Quantum gravity predictions for particle physics

Prediction of mass of Higgs boson

Asymptotic safety of gravity and the Higgs boson mass

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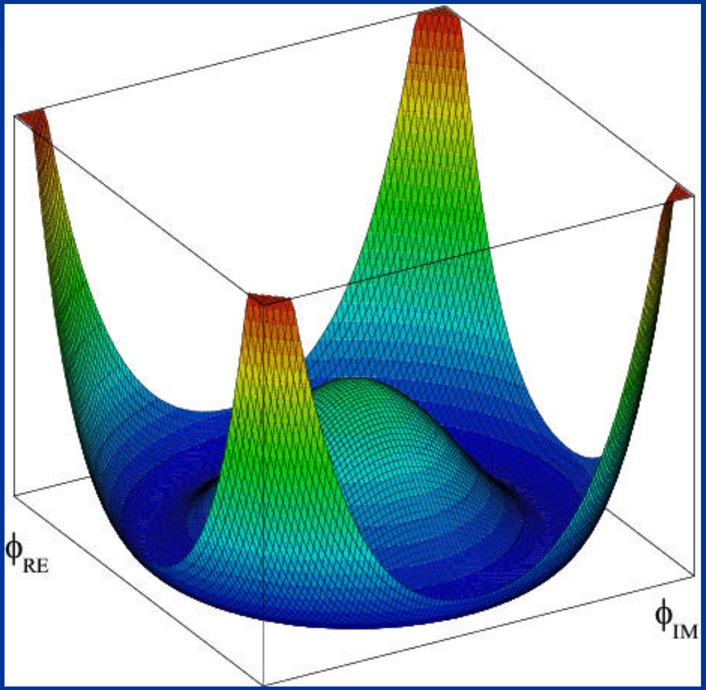
Abstract

There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalous dimension $A_\lambda > 0$ the running of the quartic scalar self interaction λ at scales beyond the Planck mass is determined by a fixed point at zero. This results in $m_H = m_{\min} = 126$ GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well.

s in $m_H = m_{\min} = 126$ GeV, with o

Why can quantum gravity make predictions for particle physics ?

Effective potential for Higgs scalar



$$V(\varphi) = -\mu^2\varphi^\dagger\varphi + \frac{1}{2}\lambda(\varphi^\dagger\varphi)^2$$

$$= \frac{1}{2}\lambda(\varphi^\dagger\varphi - \varphi_0^2)^2 + \text{const.})$$

Fermi scale

$$\varphi_0 = 175 \text{ GeV}$$

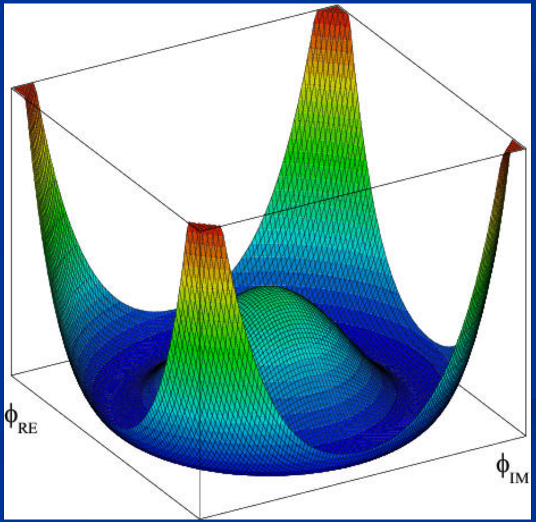
Radial mode and Goldstone mode

expand around minimum of potential

$$\varphi = \varphi_0 + \sigma + i\eta$$

$$\varphi_0, \sigma, \eta : \ \text{real}$$

$$-\mathcal{L}_\varphi = \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma + \frac{1}{2}\partial^\mu\eta\partial_\mu\eta + \frac{1}{2}m^2\sigma^2 + \ldots$$



mass term for radial mode

$$m^2 = 2\lambda\varphi_0^2$$

Quartic scalar coupling

prediction of mass of Higgs boson

=

prediction of value of quartic scalar coupling λ at Fermi scale

$$m^2 = 2\lambda\varphi_0^2$$

Why can quantum gravity make predictions for quartic scalar coupling ?

Mass scales

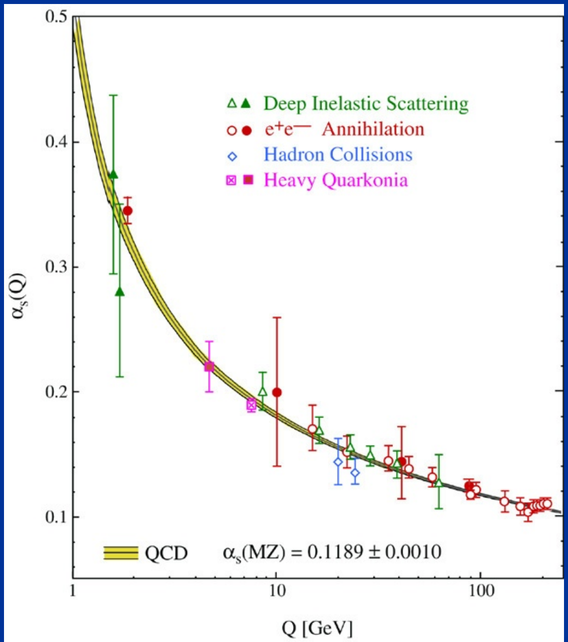
- Fermi scale φ_0 ~ 100 GeV
- Planck mass M ~ 10^{18} GeV
- Gravity at Fermi scale is very weak : How can it influence the effective potential for the Higgs scalar and the mass of the Higgs boson ?

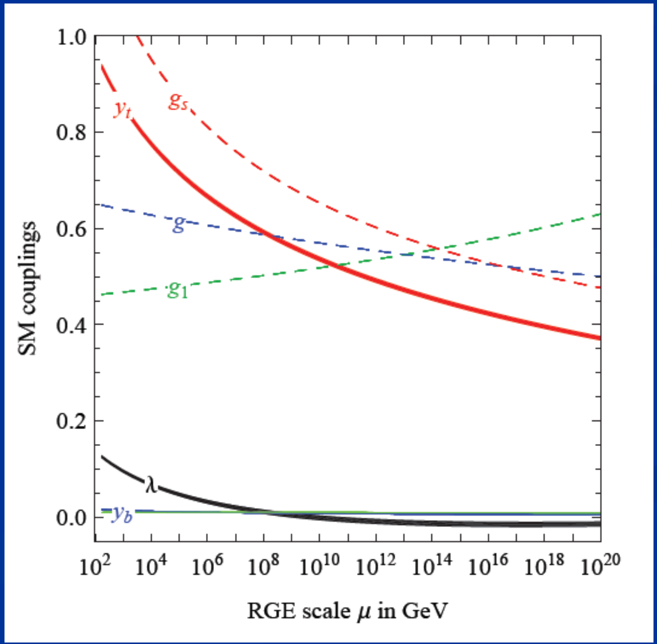
$$\varepsilon = \frac{\varphi_0^2}{M^2} = 5 \cdot 10^{-33}$$

Quantum fluctuations induce running couplings

- possible violation of scale symmetry
- well known in QCD or standard model

Bethke

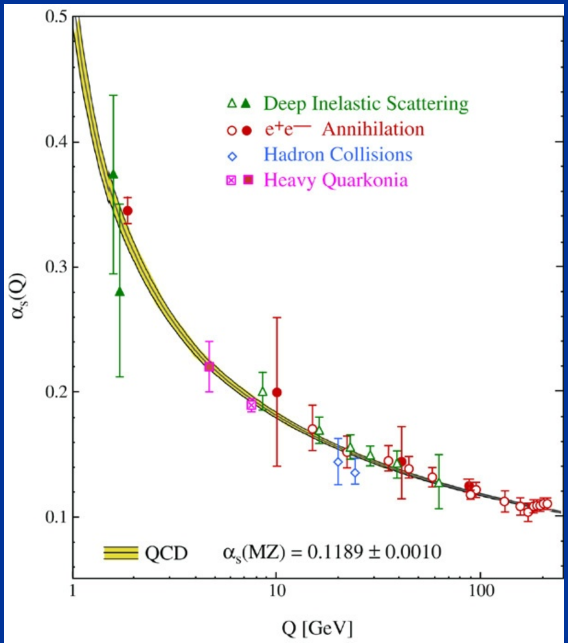


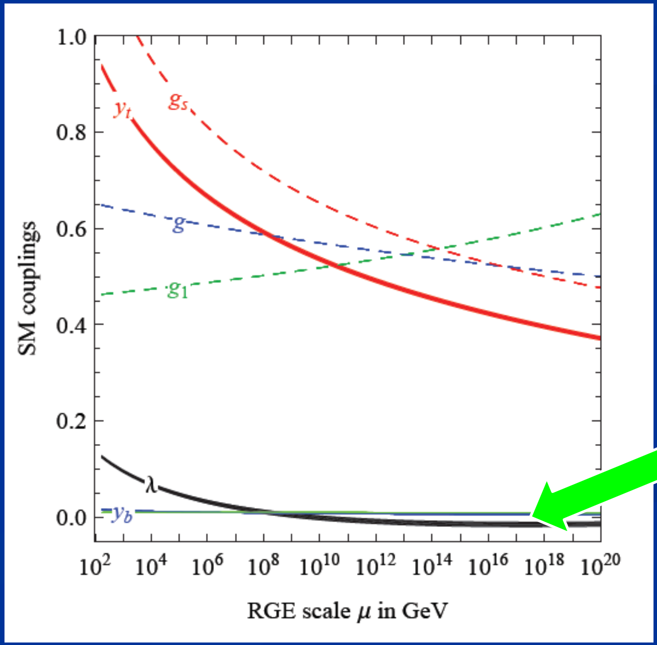


Degrassi et al

Quantum fluctuations induce running couplings

- possible violation of scale symmetry
- well known in QCD or standard model

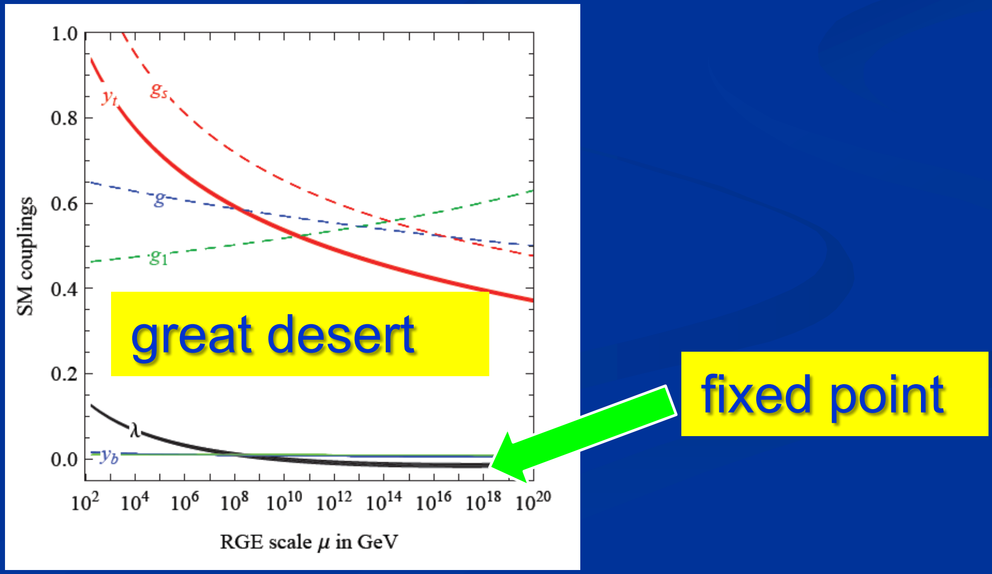


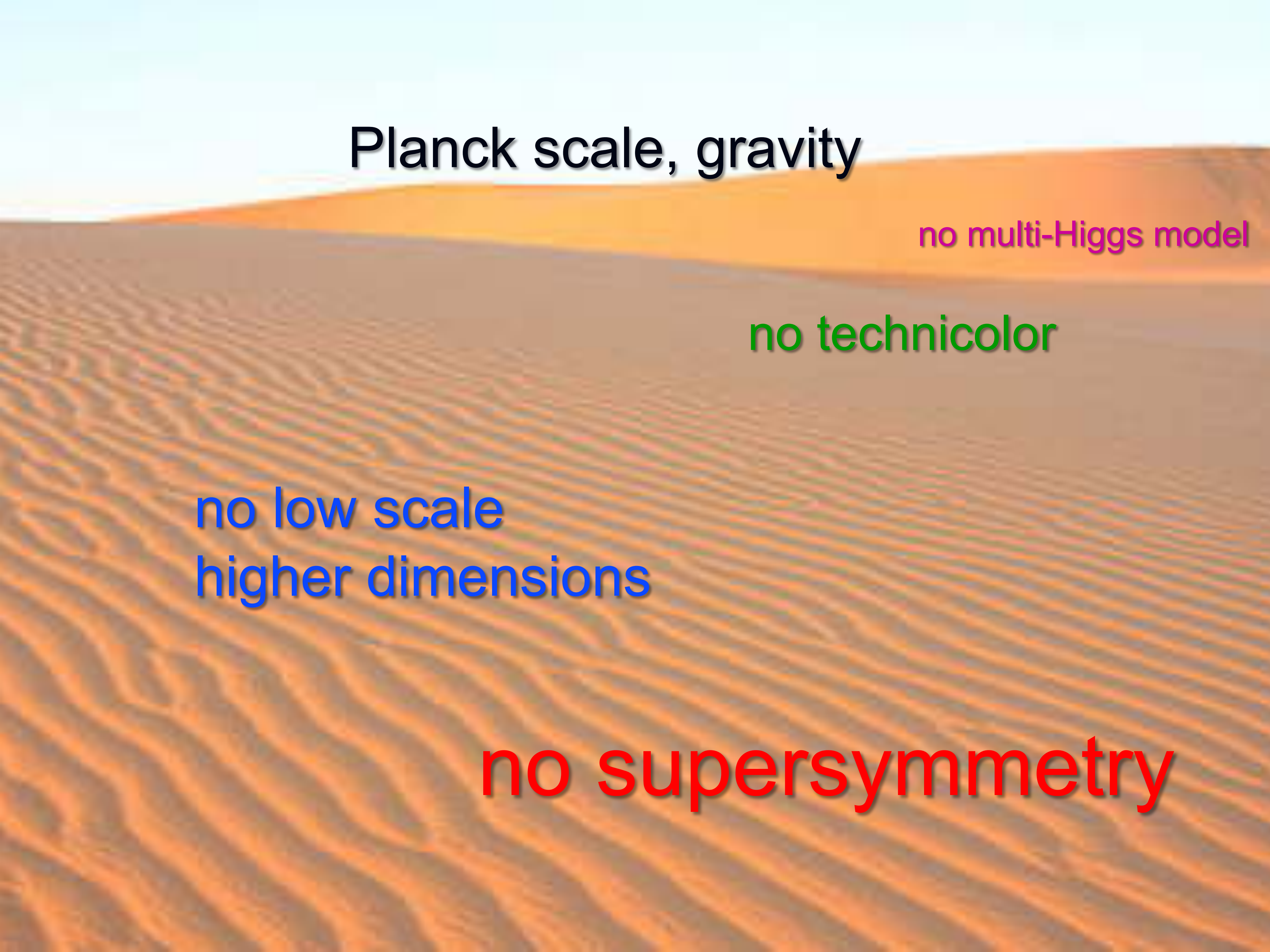


The mass of the Higgs boson, the great desert, and asymptotic safety of gravity

key points

- great desert
- solution of hierarchy problem at high scale
- high scale fixed point
- vanishing scalar coupling at fixed point

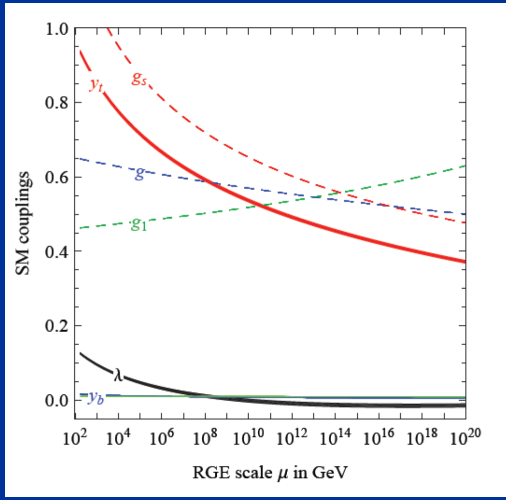



Planck scale, gravity
no multi-Higgs model
no technicolor
no low scale
higher dimensions
no supersymmetry

Essential point for prediction of Higgs boson mass:

Initial value of quartic scalar coupling near Planck mass is predicted by quantum gravity

Extrapolate perturbatively to Fermi scale



Results in prediction for ratio Higgs boson mass over W- boson mass, or Higgs boson mass over top quark mass

Near Planck mass gravity is not weak !

Predictive power !

Flowing couplings

Couplings change with momentum scale due to quantum fluctuations.

Renormalization scale k : Only fluctuations with momenta larger k are included. The scale k can be momenta, geometric quantities, or just be introduced "by hand".

Flow of k to zero : all fluctuations included, IR
Flow of k to infinity : UV

Renormalization group

How do couplings or physical laws change with scale k ?

Graviton fluctuations erase quartic scalar coupling

Renormalization scale k : Only fluctuations with momenta larger k are included

$$k \frac{\partial \lambda}{\partial k} = A \lambda$$

gravity induced anomalous dimension

A > 0

$$\lambda(k) = \lambda(\mu) \left(\frac{k}{\mu} \right)^{A}$$

$$k \to 0 \quad \Rightarrow \quad \lambda \to 0$$

Fixed point

$$k \frac{\partial \lambda}{\partial k} = A \lambda$$

$$\lambda(k) = \lambda(\mu) \left(\frac{k}{\mu} \right)^A$$

The quartic scalar coupling λ has a

fixed point at λ=0

It flows towards the fixed point as k is lowered :

irrelevant coupling

For a UV – complete theory it is predicted to assume the fixed point value

Gravitational contribution to running quartic coupling

$$\partial_t \lambda = A_\lambda \lambda$$

$$A = \frac{1}{48\pi^2 \tilde{M}_p^2}\left[\frac{20}{(1-v_0)^2} + \frac{1}{(1-v_0/4)^2}\right]$$

$$\partial_t = k\partial_k$$

running Planck mass : $$M_p^2(k) = M^2 + \tilde{M}_{p*}^2 k^2$$

dimensionless squared Planck mass $$\tilde{M}_p^2 = \frac{M_p^2}{k^2}$$

Strength of gravity

$$g_{grav} = \frac{l_p^2}{2l^2} = \frac{k^2}{2M^2}$$

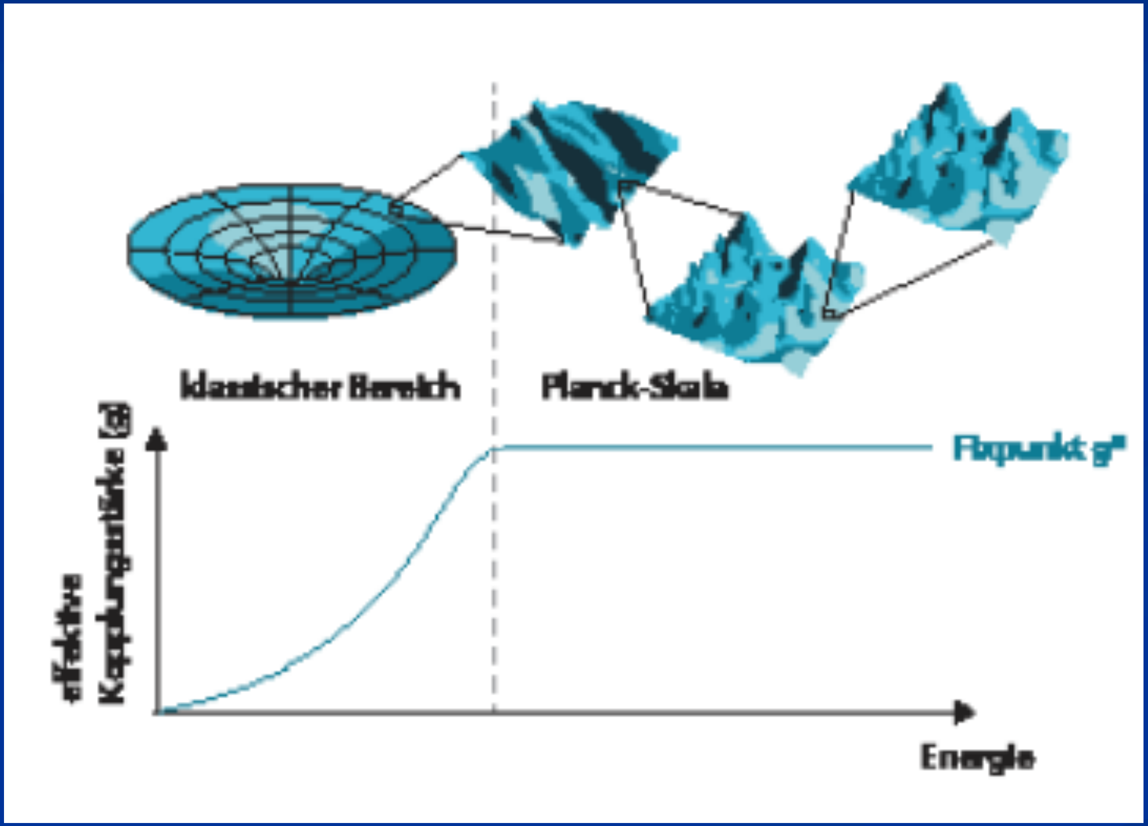
l_p : Planck length

M : Planck mass

running gravitational coupling

$$g_{grav} = \frac{k^2}{2M^2(k)} = w^{-1}(k)$$

Strength of gravity



A.Eichhorn, CW, Spektrum der Wissenschaft

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$$\partial_t = k \partial_k$$

running Planck mass :

$$M_p^2(k) = M^2 + \tilde{M}_{p*}^2 k^2$$

dimensionless squared Planck mass

$$\tilde{M}_p^2 = \frac{M_p^2}{k^2} = \tilde{M}_{p*}^2 \left[\left(\frac{k_t}{k} \right)^2 + 1 \right]$$

$$k_t^2 = \frac{M^2}{\tilde{M}_{p*}^2}$$

large k : constant A

small k: $A \sim k^2 / M^2$

transition at

$k_t \sim 10^{19}$ GeV

Flowing dimensionless Planck mass

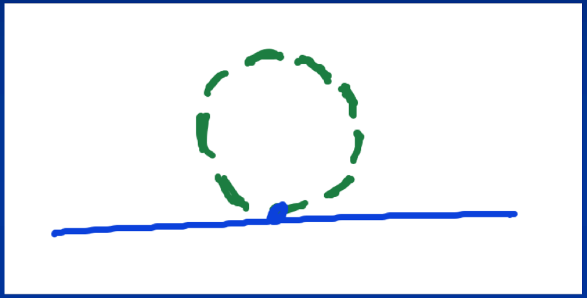
- Renormalization scale k : Only fluctuations with momenta larger k are included

Flowing
Planck mass $M^2(k)$

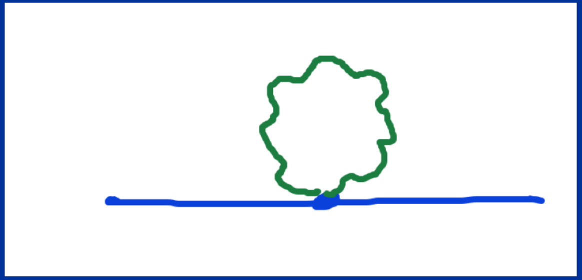
$$\partial_t M^2 = 4ck^2$$

$$\partial_t = k\partial_k$$

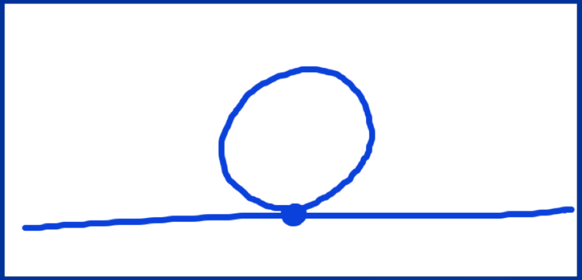
Universality of gravity



scalar loop,
fermion loop

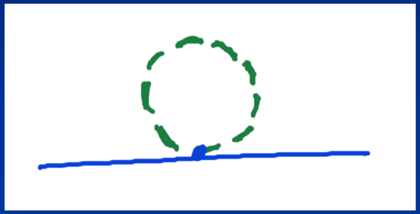


gauge boson
loop

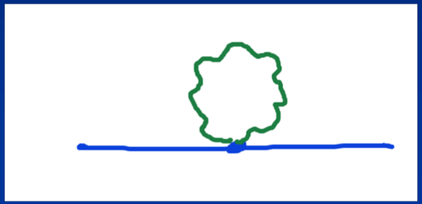


graviton
loop

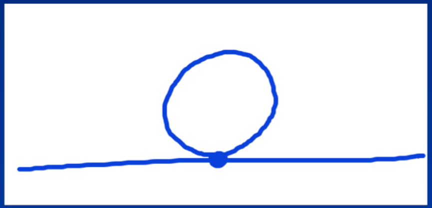
Universality of gravity



scalar loop,
fermion loop



gauge boson
loop



graviton
loop

c is independent of coupling constants

$$\partial_t M^2 = 4ck^2$$

Flowing dimensionless Planck mass

- Renormalization scale k : Only fluctuations with momenta larger k are included

Flowing Planck mass $M^2(k)$

$$\partial_t M^2 = 4ck^2$$

$$\partial_t = k\partial_k$$

matter contribution

$$c_M = \frac{\mathcal{N}_M}{192\pi^2}.$$

$$\mathcal{N}_M = 4\,N_V - N_S - N_F$$

with graviton contribution

$$c_M = \frac{1}{192\pi^2}\left(\mathcal{N}_M + \frac{43}{6} + \frac{75(1-\eta_g/6)}{2(1-v)}\right)$$

Flowing dimensionless Planck mass

Flowing
Planck mass $M^2(k)$

$$\partial_t M^2 = 4ck^2$$

$$\partial_t = k\partial_k$$

solution :

$$M^2(k) = M^2 + 2c_M k^2$$

Flowing dimensionless Planck mass

Flowing
Planck mass $M^2(k)$

$$\partial_t M^2 = 4ck^2$$

Dimensionless
squared Planck mass

$$w = \frac{M^2}{2k^2}$$

$$\partial_t w = -2w + 2c$$

solution :

$$w = c + \frac{\bar{M}^2}{2k^2}$$

$$M^2(k) = M^2 + 2c_M k^2$$

Fixed point and flow away from fixed point

UV - fixed point $w_* = c$

$\partial_t w = -2w + 2c$

approached for k → ∞

$$w = c + \frac{\bar{M}^2}{2k^2}$$

$$M^2(k) = M^2 + 2c_M k^2$$

Near UV – fixed point : M ~ k

$\tilde{M}^2_{p*}$ = $2c$

Transition to constant M for small k, gravity gets weak, w^{-1} decreases to zero

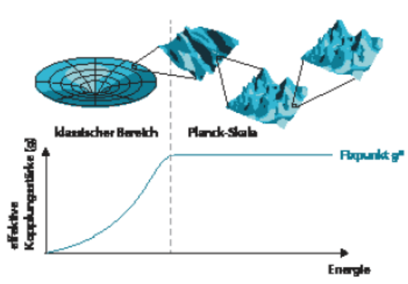
M is relevant parameter, cannot be predicted

Weak and constant gravity

$$M_{\mathrm{p}}^2(k) = \begin{cases} \tilde{M}_{\mathrm{p}*}^2\, k^2 & \text{for } k > k_t \\ M^2 & \text{for } k < k_t \end{cases}$$

$$k_t^2 = \frac{M^2}{\tilde{M}_{\mathrm{p}*}^2}$$

$$M_{\mathrm{p}}^2(k) = M^2 + \tilde{M}_{\mathrm{p}*}^2 k^2$$



Two regimes for the (inverse) strength of gravity

$$\tilde{M}_{\mathrm{p}}^2 = \frac{M_{\mathrm{p}}^2}{k^2} = \tilde{M}_{\mathrm{p}*}^2 \left[\left(\frac{k_t}{k} \right)^2 + 1 \right]$$

Gravitational contribution to running quartic coupling

$$\partial_t \lambda = A_\lambda \lambda$$

$$A = \frac{1}{48\pi^2 \tilde{M}_{\rm p}^2}\left[\frac{20}{(1-v_0)^2} + \frac{1}{(1-v_0/4)^2}\right]$$

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Running Planck mass :

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$$\tilde{M}_{\rm p}^2 = \frac{M_{\rm p}^2}{k^2} = \tilde{M}_{{\rm p}*}^2\left[\left(\frac{k_t}{k}\right)^2 + 1\right]$$

$$k_t^2 = \frac{M^2}{\tilde{M}_{{\rm p}*}^2}$$

large k : constant A

small k: $A \sim k^2 / M^2$

transition at

$k_t \sim 10^{19}$ GeV

UV – fixed point for quartic coupling

Flow equation for λ : $$\partial_t \lambda_H = A\lambda_H - C_H$$

$$C_H^{(p)} = -\beta_\lambda^{(SM)}$$

$$\approx -\frac{1}{16\pi^2}\left\{12\lambda_H^2 + 12y_t^2\lambda_H - 12y_t^4 + \frac{9}{4}g_2^2 + \frac{9}{10}g_2^2g_1^2 + \frac{27}{100}g_1^4 - \left(9g_2^2 + \frac{9}{5}g_1^2\right)\lambda_H\right\}$$

Fixed point : λ = C / A

$$\lambda(k_{tr}) \approx 0\ ,\ \beta_\lambda(k_{tr}) \approx 0$$

Quantum Gravity

Quantum Gravity is a renormalisable quantum field theory

Asymptotic safety

Asymptotic safety of quantum gravity

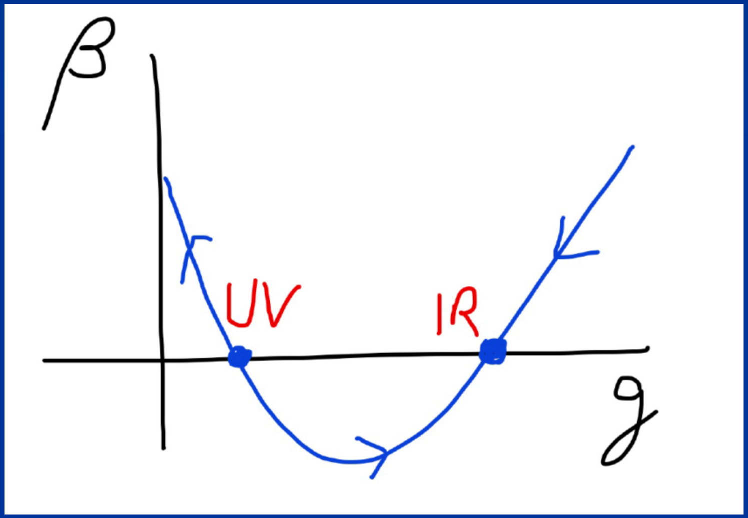
if UV fixed point exists :

quantum gravity is

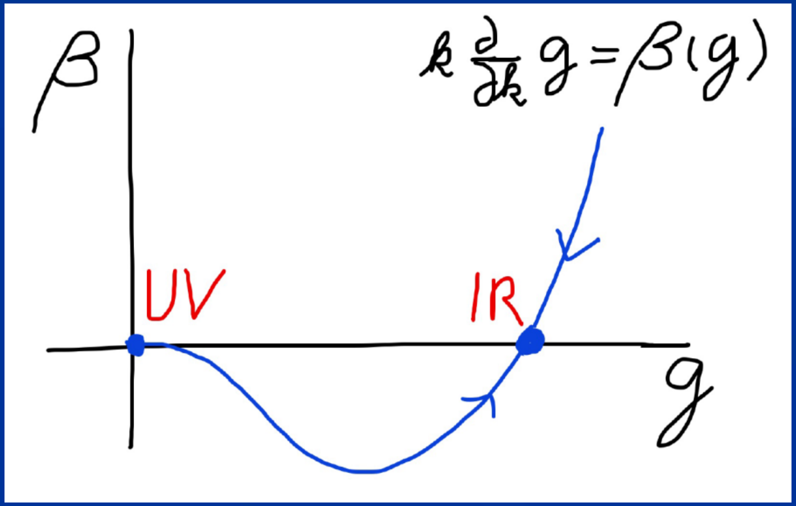
non-perturbatively renormalizable !

S. Weinberg , M. Reuter

Asymptotic safety

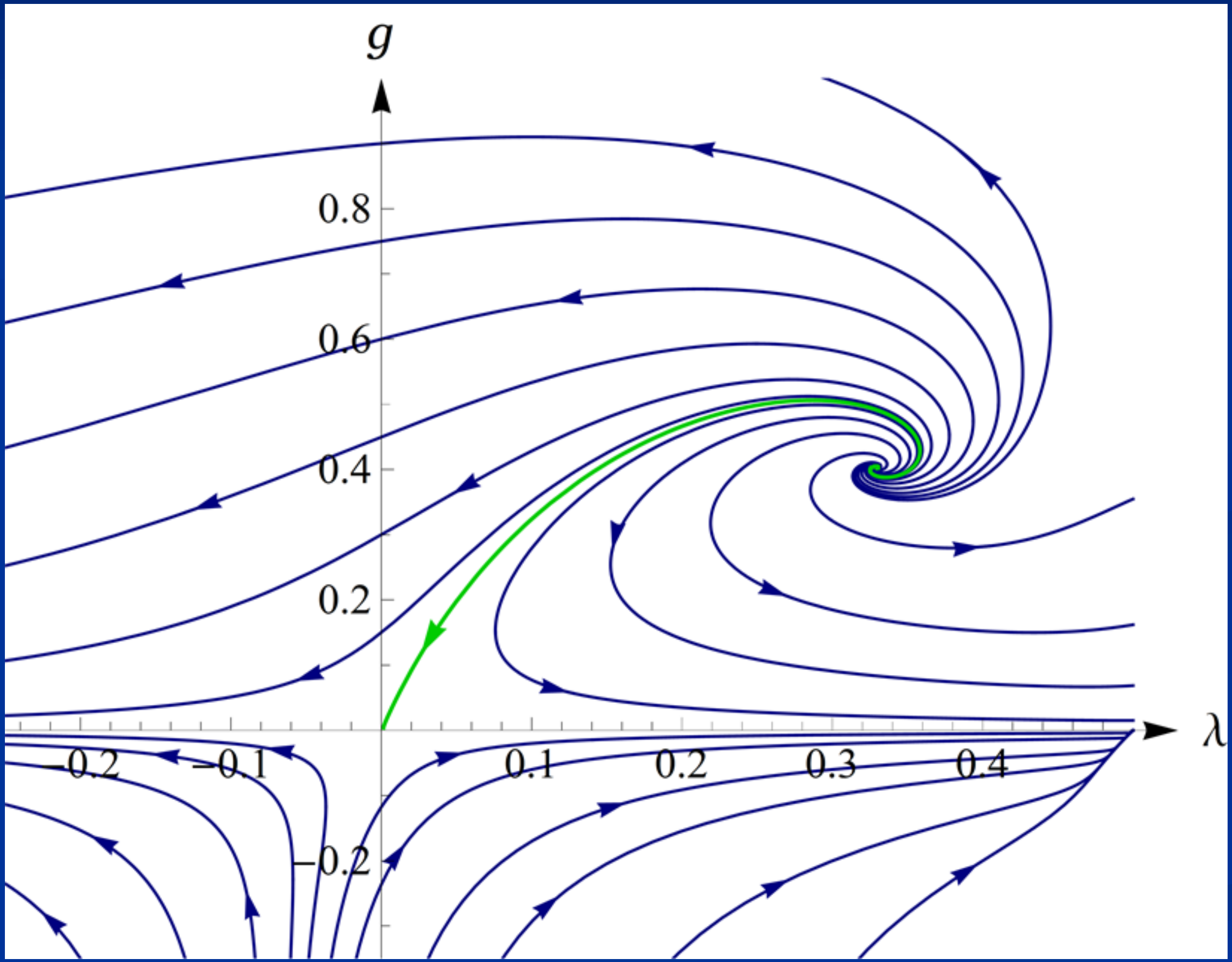


Asymptotic freedom



Relevant parameters yield undetermined couplings. Quartic scalar coupling is not relevant and can therefore be predicted.

UV- fixed point for quantum gravity



Wikipedia

Enhanced predictivity for UV – fixed point

- Free parameters of a theory correspond to relevant parameters for small deviations from fixed point.
- If the number of relevant parameters at the UV-fixed point is smaller than the number of free parameters (renormalizable couplings) in the standard model:
- Relations between standard model parameters become predictable !

Fixed points

$$g = \{g_1, \ldots, g_i, \ldots\}$$

couplings

$$\tilde{g}_i = g_i k^{-d_i}$$

dimensionless

Flow equation $$\partial_t \tilde{g}_i = \beta_i(\tilde{g})$$

Fixed points: zeros of beta-function

No running → No scale

Quantum scale symmetry

Stability matrix

$$g = \{g_1, \ldots, g_i, \ldots\}$$

couplings

$$\tilde{g}_i = g_i k^{-d_i}$$

dimensionless

Flow equation

$$\partial_t \tilde{g}_i = \beta_i(\tilde{g}) = -d_i \tilde{g}_i + f_i(\tilde{g})$$

Expand in vicinity of fixed point

$$\partial_t \tilde{g}_i = \sum_j \frac{\partial \beta_i}{\partial \tilde{g}_j}\bigg|_{\tilde{g}=\tilde{g}_*} (\tilde{g}_j - \tilde{g}_{j*}) = -T_{ij}(\tilde{g}_j - \tilde{g}_{j*})$$

T : stability matrix

Critical exponents

$$\partial_t \tilde{g}_i = \sum_j \frac{\partial \beta_i}{\partial \tilde{g}_j}\bigg|_{\tilde{g}=\tilde{g}_*} (\tilde{g}_j - \tilde{g}_{j*}) = -T_{ij}(\tilde{g}_j - \tilde{g}_{j*})$$

θ_l : Eigenvalues of stability matrix T
= Critical exponents

Linearized solution

$$\tilde{g}_i = \tilde{g}_{i*} + \sum_l C_l V_i^l \left(\frac{k}{\mu}\right)^{-\theta_l}$$

Irrelevant parameters: eigenvectors in coupling constant space with $\theta_l < 0$

flow towards fixed point values as k is lowered

Irrelevant parameters

- "Forget" information about initial values
- Central ingredient for

  predictivity of quantum field theories
- For UV – complete theories : irrelevant parameters have to take precisely the fixed point values
- **Relevant parameters** flow away from fixed point as k is lowered – they are the only free parameters

Predictivity at fixed point

- Irrelevant parameters are predicted to take fixed point values
- Only relevant parameters are free
- Number of free parameters of a renormalizable quantum field theory = number of relevant parameters at the fixed point

a prediction…

Asymptotic safety of gravity and the Higgs… mass

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Christof Wette…

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12 Ja…

Abstract

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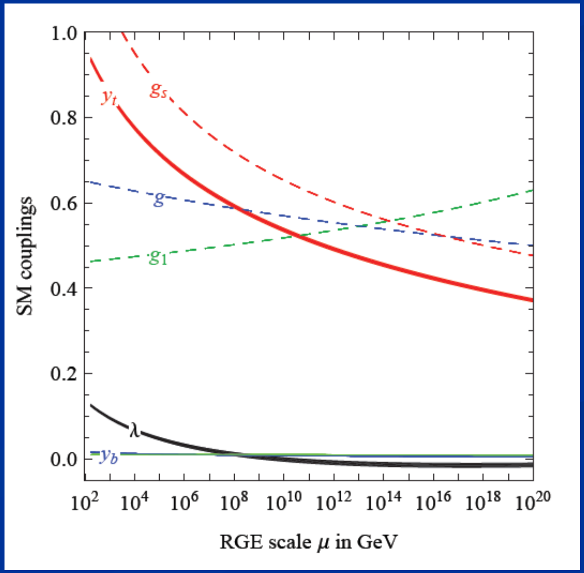
s in $m_H = m_{\text{min}} = 126$ GeV, with o

Quartic scalar coupling is irrelevant coupling

Prediction of Higgs boson mass:

- Value of quartic scalar coupling near Planck mass is predicted by UV- fixed point
- Gravity decouples below Planck mass , resulting in perturbative flow

Extrapolate perturbatively to Fermi scale :



How to compute non-perturbative quantum gravity effects ?

Quantum gravity computation by functional renormalization

Introduce infrared cutoff with scale k, such that only fluctuations with (covariant) momenta larger than k are included.

Then lower k towards zero

Exact renormalization group equation

Exact flow equation

for scale dependence of average action

$$\partial_k \Gamma_k[\varphi] = \frac{1}{2}\mathrm{Tr}\left\{\left(\Gamma_k^{(2)}[\varphi] + R_k\right)^{-1} \partial_k R_k\right\}$$

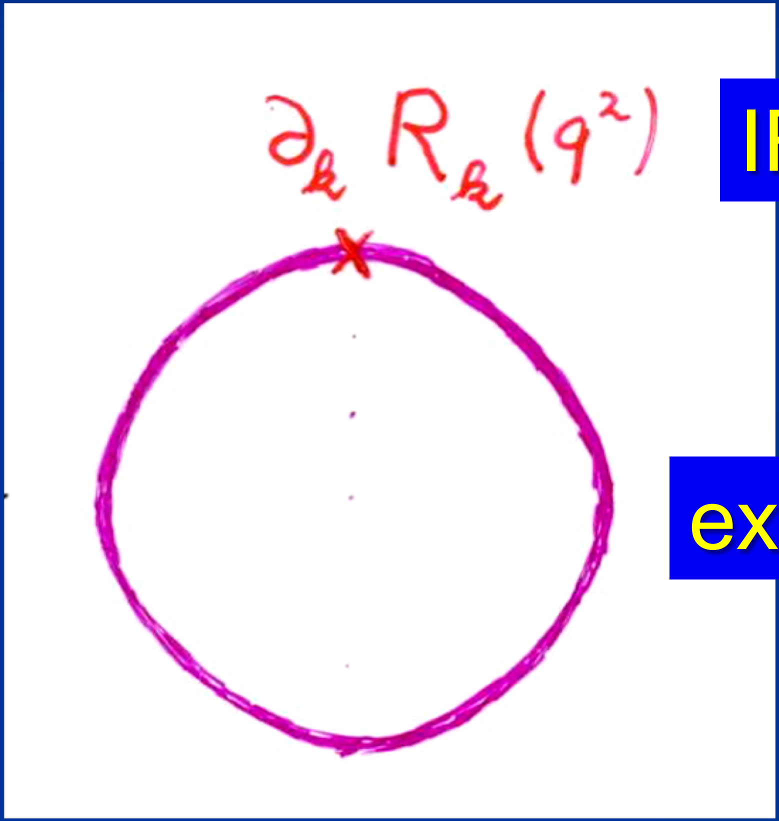
'92

$$\left(\Gamma_k^{(2)}\right)_{ab}(q,q') = \frac{\delta^2 \Gamma_k}{\delta\varphi_a(-q)\delta\varphi_b(q')}$$

$$\mathrm{Tr} : \sum_a \int \frac{d^d q}{(2\pi)^d}$$

(fermions : STr)

Functional flow equation for scale dependent effective action



IR cutoff

exact propagator

Exact renormalization group equation

Exact flow equation

for scale dependence of average action

$$\partial_k \Gamma_k[\varphi] = \frac{1}{2} \mathrm{Tr} \left\{ \left(\Gamma_k^{(2)}[\varphi] + R_k \right)^{-1} \partial_k R_k \right\}$$

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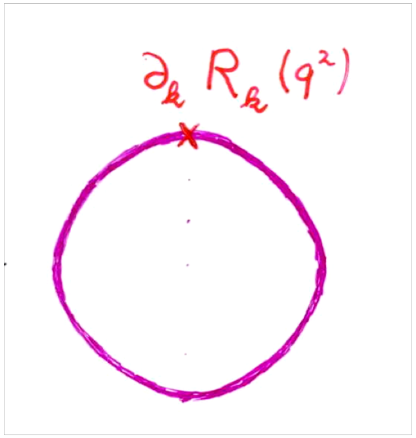


From

Microscopic Laws

(Interactions, classical action)

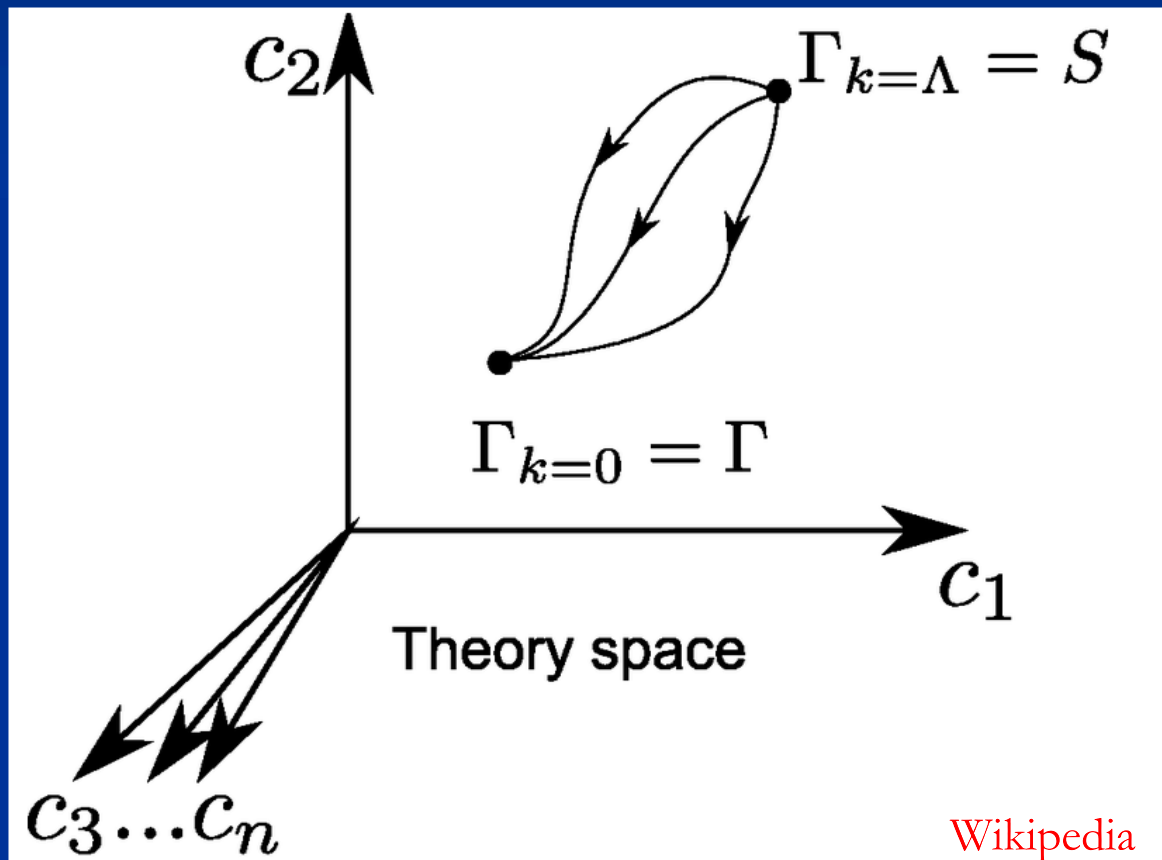
to

Fluctuations!

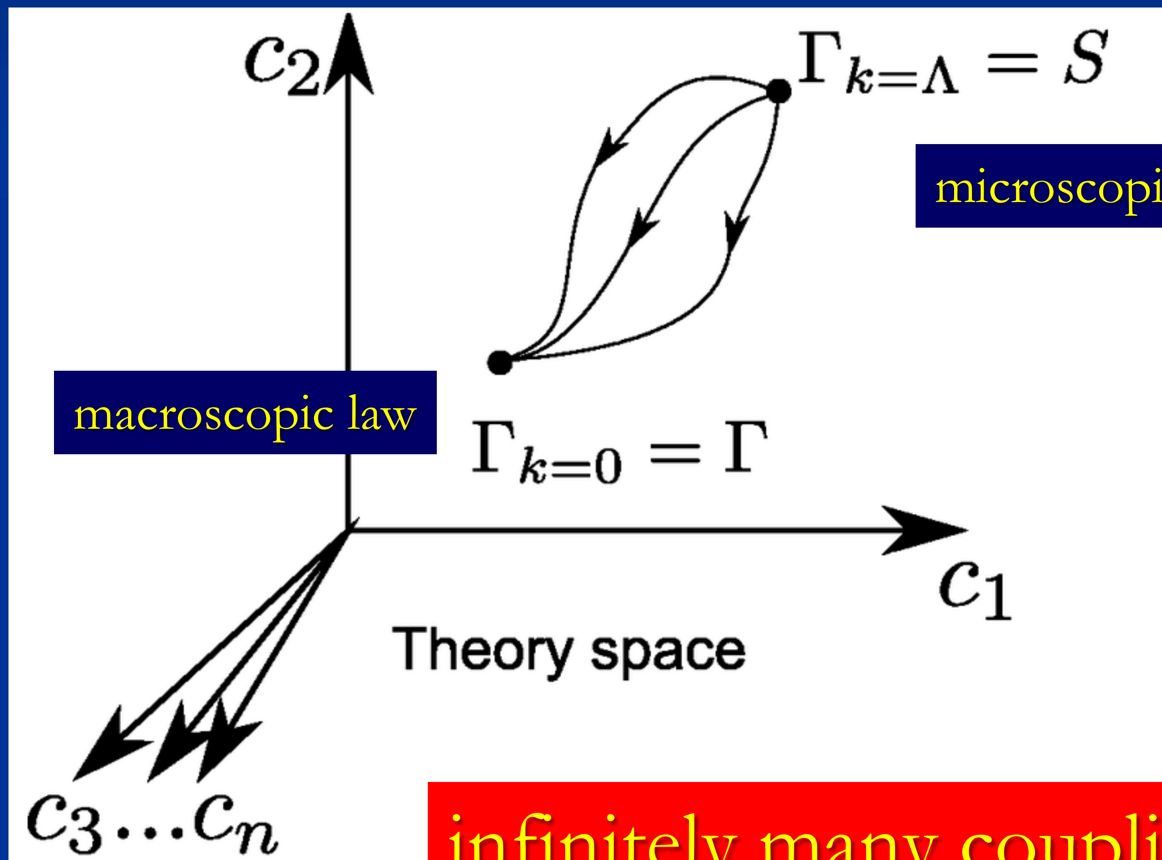
Macroscopic Observation

(Free energy functional, effective action)

functional renormalization : flowing action



flowing action

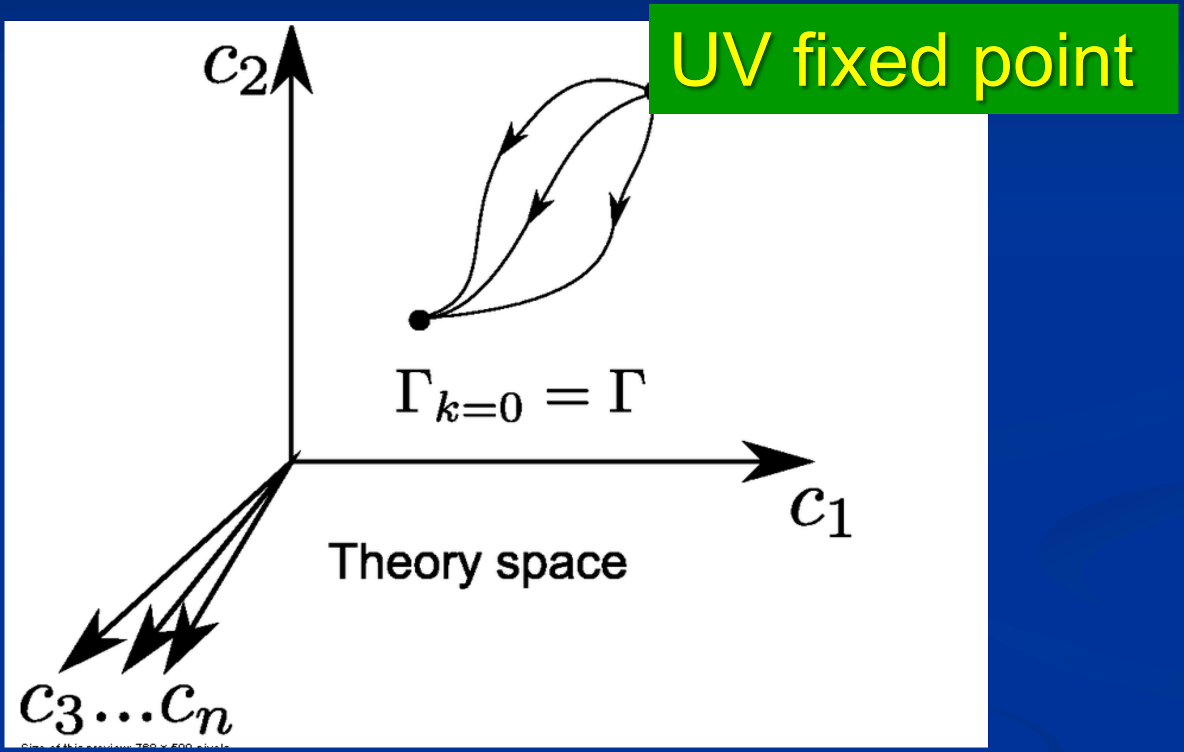


microscopic law

macroscopic law

infinitely many couplings

Ultraviolet fixed point



Extrapolation of microscopic law to infinitely short distances is possible.

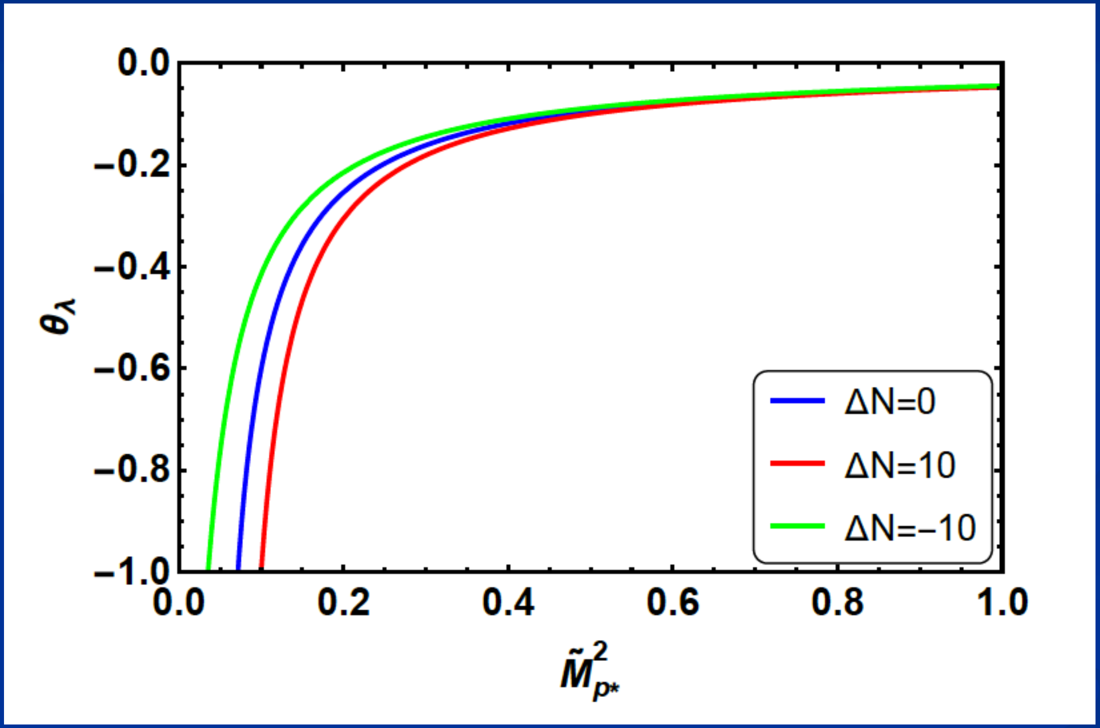
Complete theory

Prediction of mass of Higgs boson ?

Quartic scalar coupling irrelevant ?

needs $\theta_l < 0$ or A>0

Quartic scalar coupling is irrelevant parameter



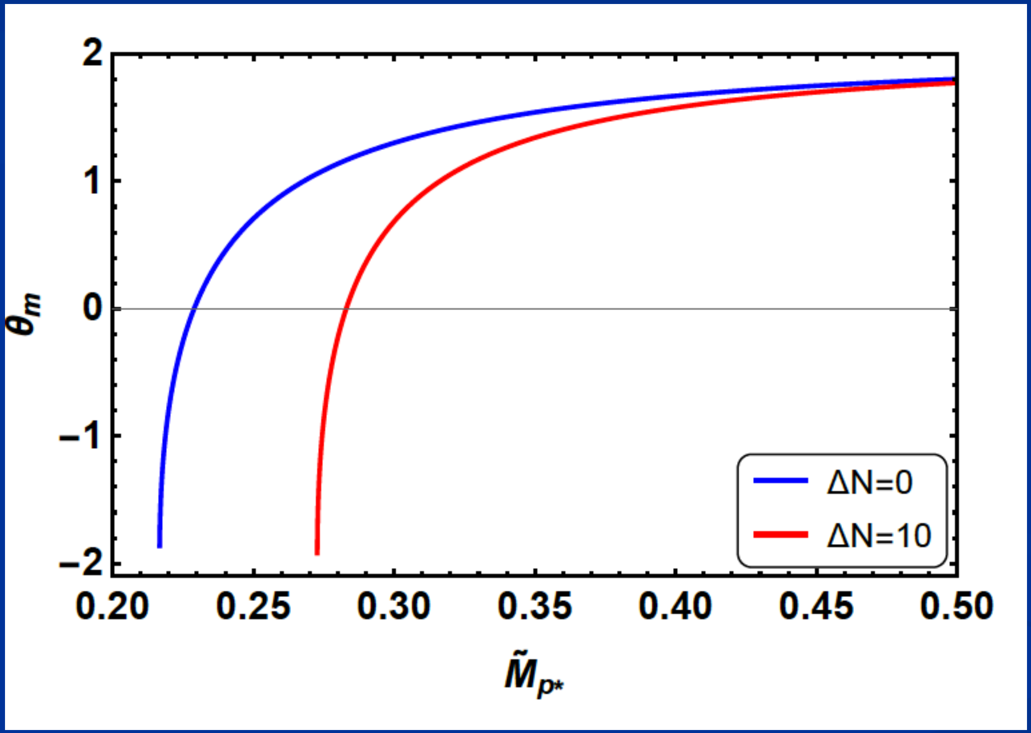
Can be predicted !

Pawlowski, Reichert, Yamada,…

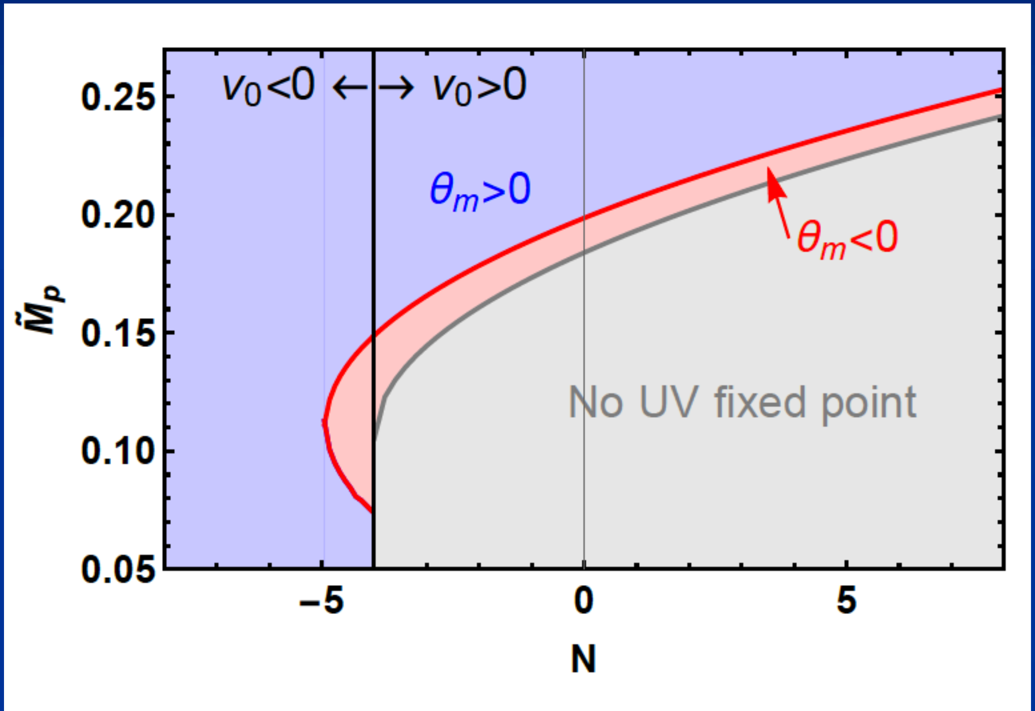
Predictivity for Fermi scale ?

Scalar mass term irrelevant ?

Higgs mass term is irrelevant for strong enough gravity



Critical exponent for Higgs mass term



For suitable particle content of model:

Higgs mass term is an irrelevant parameter

Gauge hierarchy

Possible explanation of small parameter : distance from second order vacuum electroweak phase transition is

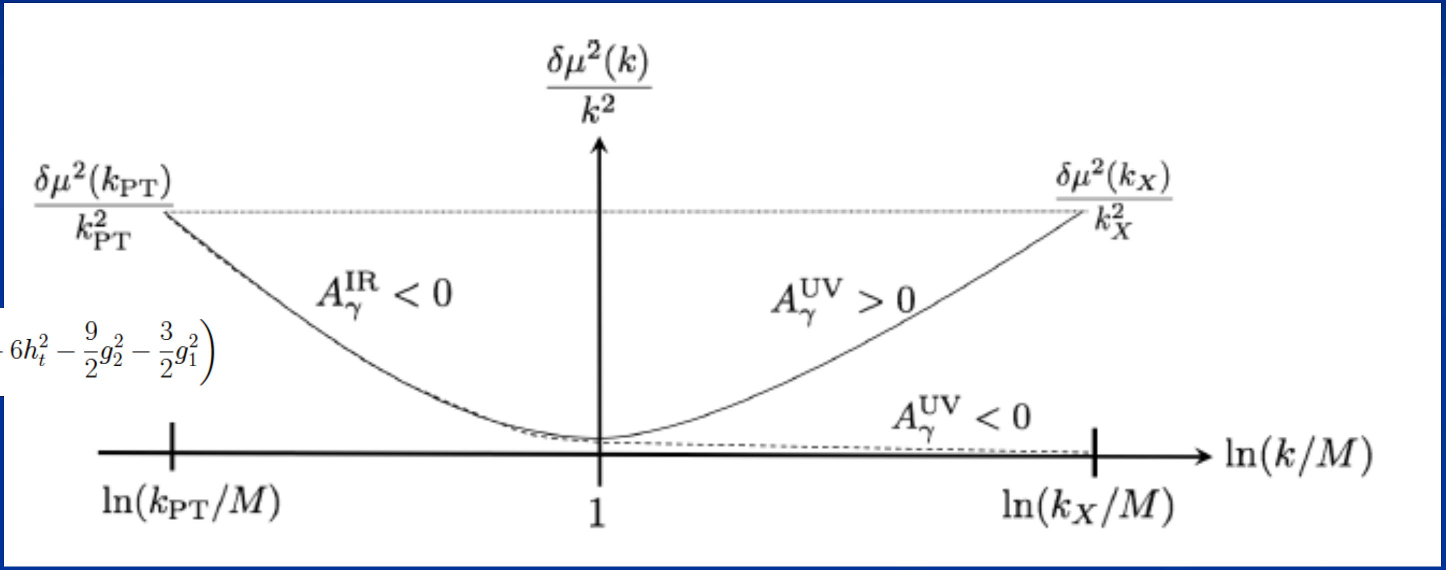
irrelevant parameter

at UV – fixed point

Possible explanation of gauge hierarchy

$$A_\gamma^{\mathrm{IR}} = -2 + A,$$

$$A = \frac{1}{16\pi^2}\left(2\lambda_H + 6h_t^2 - \frac{9}{2}g_2^2 - \frac{3}{2}g_1^2\right)$$



Gauge hierarchy problem in asymptotically safe gravity –the resurgence mechanism

Christof Wetterich[1] and Masatoshi Yamada[1]

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Prediction of Fermi scale

- If scalar mass term is irrelevant and vacuum electroweak phase transition would be precisely second order:
- The Fermi scale would be predicted to be zero !
- Running gauge and Yukawa couplings in standard model imply that vacuum electroweak phase transition is not precisely second order. Small effect.
- Small Fermi scale and huge gauge hierarchy expected.
- May be a couple of orders too small as compared to observation ? Not known definitely.

Predictions of quantum gravity ?

Simple approximation for graviton contribution to scalar potential:

- Predicts mass of Higgs scalar
- Solves Gauge Hierarchy problem ?
- Solves cosmological constant problem

Conclusions

- Quantum gravity is a renormalizable quantum field theory, realized by UV - fixed point of running couplings or flowing effective action

- Quantum gravity is predictive :

  Mass of the Higgs boson (and more …?)

  Properties of inflation

  Properties of dark energy

end

Quantum gravity prediction for the cosmological "constant" ?

Variable Gravity

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2 \chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu \chi \partial_\mu \chi \right\}$$

quantum effective action,
variation yields field equations

Asymptotically vanishing cosmological „constant"

- What matters : Ratio of potential divided by fourth power of Planck mass

$$\frac{V}{\chi^4} = \frac{\mu^2 \chi^2}{\chi^4} = \frac{\mu^2}{\chi^2}$$

- vanishes for $\chi \to \infty$!

small dimensionless number ?

- needs two intrinsic mass scales
- standard approach :V and M (cosmological constant and Planck mass)
- variable gravity : Planck mass moving to infinity , with fixed V ➡ ratio vanishes asymptotically !

Variable Gravity in scaling frame

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2 \chi^2 + \frac{1}{2}\big(B(\chi/\mu) - 6\big)\partial^\mu \chi \partial_\mu \chi \right\}$$

Variable gravity in Einstein frame

Weyl scaling :

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} \ , \ \varphi = \frac{2M}{\alpha} \ln\left(\frac{\chi}{\mu}\right)$$

effective action in Einstein frame :

$$\Gamma = \int_x \sqrt{g'} \left\{ -\frac{1}{2} M^2 R' + V'(\varphi) + \frac{1}{2} k^2(\varphi) \partial^\mu \varphi \partial_\mu \varphi \right\}$$

$$V'(\varphi) = M^4 \exp\left(-\frac{\alpha\varphi}{M}\right)$$

$$k^2 = \frac{\alpha^2 B}{4}$$

Quintessence

Dynamical dark energy ,

generated by scalar field (cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87
P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Prediction :

homogeneous dark energy influences recent cosmology

- of same order as dark matter -

Original models do not fit the present observations

…. modifications

(different growth of neutrino mass)

Quantum gravity restricts the increase of scalar potential for large fields

In quantum gravity,
the graviton fluctuations can
play an important role on
distances as large as the
size of the Universe

- for long range scalar fields and dynamical dark energy
- not for all quantities

Graviton barrier

Quantum gravity computation :

For $\chi \to \infty$

V cannot increase stronger than M^2 !

Instability of graviton propagator is avoided

Graviton barrier and solution of the cosmological constant problem

V cannot increase stronger than M^2 !

If M increases with χ, and for cosmological solutions where χ asymptotically diverges for time going to infinity:

Effective cosmological constant vanishes in infinite future

$$\mathbf{M} = \chi \quad : \quad \mathbf{V} = \mu^2 \chi^2$$

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