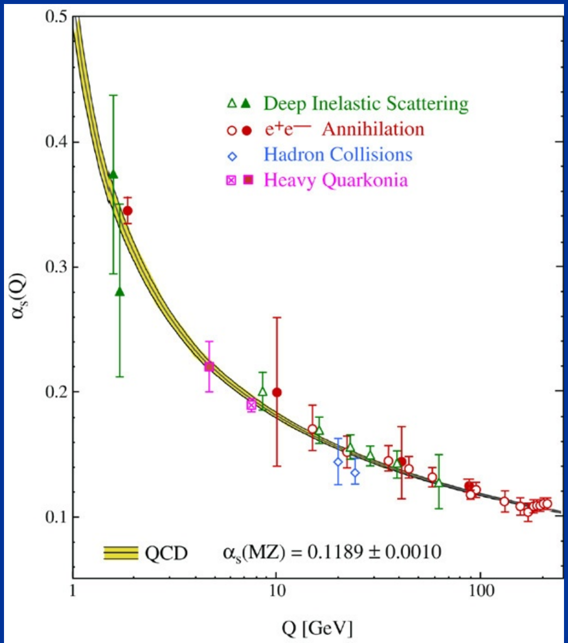
- Fermi scale φ_0 ~ 100 GeV
- Planck mass M ~ 10^{18} GeV
- Gravity at Fermi scale is very weak : How can it influence the effective potential for the Higgs scalar and the mass of the Higgs boson ?

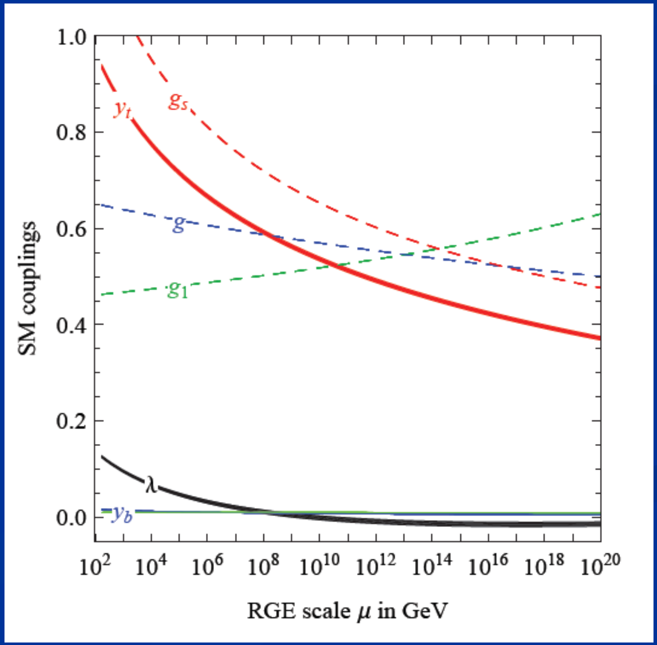
$$\varepsilon = \frac{\varphi_0^2}{M^2} = 5 \cdot 10^{-33}$$

Quantum fluctuations induce running couplings

- possible violation of scale symmetry
- well known in QCD or standard model

Bethke

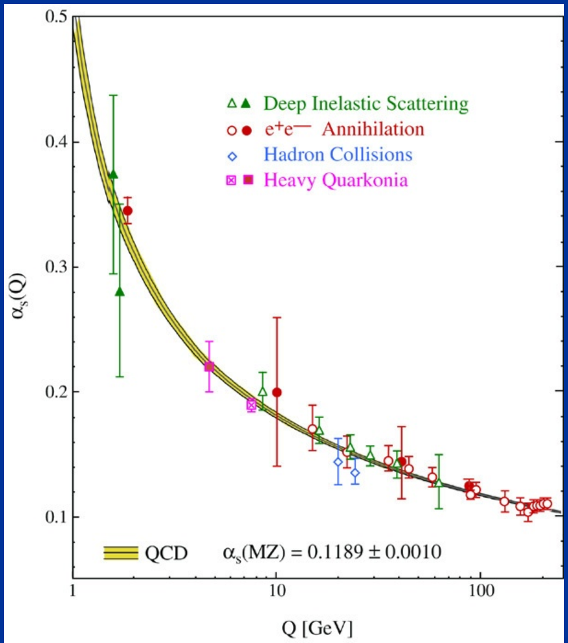


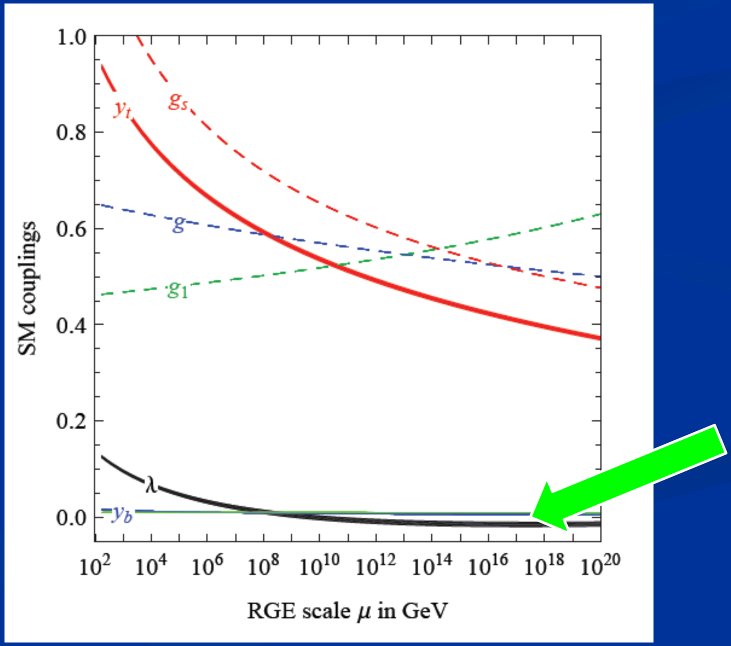


Degrassi et al

Quantum fluctuations induce running couplings

- possible violation of scale symmetry
- well known in QCD or standard model

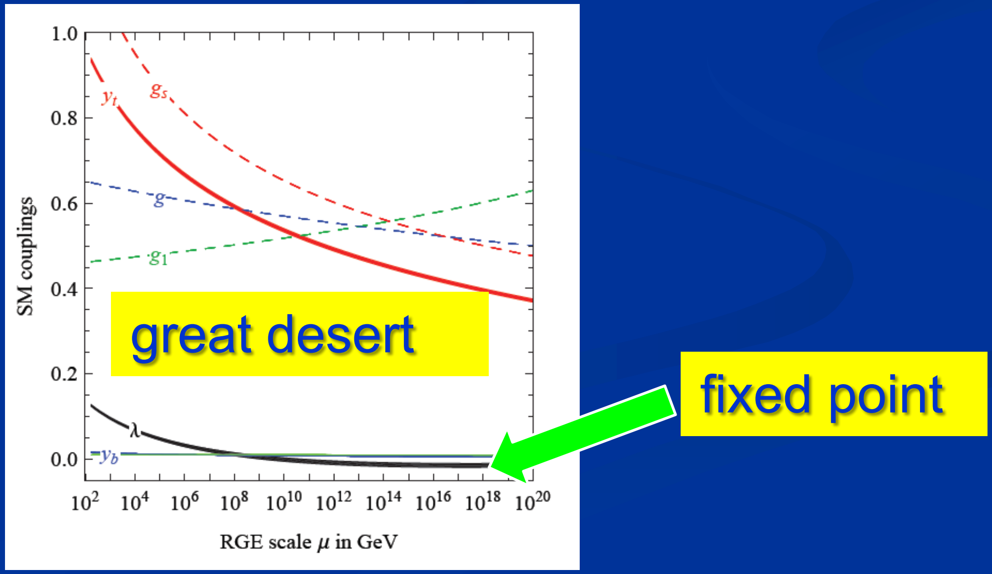


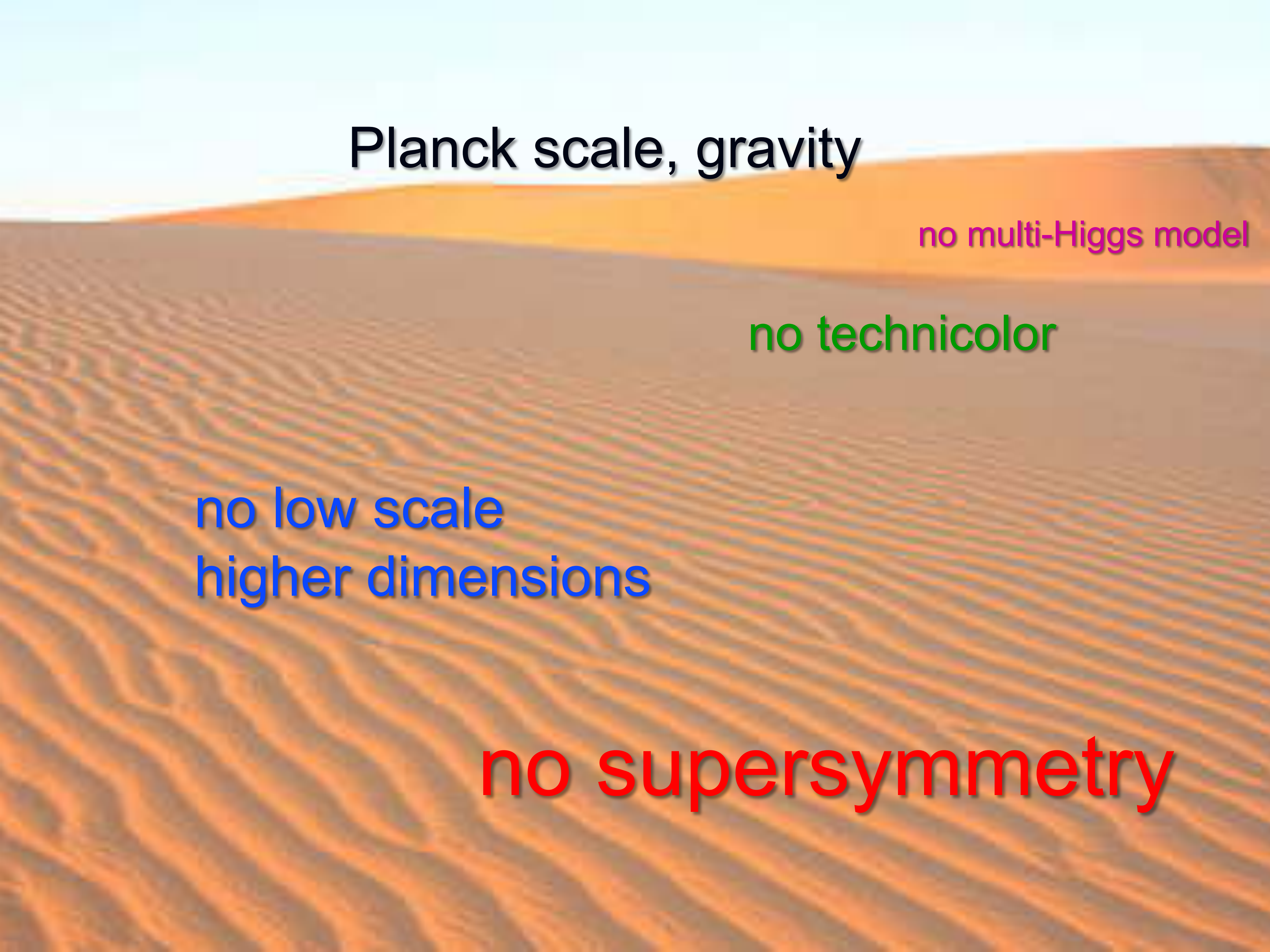


The mass of the Higgs boson, the great desert, and asymptotic safety of gravity

key points

- great desert
- solution of hierarchy problem at high scale
- high scale fixed point
- vanishing scalar coupling at fixed point

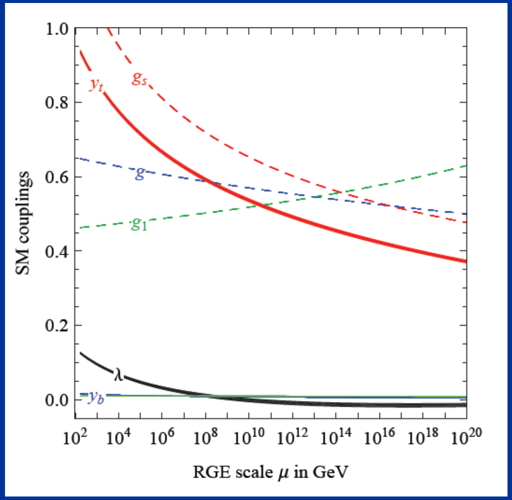



Planck scale, gravity
no multi-Higgs model
no technicolor
no low scale
higher dimensions
no supersymmetry

Essential point for quantum gravity prediction of Higgs boson mass:

Initial value of quartic scalar coupling near Planck mass is predicted by quantum gravity

Extrapolate perturbatively to Fermi scale



Results in prediction for ratio Higgs boson mass over W- boson mass, or Higgs boson mass over top quark mass

Near Planck mass gravity is not weak !

Predictive power !

Flowing couplings

Couplings change with momentum scale due to quantum fluctuations.

Renormalization scale k : Only fluctuations with momenta larger k are included. The scale k can be momenta, geometric quantities, or just be introduced "by hand".

Flow of k to zero : all fluctuations included, IR
Flow of k to infinity : UV

Renormalization group

How do couplings or physical laws change with scale k ?

Graviton fluctuations erase quartic scalar coupling

Renormalization scale k : Only fluctuations with momenta larger k are included.

Consider first only fluctuations of metric or graviton :

$$k \frac{\partial \lambda}{\partial k} = A \lambda$$

gravity induced anomalous dimension

A > 0

for constant A :

$$\lambda(k) = \lambda(\mu) \left(\frac{k}{\mu} \right)^A$$

$$k \to 0 \quad \Rightarrow \quad \lambda \to 0$$

Fixed point

$$k \frac{\partial \lambda}{\partial k} = A \lambda$$

$$\lambda(k) = \lambda(\mu) \left(\frac{k}{\mu} \right)^{A}$$

The quartic scalar coupling λ has a

fixed point at $\lambda=0$

For A>0 it flows towards the fixed point as k is lowered:

irrelevant coupling

For a UV – complete theory it is predicted to assume the fixed point value

Gravitational contribution to running quartic coupling

$$\partial_t \lambda = A_\lambda \lambda$$

$$A = \frac{1}{48\pi^2 \tilde{M}_{\rm p}^2}\left[\frac{20}{(1-v_0)^2} + \frac{1}{(1-v_0/4)^2}\right]$$

$$\partial_t = k\partial_k$$

running Planck mass :

$$M_{\rm p}^2(k) = M^2 + \tilde{M}_{\rm p*}^2 k^2$$

dimensionless
squared Planck mass

$$\tilde{M}_{\rm p}^2 = \frac{M_{\rm p}^2}{k^2}$$

for length scales smaller than the Planck length:
metric fluctuations dominate, constant A

Strength of gravity

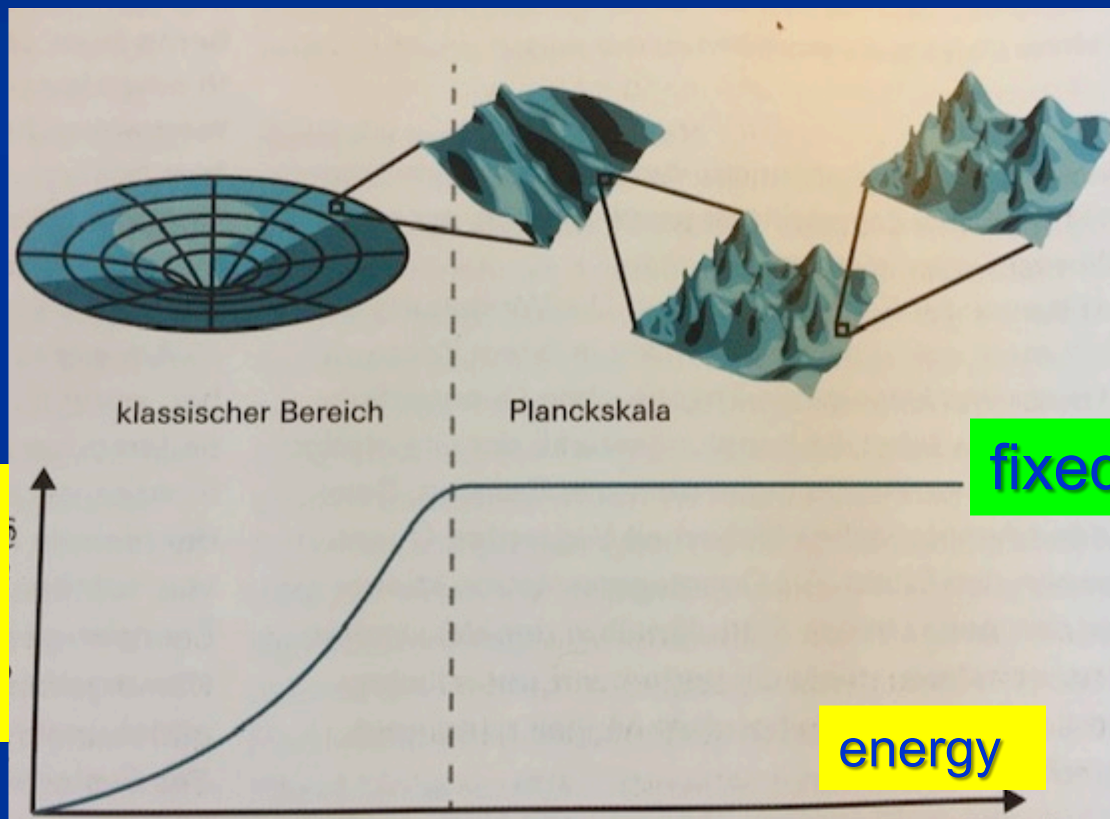
$$g_{grav} = \frac{l_p^2}{2l^2} = \frac{k^2}{2M^2}$$

l_p : Planck length
M : Planck mass

running gravitational coupling

$$g_{grav} = \frac{k^2}{2M^2(k)} = w^{-1}(k)$$

Strength of gravity



A. Eichhorn, CW, Spektrum der Wissenschaft

Flowing dimensionless Planck mass

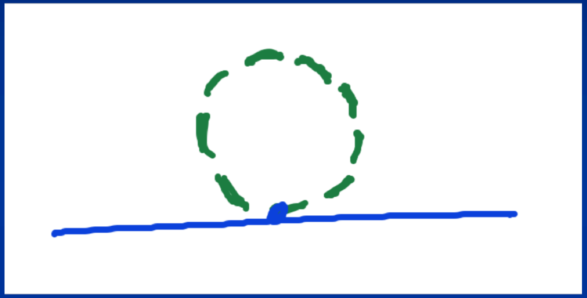
- Renormalization scale k : Only fluctuations with momenta larger k are included

Flowing
Planck mass $M^2(k)$

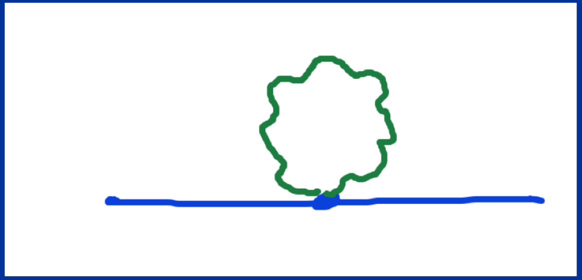
$$\partial_t M^2 = 4ck^2$$

$$\partial_t = k\partial_k$$

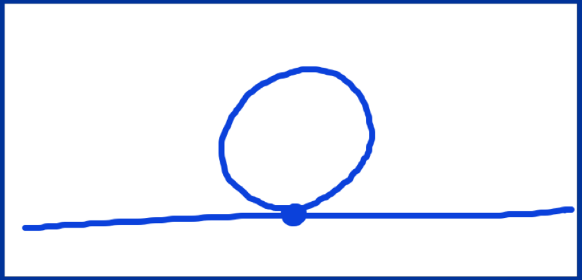
Universality of gravity



scalar loop,
fermion loop

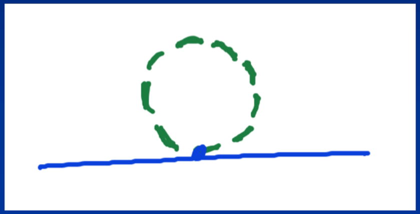


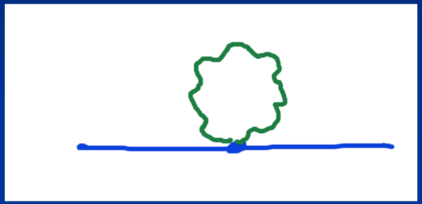
gauge boson
loop

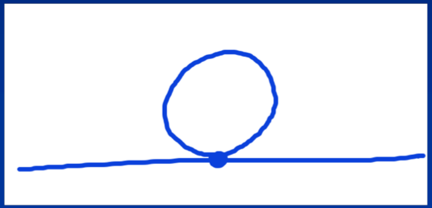


graviton
loop

Universality of gravity







scalar loop,
fermion loop

gauge boson
loop

graviton
loop

c is independent of coupling constants

$$\partial_t M^2 = 4ck^2$$

Flowing dimensionless Planck mass

- Renormalization scale k : Only fluctuations with momenta larger k are included

Flowing
Planck mass $M^2(k)$

$$\partial_t M^2 = 4ck^2$$

$$\partial_t = k\partial_k$$

matter contribution

$$c_M = \frac{\mathcal{N}_M}{192\pi^2}.$$

$$\mathcal{N}_M = $$ 4 N_V – N_S - N_F

with graviton contribution

$$c_M = \frac{1}{192\pi^2}\left(\mathcal{N}_M + \frac{43}{6} + \frac{75(1-\eta_g/6)}{2(1-v)}\right)$$

Flowing Planck mass

Flowing
Planck mass $M^2(k)$

$$\partial_t M^2 = 4ck^2$$

$$\partial_t = k\partial_k$$

solution :

$$M^2(k) = M^2 + 2c_M k^2$$

Flowing dimensionless Planck mass

Flowing
Planck mass $M^2(k)$

$$\partial_t M^2 = 4ck^2$$

Dimensionless
squared Planck mass

$$w = \frac{M^2}{2k^2}$$

$$\partial_t w = -2w + 2c$$

solution :

$$w = c + \frac{\bar{M}^2}{2k^2}$$

$$M^2(k) = M^2 + 2c_M k^2$$

Fixed point and flow away from fixed point

UV - fixed point $w_* = c$

$$\partial_t w = -2w + 2c$$

approached for k → ∞ $w = c + \frac{\bar{M}^2}{2k^2}$

$$M^2(k) = M^2 + 2c_M k^2$$

Near UV – fixed point : M ~ k

$$\tilde{M}^2_{p*} = 2c$$

Transition to constant M for small k, gravity gets weak, w^{-1} decreases to zero

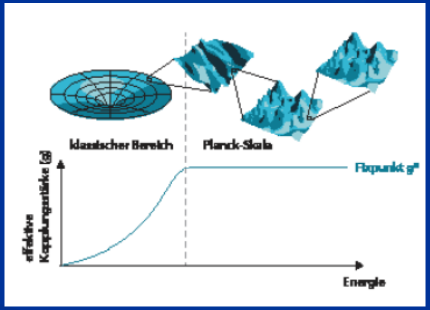
M is relevant parameter, cannot be predicted

Weak and constant gravity

$$M_{\mathrm{p}}^2(k) = \begin{cases} \tilde{M}_{\mathrm{p}*}^2\, k^2 & \text{for } k > k_t \\ M^2 & \text{for } k < k_t \end{cases}$$

$$k_t^2 = \frac{M^2}{\tilde{M}_{\mathrm{p}*}^2}$$

$$M_{\mathrm{p}}^2(k) = M^2 + \tilde{M}_{\mathrm{p}*}^2 k^2$$



Two regimes for the (inverse) strength of gravity

$$\tilde{M}_{\mathrm{p}}^2 = \frac{M_{\mathrm{p}}^2}{k^2} = \tilde{M}_{\mathrm{p}*}^2 \left[\left(\frac{k_t}{k} \right)^2 + 1 \right]$$

Gravitational contribution to running quartic coupling

$$\partial_t \lambda = A_\lambda \lambda$$

$$A = \frac{1}{48\pi^2 \tilde{M}_p^2} \left[\frac{20}{(1-v_0)^2} + \frac{1}{(1-v_0/4)^2} \right]$$

$$\partial_t = k\partial_k$$

Running Planck mass : $M_p^2(k) = M^2 + \tilde{M}_{p*}^2 k^2$

$$\tilde{M}_p^2 = \frac{M_p^2}{k^2} = \tilde{M}_{p*}^2 \left[\left(\frac{k_t}{k} \right)^2 + 1 \right]$$

$$k_t^2 = \frac{M^2}{\tilde{M}_{p*}^2}$$

large k : constant A

small k: $A \sim k^2 / M^2$

transition at

$k_t \sim 10^{19}$ GeV

UV – fixed point for quartic coupling

Flow equation for λ : $\partial_t \lambda_H = A\lambda_H - C_H$

$$C_H^{(p)} = -\beta_\lambda^{(SM)}$$

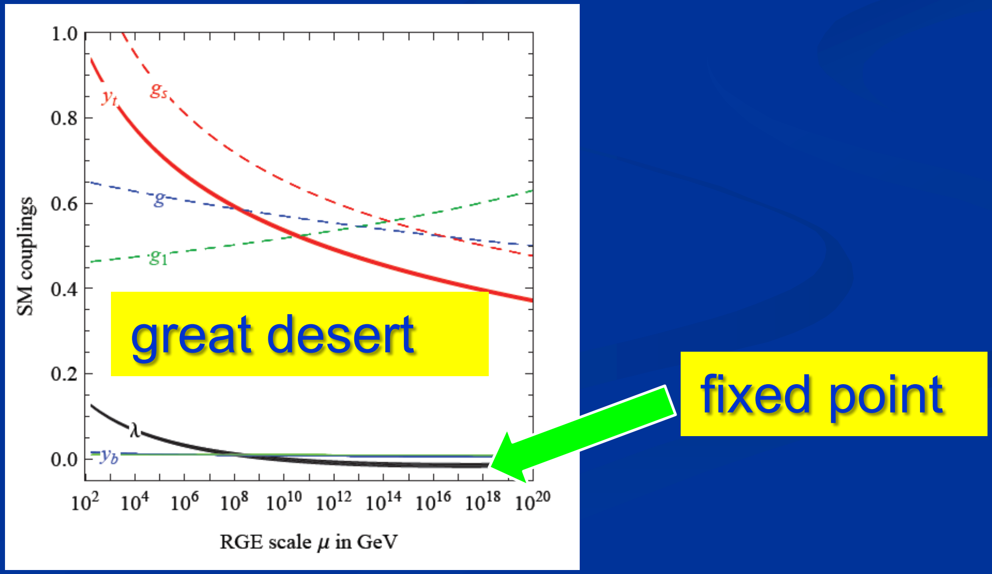
$$\approx -\frac{1}{16\pi^2}\left\{12\lambda_H^2 + 12y_t^2\lambda_H - 12y_t^4 + \frac{9}{4}g_2^2 + \frac{9}{10}g_2^2 g_1^2 + \frac{27}{100}g_1^4 - \left(9g_2^2 + \frac{9}{5}g_1^2\right)\lambda_H\right\}$$

Fixed point : λ = C / A

$$\lambda(k_{tr}) \approx 0\ ,\ \beta_\lambda(k_{tr}) \approx 0$$

Prediction for quartic Higgs coupling

- great desert
- high scale fixed point
- quartic scalar coupling predicted to be very small at transition scale where gravity decouples



Quantum Gravity

Quantum Gravity is a renormalisable quantum field theory

Asymptotic safety

Asymptotic safety of quantum gravity

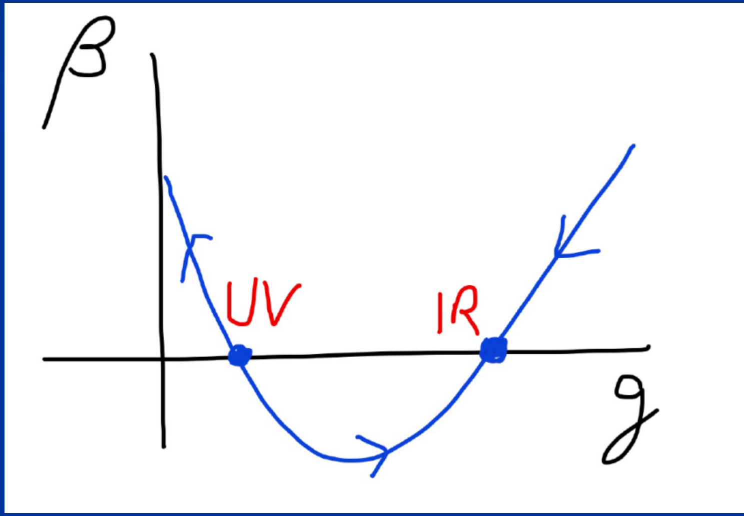
if UV fixed point exists :

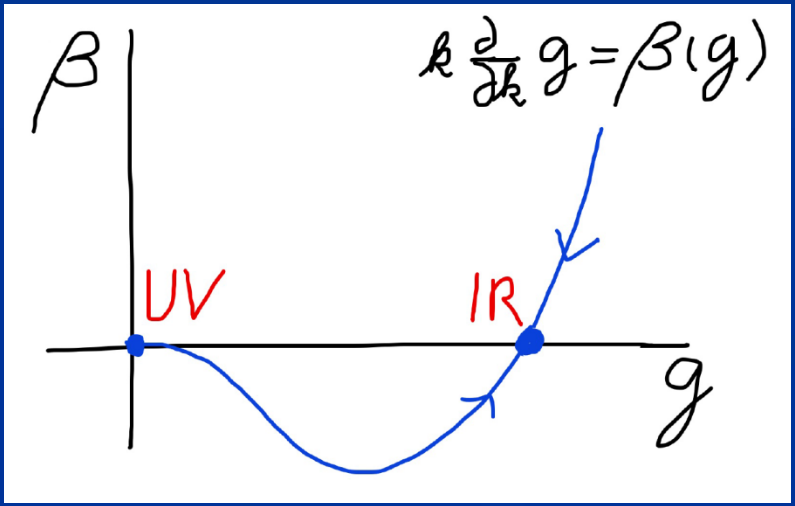
quantum gravity is

non-perturbatively renormalizable !

S. Weinberg , M. Reuter

Asymptotic safety Asymptotic freedom





Relevant parameters yield undetermined couplings. Quartic scalar coupling is not relevant and can therefore be predicted.

Enhanced predictivity for UV – fixed point

- Free parameters of a theory correspond to relevant parameters for small deviations from fixed point.
- If the number of relevant parameters at the UV-fixed point is smaller than the number of free parameters (renormalizable couplings) in the standard model:
- Relations between standard model parameters become predictable !

How to compute non-perturbative quantum gravity effects ?

Quantum gravity computation by functional renormalization

Introduce infrared cutoff with scale k, such that only fluctuations with (covariant) momenta larger than k are included.

Then lower k towards zero

Exact renormalization group equation

Exact flow equation

for scale dependence of average action

$$\partial_k \Gamma_k[\varphi] = \frac{1}{2} \mathrm{Tr} \left\{ \left(\Gamma_k^{(2)}[\varphi] + R_k \right)^{-1} \partial_k R_k \right\}$$

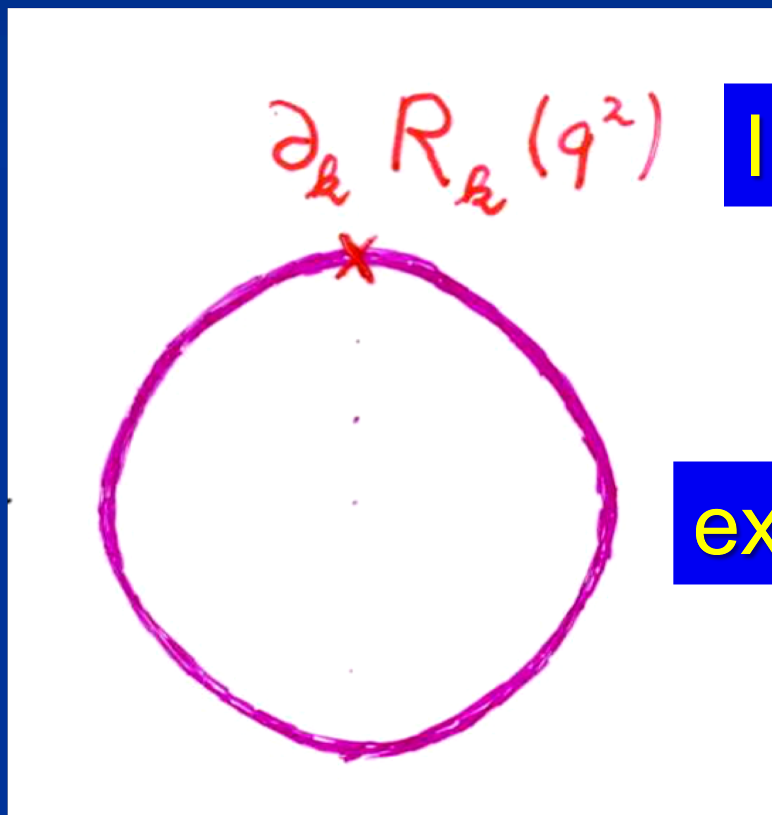
'92

$$\left(\Gamma_k^{(2)} \right)_{ab} (q, q') = \frac{\delta^2 \Gamma_k}{\delta \varphi_a(-q) \delta \varphi_b(q')}$$

$$\mathrm{Tr} : \sum_a \int \frac{d^d q}{(2\pi)^d}$$

(fermions : STr)

Functional flow equation for scale dependent effective action



IR cutoff

exact propagator

Exact renormalization group equation

Exact flow equation

for scale dependence of average action

$$\partial_k \Gamma_k[\varphi] = \frac{1}{2} \mathrm{Tr} \left\{ \left(\Gamma_k^{(2)}[\varphi] + R_k \right)^{-1} \partial_k R_k \right\}$$

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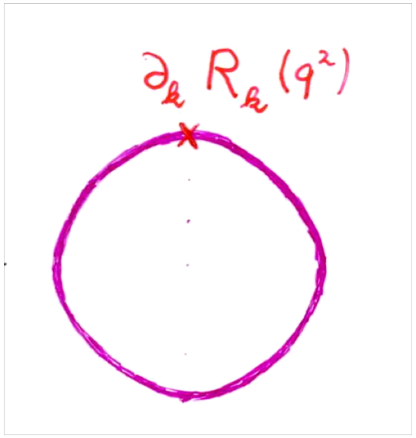


From

Microscopic Laws

(Interactions, classical action)

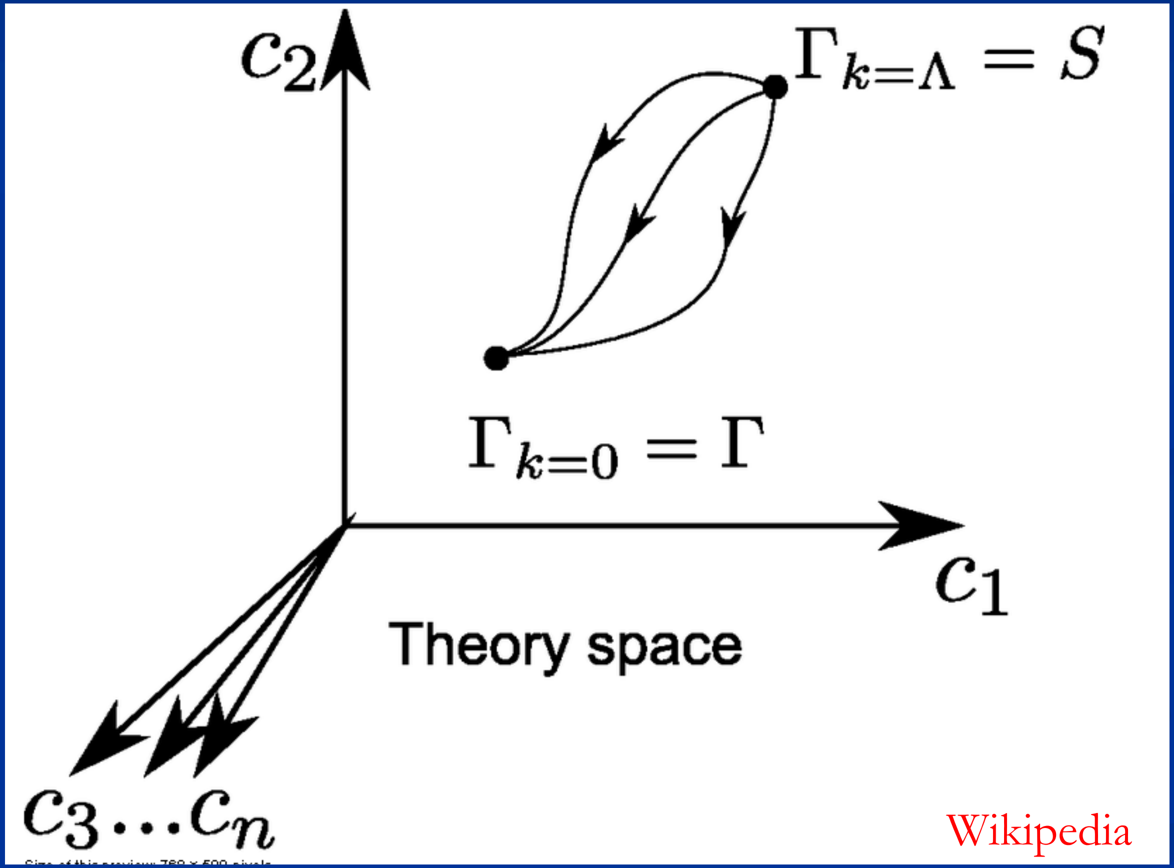
to

Fluctuations!

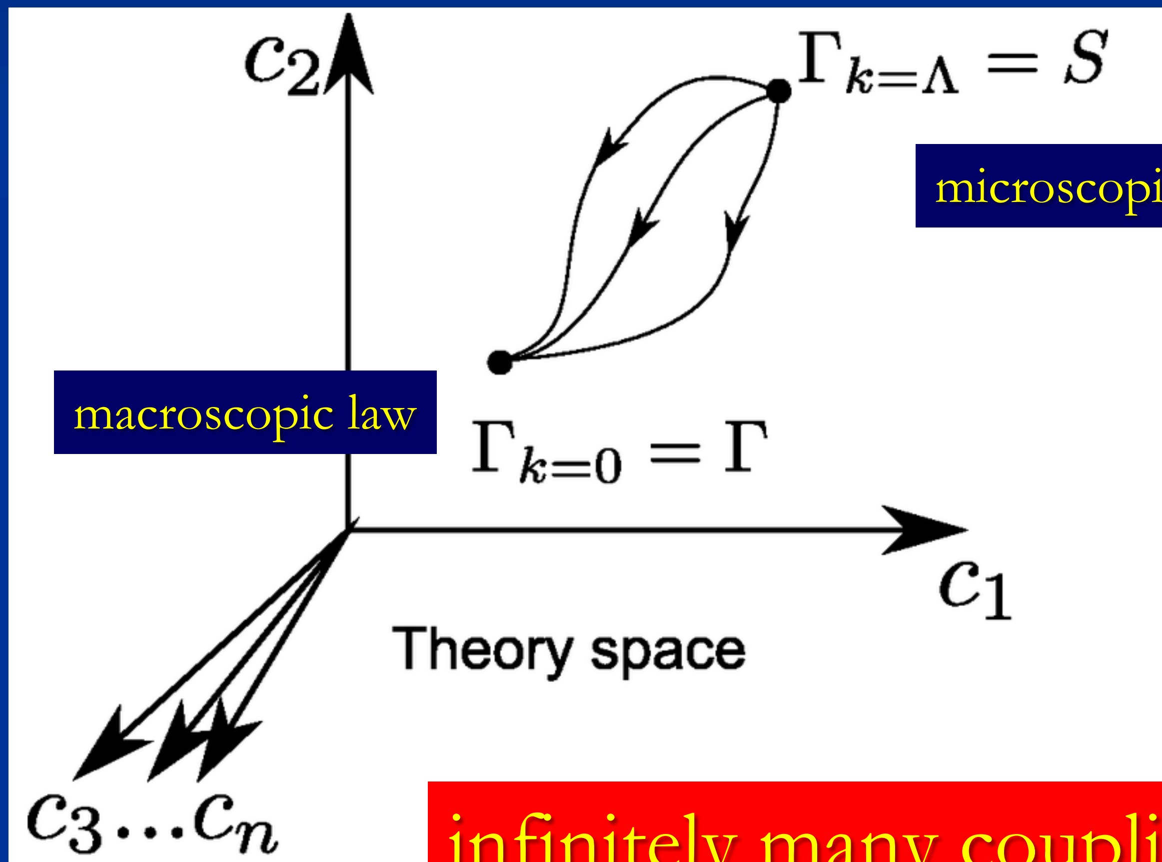
Macroscopic Observation

(Free energy functional, effective action)

functional renormalization : flowing action



flowing action

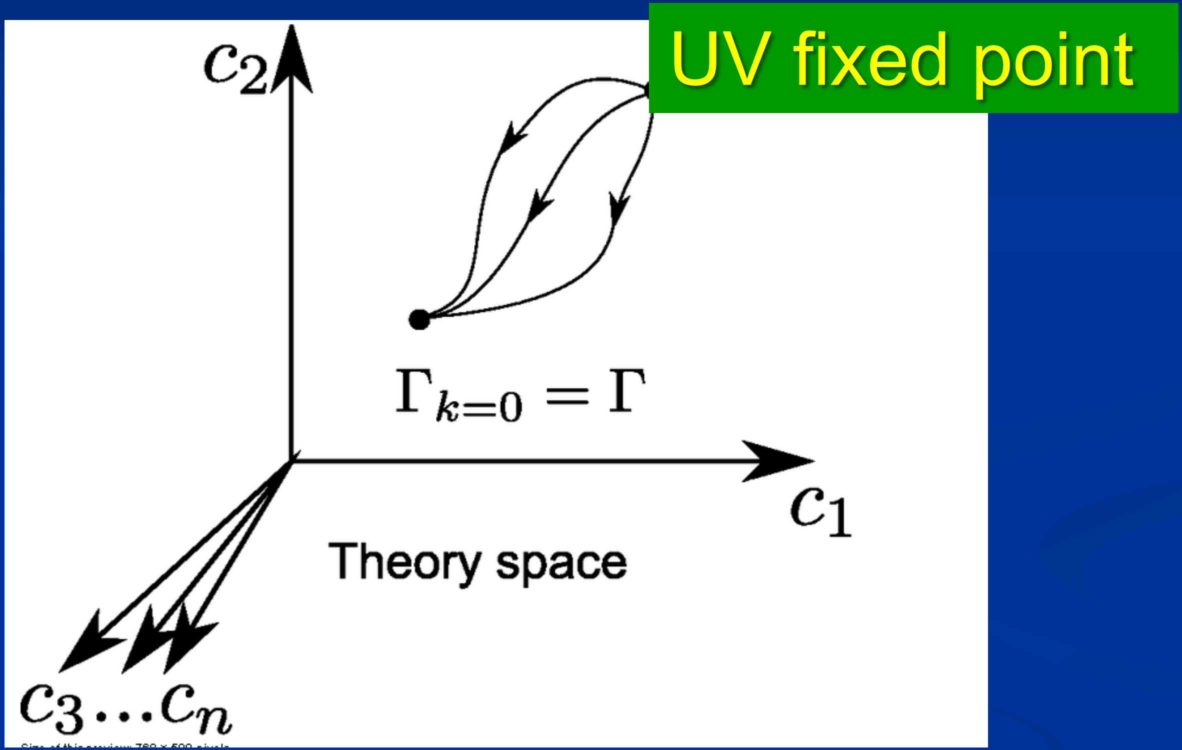


microscopic law

macroscopic law

infinitely many couplings

Ultraviolet fixed point



Extrapolation of microscopic law to infinitely short distances is possible.

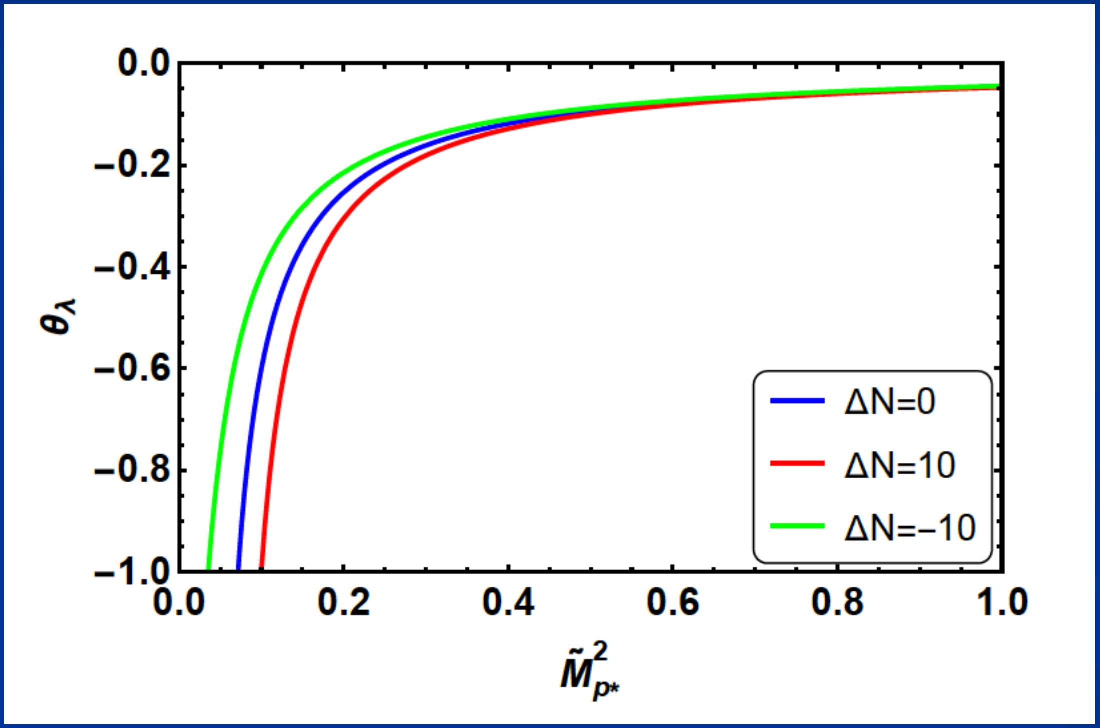
Complete theory

Prediction of mass of Higgs boson ?

Quartic scalar coupling irrelevant ?

needs $\theta_l < 0$ or A>0

Quartic scalar coupling is irrelevant parameter



Can be predicted !

Pawlowski, Reichert, Yamada,…

Conclusion (1)

- Quantum gravity is a renormalizable quantum field theory, realized by UV - fixed point of running couplings or flowing effective action

- Quantum gravity is predictive :

  Mass of the Higgs boson (and more …?)

  Properties of inflation

  Properties of dark energy

Quantum scale symmetry

Exactly on fixed point:
No parameter with dimension of length or mass is present in the quantum effective action.

Then invariance under dilatations or global scale transformations is realized as a quantum symmetry.

Continuous global symmetry

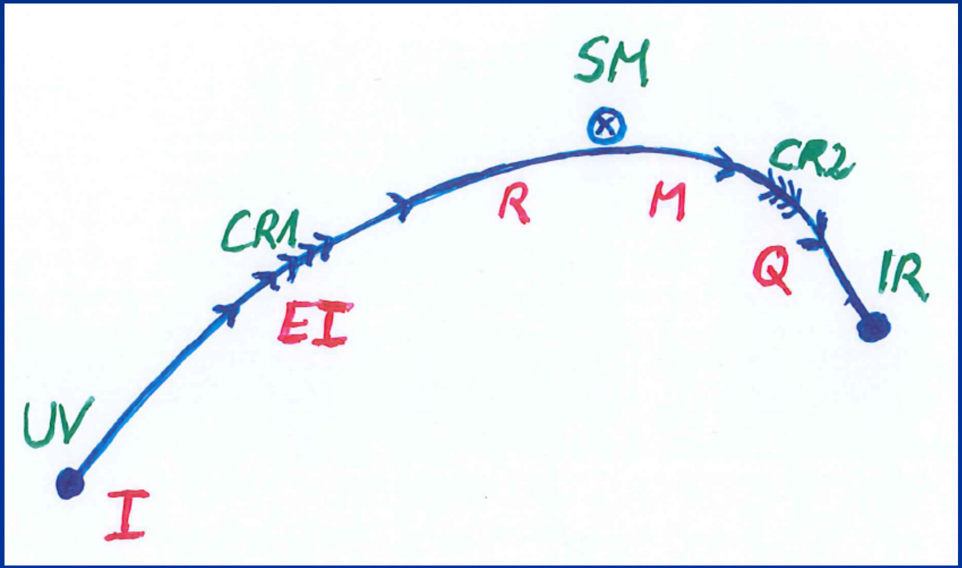
Scale symmetry in cosmology

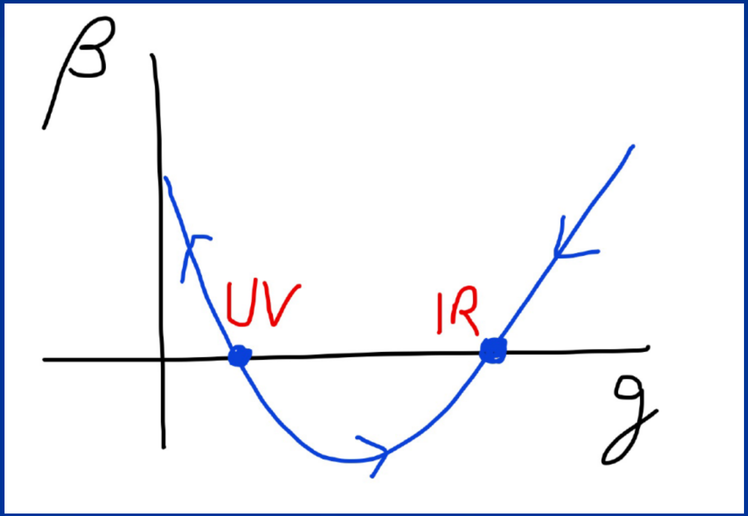
Approximate scale symmetry near fixed points

- UV : approximate scale invariance of primordial fluctuation spectrum from inflation

- IR : cosmon is pseudo Goldstone boson of
  spontaneously broken scale symmetry,
  tiny mass,
  responsible for dynamical Dark Energy

Possible consequences of crossover in quantum gravity





Realistic model for inflation and dark energy with single scalar field

Inflation :

the vicinity of the UV - fixed point

Starobinski inflation

$$\mathcal{L} = \sqrt{g}\left\{-\frac{C}{2}R^2 - \frac{M^2}{2}R + V\right\}$$

Scale symmetry if M^2/R (and V/R^2) go to zero.

Cosmological solution : R decreases

Early stages : very large R, close to scale symmetry

Starobinski inflation

$$\mathcal{L} = \sqrt{g}\left\{-\frac{C}{2}R^2 - \frac{M^2}{2}R + V\right\}$$

Scale symmetry for large R/M^2

End of inflation : C R near M^2
Substantial violation of scale symmetry

Primordial fluctuation spectrum:
frozen long before end of inflation
approximate scale symmetry of fluctuation spectrum

Eternal Universe

- Universe exists since infinite past (in physical time), close to fixed point.
- Big Bang is field – singularity, due to inappropriate choice of field variables for the metric.

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu}$$

This is similar, but not identical,

to coordinate - singularity

Conclusion (2)

Fixed points of quantum gravity

and associated quantum scale symmetry

are crucial for understanding the

evolution of our Universe

variable gravity

“Newton’s constant is not constant – and particle masses are not constant”

Variable Gravity

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi \right\}$$

quantum effective action,
variation yields field equations

Einstein gravity : $\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} M^2 R \right\}$

+ scale symmetric standard model

- Replace all mass scales by scalar field χ

(1) Higgs potential

$$U = \frac{\lambda_H}{2}(\varphi^\dagger\varphi - \epsilon\chi^2)^2 \;\Rightarrow\; \varphi_0^2 = \epsilon\chi^2$$

(2) Strong gauge coupling, normalized at $\mu = \chi$, is independent of χ

$$g(\chi) = \bar{g} \;\Rightarrow\; \Lambda_{QCD} = \chi \exp\left(-\frac{1}{b_0\bar{g}^2}\right) \quad b_0 = \frac{1}{16\pi^2}\left(22 - \frac{4}{3}N_f\right)$$

+ scale invariant action for dark matter

Variable Gravity

- Scalar field coupled to gravity
- Effective Planck mass depends on scalar field
- Simple quadratic scalar potential involves intrinsic mass μ
- Nucleon and electron mass proportional to dynamical Planck mass

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi \right\}$$

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Scale symmetry in variable gravity (IR – fixed point)

$$\Gamma = \int_x \sqrt{g}\left\{ -\frac{1}{2}\chi^2 R + \xcancel{\mu^2\chi^2} + \frac{1}{2}\left(B(\xcancel{\chi/\mu}) - 6\right)\partial^\mu\chi\partial_\mu\chi \right\}$$

IR fixed point for $\mu/\chi = 0$:
quantum scale symmetry

Tiny violation of scale symmetry
for tiny μ/χ .

Cosmic scale symmetry and the cosmological constant problem

- IR – fixed point reached for $\chi \to \infty$
- Impact of intrinsic mass scale disappears

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi \right\}$$

asymptotically vanishing cosmological „constant“

- What matters : Ratio of potential divided by fourth power of Planck mass

$$\frac{V}{\chi^4} = \frac{\mu^2\chi^2}{\chi^4} = \frac{\mu^2}{\chi^2}$$

- vanishes for $\chi \to \infty$!

Quintessence

Dynamical dark energy ,

generated by scalar field (cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87
P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Prediction :

homogeneous dark energy influences recent cosmology

- of same order as dark matter -

Original models do not fit the present observations

…. modifications

(different growth of neutrino mass)

A new view from quantum gravity

Simple approximation for graviton contribution to scalar potential:

- Predicts mass of Higgs scalar
- Solves Gauge Hierarchy problem ?
- Solves cosmological constant problem

Quantum scale symmetry at fixed points

Quantum scale symmetry plays important role in particle physics and cosmology

- Particle scale symmetry is crucial for understanding of gauge hierarchy

  SM- fixed point

- Cosmic scale symmetry is crucial for dynamical dark energy

  IR- fixed point

- Gravity scale symmetry rules beginning of cosmology

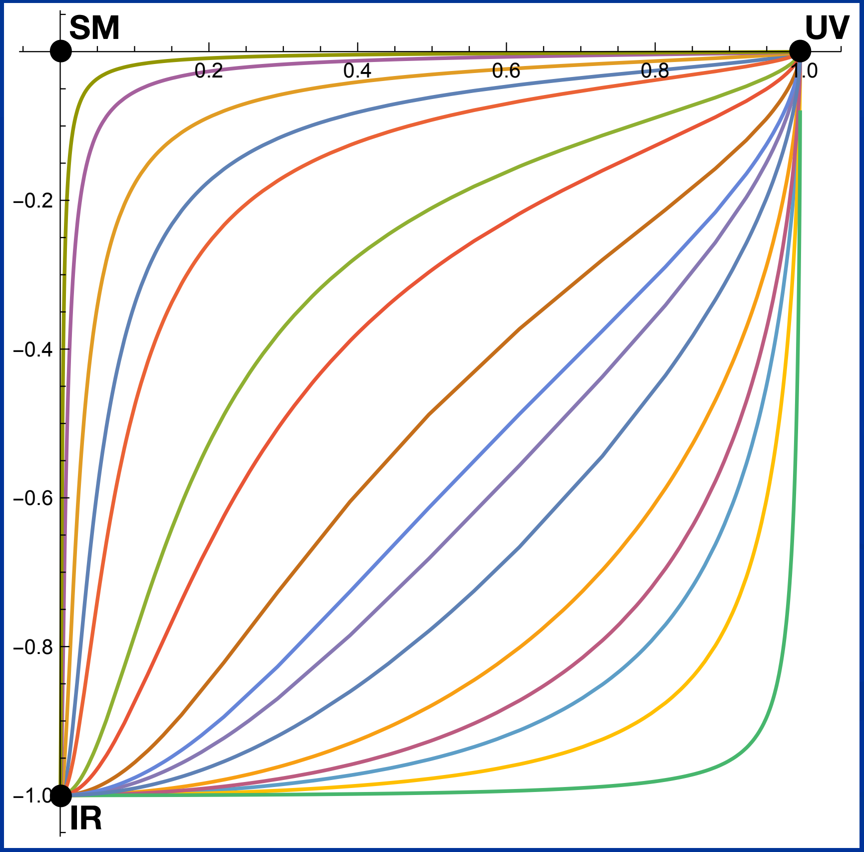
  UV- fixed point

Scale symmetry and fixed points

Relative strength of gravity

Particle scale symmetry

Gravity scale symmetry



Distance from electroweak phase transition

Cosmic scale symmetry

Conclusions (3)

Many incorrect statements on naturalness neglect the important consequences of quantum scale symmetry and associated fixed points.

Symmetries crucial for naturalness

Near fixed points : Individual contributions do not represent a natural value for the total effect end