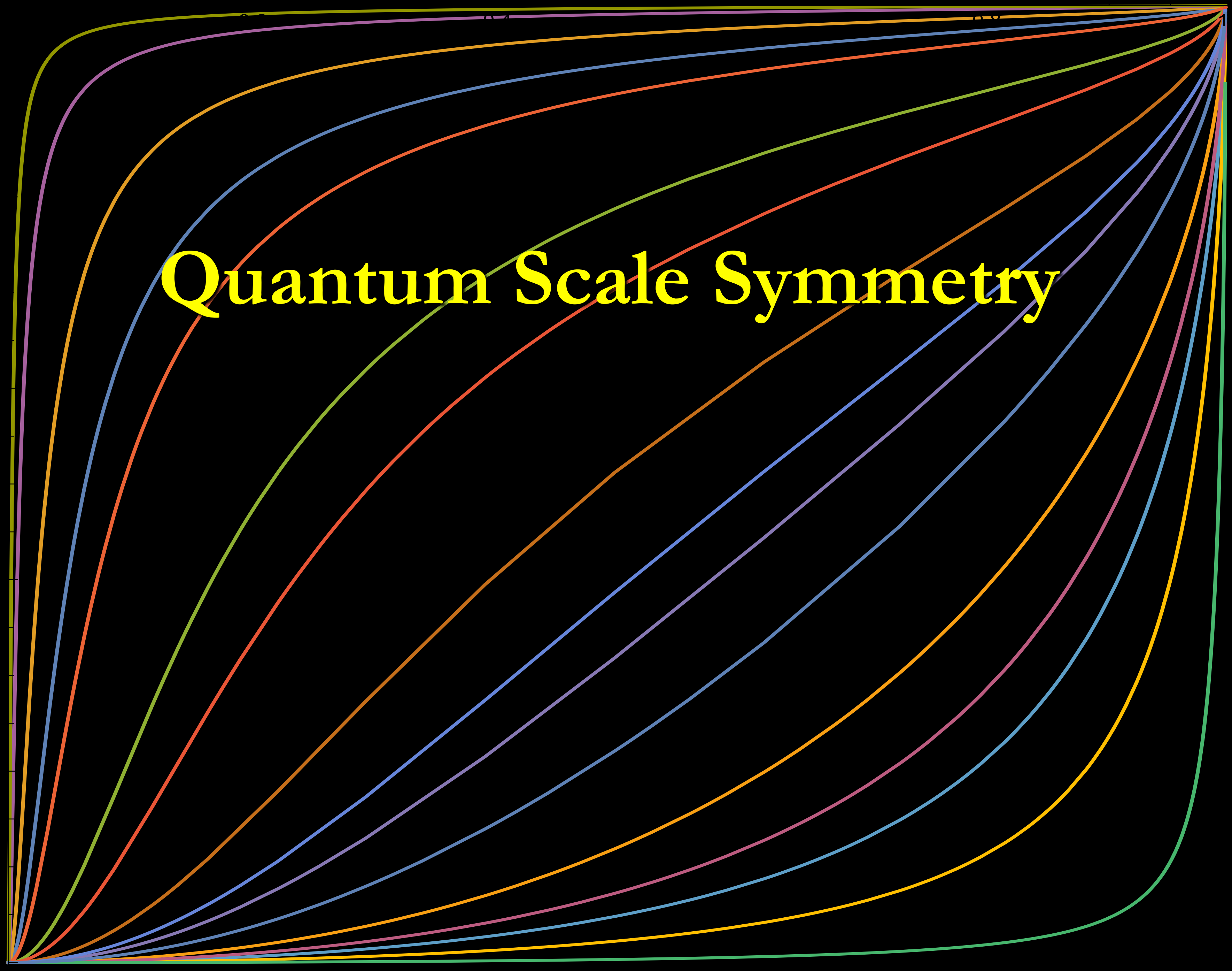
Quantum Scale Symmetry



Quantum scale symmetry

No parameter with dimension of length or mass is present in the quantum effective action.

Then invariance under dilatations or global scale transformations is realized.

Continuous global symmetry

Scale transformation of renormalized fields

$$g'_{\mu\nu} = \alpha^{-2} g_{\mu\nu}\,, \quad \sqrt{g'} = \alpha^{-4}\sqrt{g}$$

$$A'_\mu = A_\mu\,, \quad \psi' = \alpha^{3/2}\psi$$

$$\chi' = \alpha\chi$$

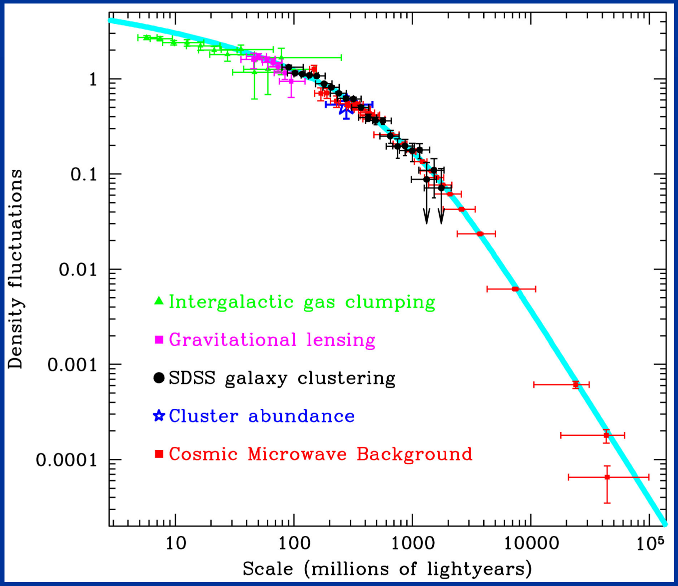
$$\mathcal{L}_\chi = \frac{1}{2} g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + \tilde{\lambda}\chi^4$$

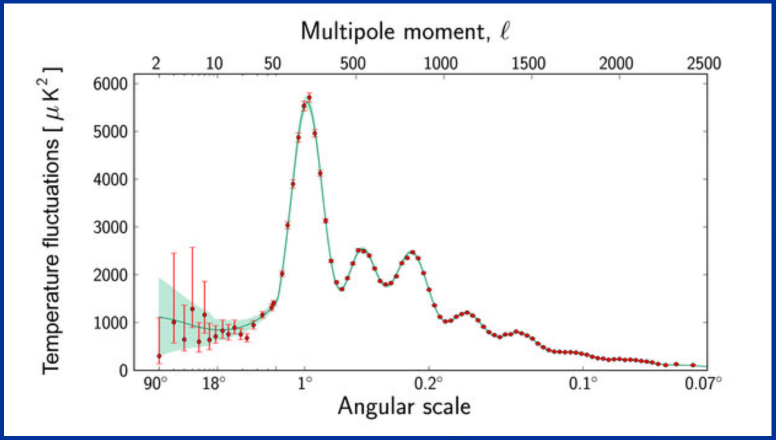
Classical scale symmetry

No parameter with dimension of length or mass is present in the classical action.

Scale symmetry in cosmology ?

Almost scale invariant primordial fluctuation spectrum





scales are present in cosmology

Scale symmetry in elementary particle physics ?

proton mass , electron mass

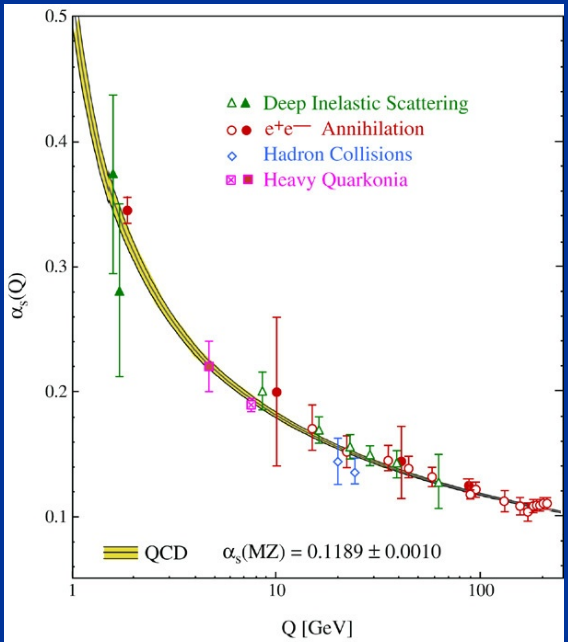
Scales are present in particle physics,
but very small as compared to Planck mass

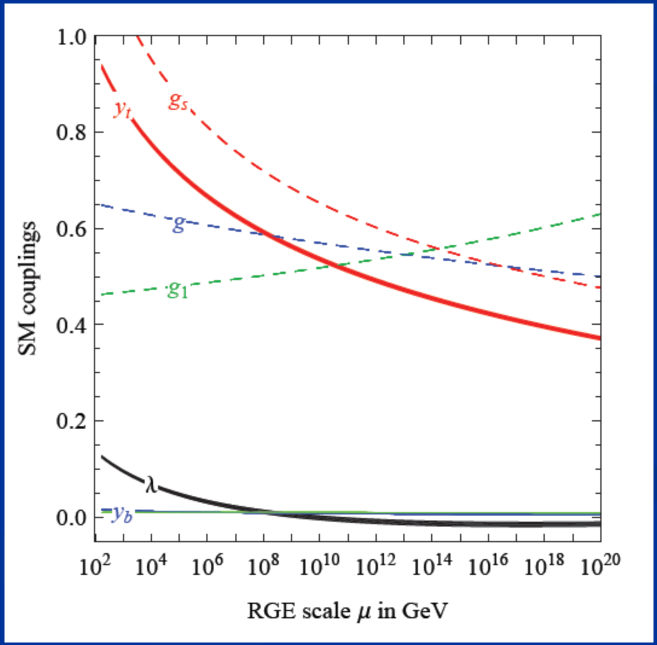
High momentum scattering almost scale invariant

Quantum scale symmetry

Quantum fluctuations induce running couplings

- violation of scale symmetry
- well known in QCD or standard model

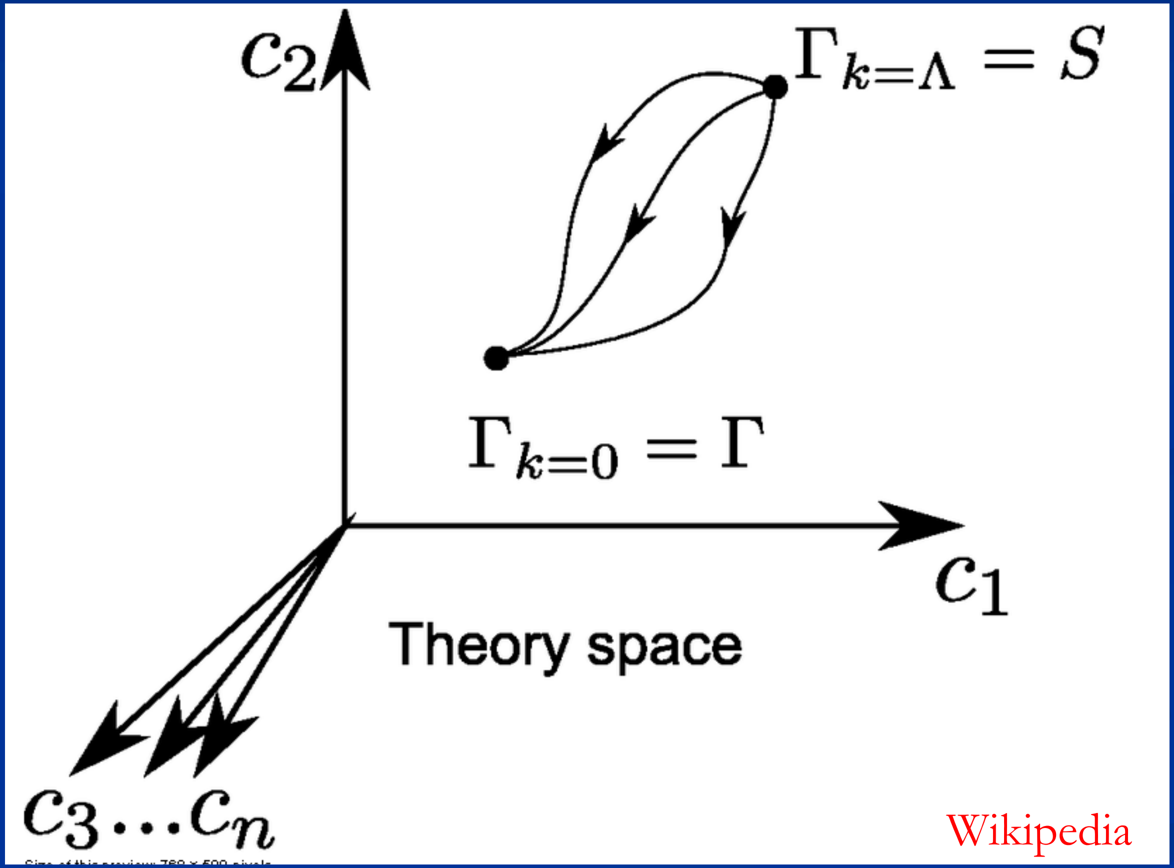




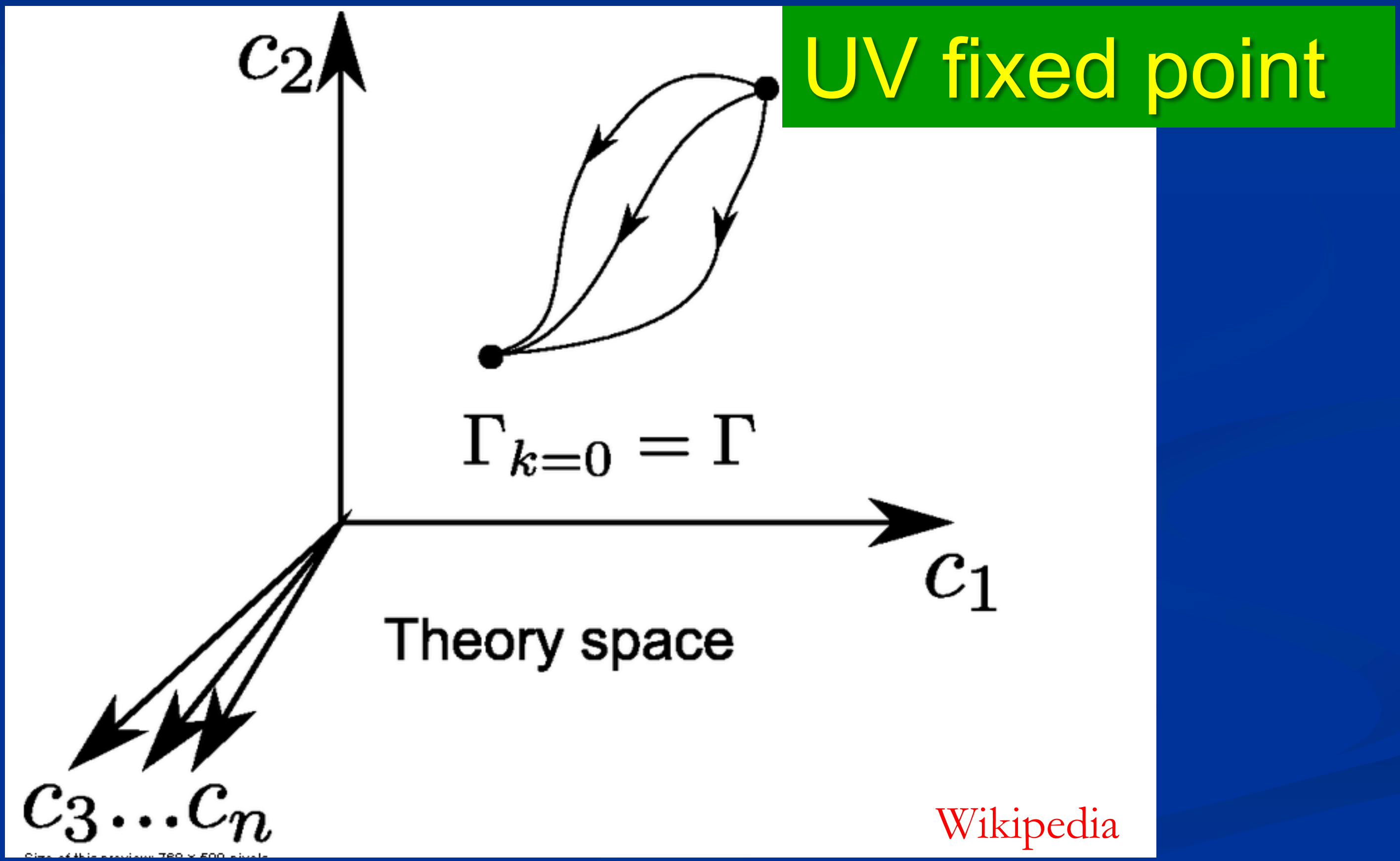
Quantum scale symmetry

- quantum fluctuations can violate scale symmetry
- running dimensionless couplings
- at fixed points , scale symmetry is exact !
- quantum fluctuations can generate scale symmetry !

Functional renormalization : flowing action



Ultraviolet fixed point



Quantum scale symmetry

Exactly on fixed point:
No parameter with dimension of length or mass is present in the quantum effective action.

Then invariance under dilatations or global scale transformations is realized as a quantum symmetry.

Continuous global symmetry

Three scale symmetries

Gravity scale symmetry:

includes transformation of fields for particles, metric and scalar singlet **UV - fixed point**

Particle scale symmetry:

metric and scalar singlet kept fixed

relative scaling of momenta with respect to Planck mass **SM - fixed point**

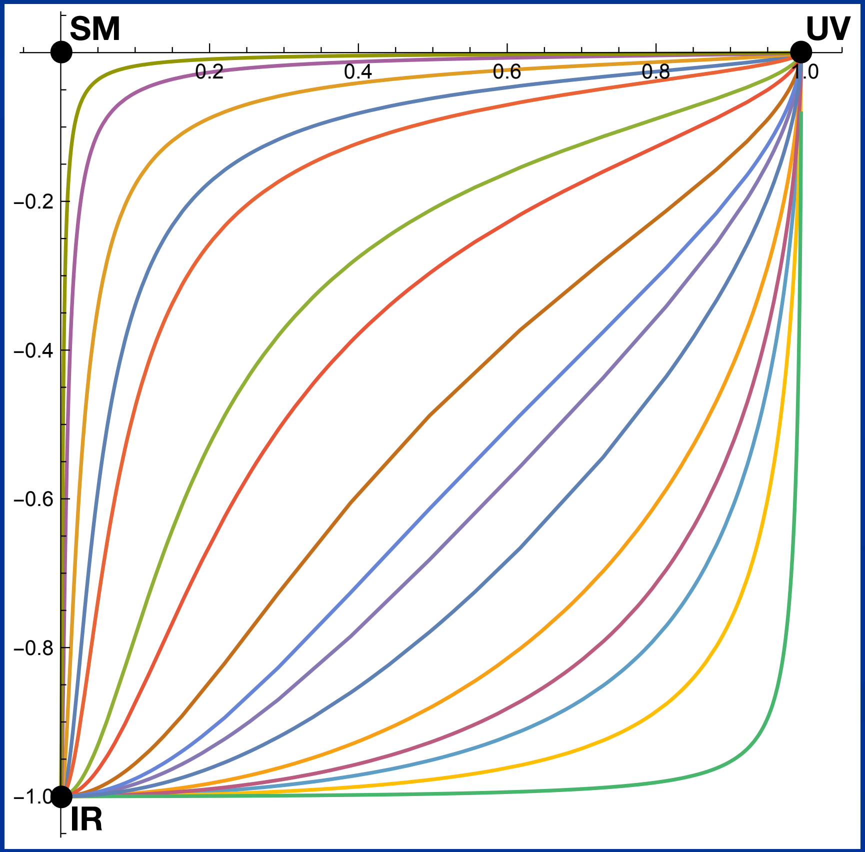
Cosmic scale symmetry :

involves metric and cosmon (pseudo Goldstone boson of spontaneously broken scale symmetry) **IR - fixed point**

Scale symmetry and fixed points

Relative strength of gravity

Particle scale symmetry

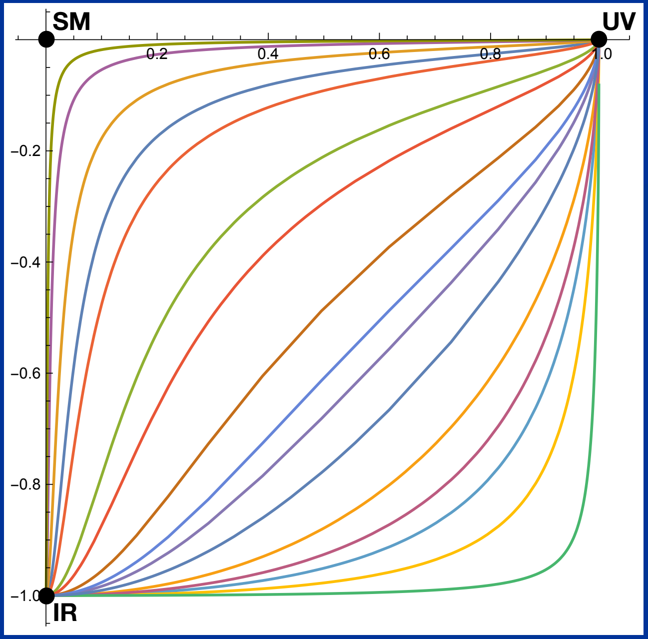


Gravity scale symmetry

Distance from electroweak phase transition

Cosmic scale symmetry

Gravity scale symmetry



Gravity scale symmetry

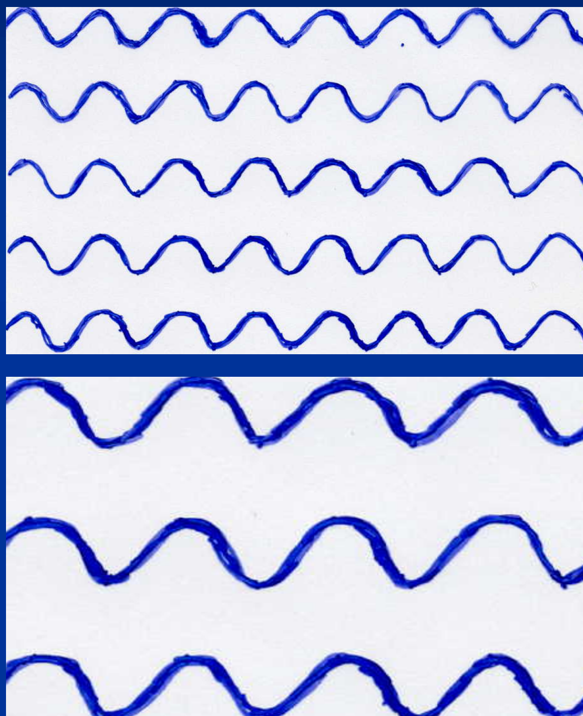
Replace Planck mass by scalar field

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \ldots \right\}$$

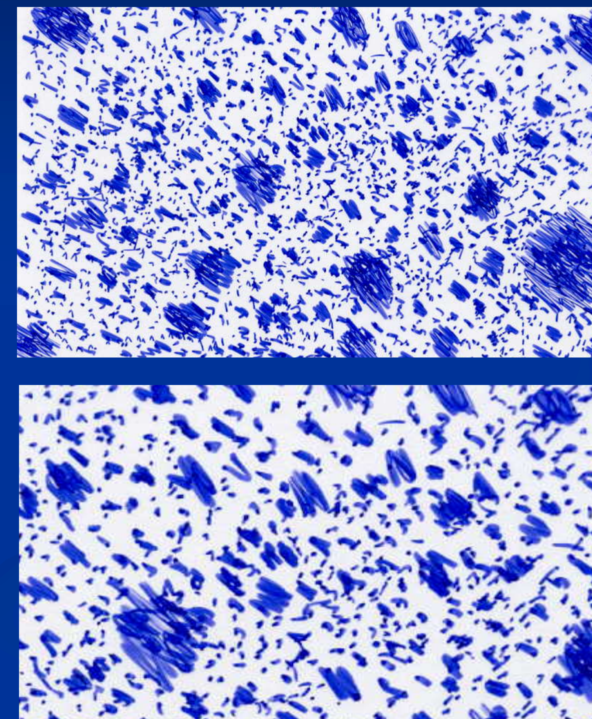
Spontaneous breaking of scale symmetry

- expectation value of scalar field breaks scale symmetry spontaneously
- massive particles are compatible with scale symmetry
- in presence of massive particles : sign of exact scale symmetry is exactly massless Goldstone boson – the dilaton

Scale symmetry



no scale symmetry



scale symmetry

only if no spontaneous symmetry breaking!

Scale symmetric standard model

- Replace all mass scales by scalar field χ

(1) Higgs potential $U = \frac{\lambda_H}{2}(\varphi^\dagger\varphi - \epsilon\chi^2)^2$ $\longrightarrow$ $\varphi_0^2 = \epsilon\chi^2$

Fujii
Englert
Zee

(2) Strong gauge coupling, normalized at $\mu = \chi$, is independent of χ

$g(\chi) = \bar{g}$ $\longrightarrow$ $\Lambda_{\mathrm{QCD}} = \chi \exp\left(-\frac{1}{b_0 \bar{g}^2}\right)$ $b_0 = \frac{1}{16\pi^2}\left(22 - \frac{4}{3}N_f\right)$

CW

(3) Similar for all dimensionless couplings

Quantum effective action for standard model does not involve intrinsic mass or length

Quantum scale symmetric standard model CW'87

For $\chi_0 \neq 0$: massless Goldstone boson

Gravity scale symmetry does not protect small Fermi scale

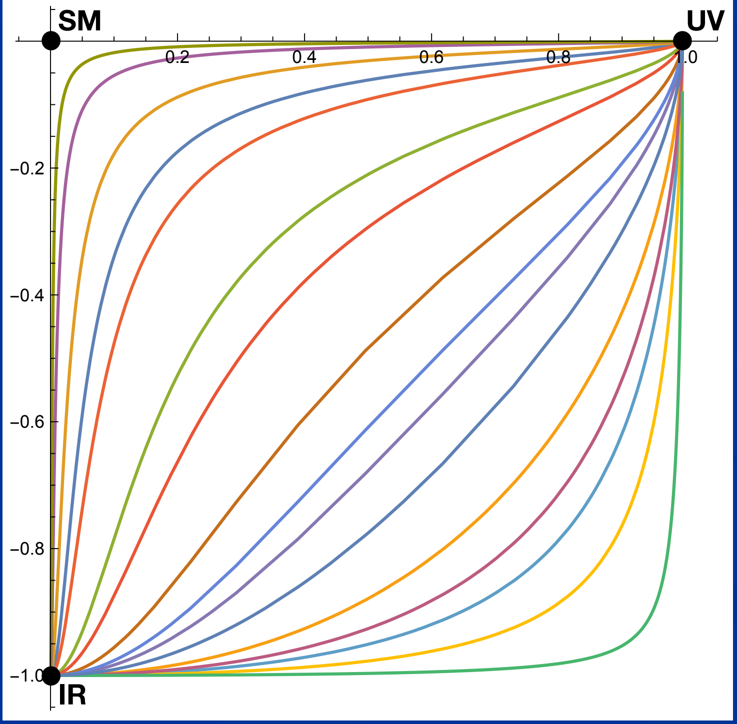
Effective potential

$$U = \frac{\lambda_H}{2}(\varphi^\dagger\varphi - \epsilon\chi^2)^2$$

is scale invariant for arbitrary ε

$$\varphi_0^2 = \epsilon\chi^2$$

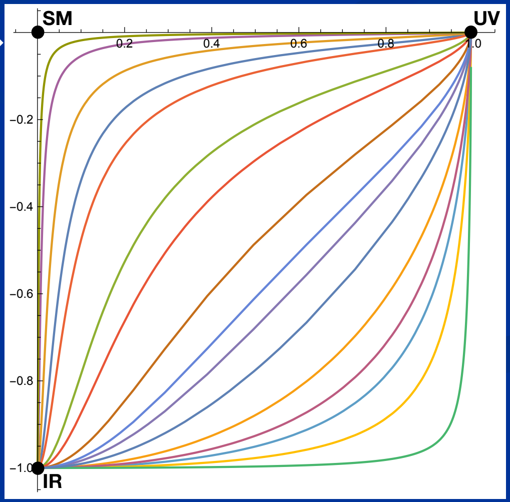
Particle scale symmetry



Particle scale symmetry

is the scale symmetry for the

effective low energy theory below the Planck mass



Second order

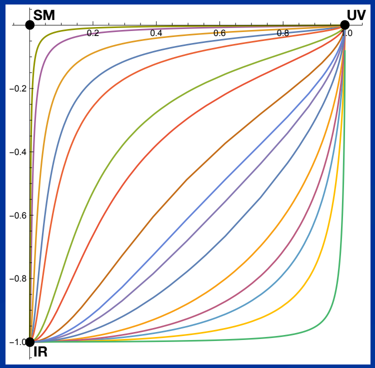
vacuum electroweak phase transition

→

fixed point

→

quantum scale symmetry



Scale symmetry and Fermi scale

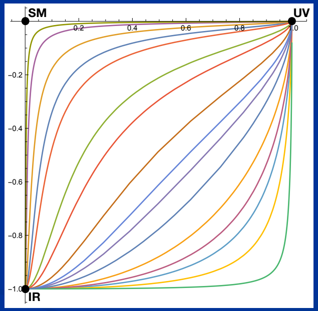
- Vacuum electroweak phase transition is (almost) second order, **including all effects from quantum fluctuations**
- Critical surface of second order phase transition: exact fixed point, **quantum scale symmetry**
- Scale symmetry guarantees **“naturalness”** of gauge hierarchy

C. Wetterich, Phys. Lett.B140(1984)215,
W. A. Bardeen, FERMILAB-CONF-95-391-T(1995)

Scale symmetry and Fermi scale

- Vacuum electroweak phase transition is (almost) second order
- Critical surface of second order phase transition: exact fixed point,
  quantum scale symmetry
- Scale symmetry guarantees **"naturalness"** of small φ_0/M (gauge hierarchy)

C. Wetterich, Phys. Lett.B140(1984)215,
W. A. Bardeen, FERMILAB-CONF-95-391-T(1995)



- No flow trajectory crosses critical trajectory

- No fine tuning for renormalisation group improved perturbation theory for deviation from critical surface

$$\mu\frac{\partial}{\partial\mu}\delta = A\delta \qquad A = \frac{1}{16\pi^2}\left(2\lambda_H + 6h_t^2 - \frac{9}{2}g_2^2 - \frac{3}{2}g_1^2\right)$$

Fine tuning ?

Fine tuning of parameters , quadratic divergences concern bare perturbation theory for location of critical surface in coupling constant space.

not relevant for observation,

not particularly interesting,

regularization dependent, not universal,

always depends on unknown microscopic details

bare perturbation theory is bad expansion

Quantum Gravity

Quantum Gravity is a renormalisable quantum field theory

Asymptotic safety

Asymptotic safety of quantum gravity

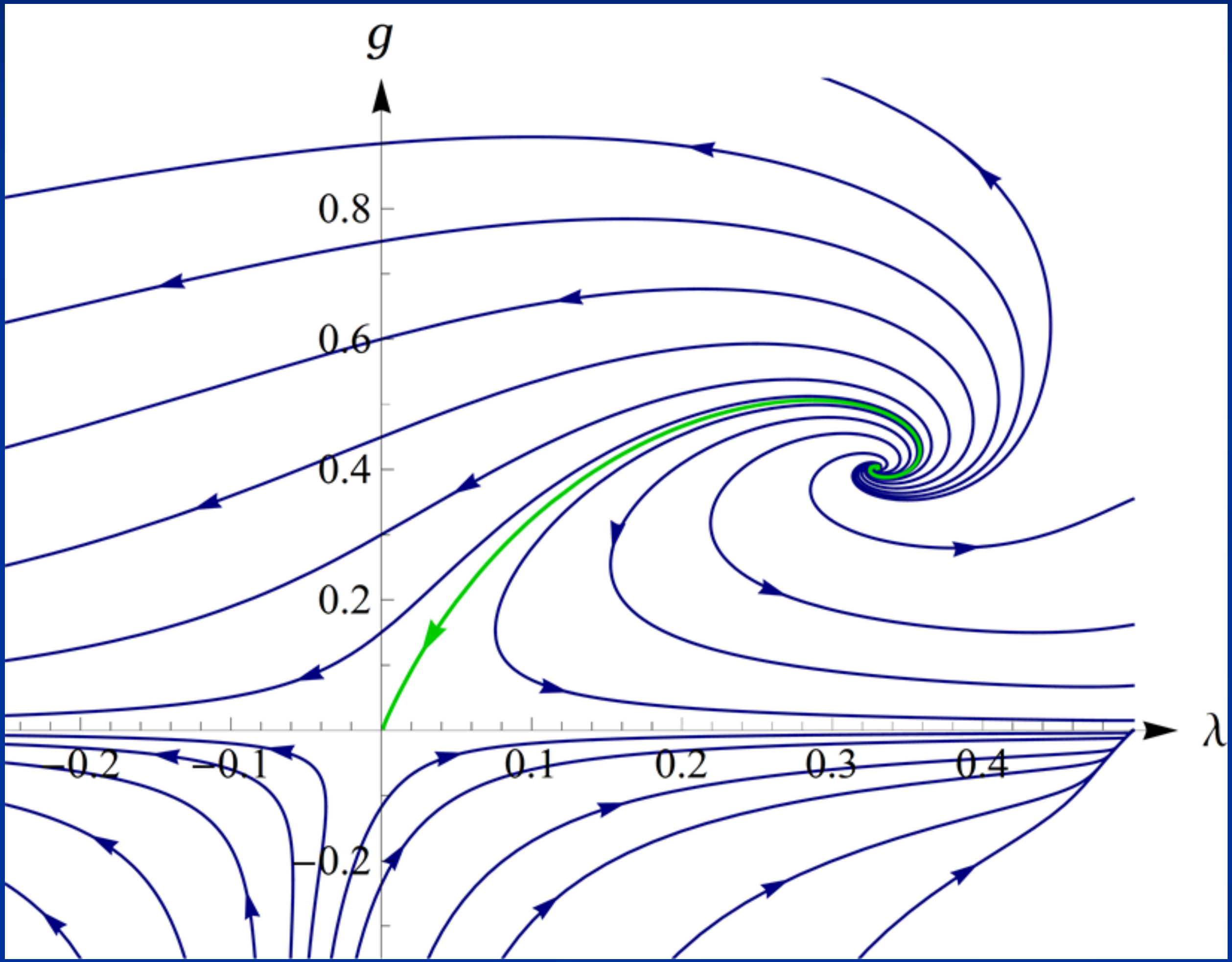
if UV fixed point exists :

quantum gravity is

non-perturbatively renormalizable !

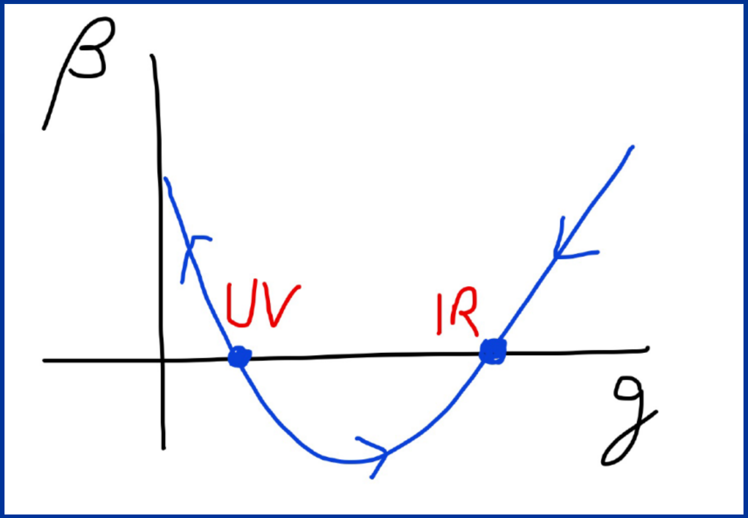
S. Weinberg , M. Reuter

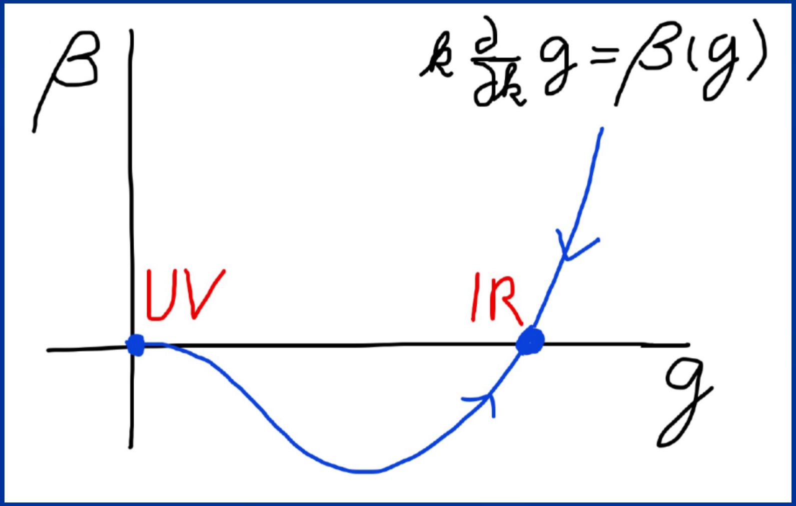
UV- fixed point for quantum gravity



Wikipedia

Asymptotic safety Asymptotic freedom





Relevant parameters yield undetermined couplings.
Quartic scalar coupling is not relevant and can therefore be predicted.

a prediction…

Asymptotic safety of gravity and the Higgs boson mass

Mikhail Shaposhnikov
Institut de Théorie des Phénomènes Physiques, École Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland

Christof Wetterich
Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, D-69120 Heidelberg, Germany

12 January 2010

Abstract

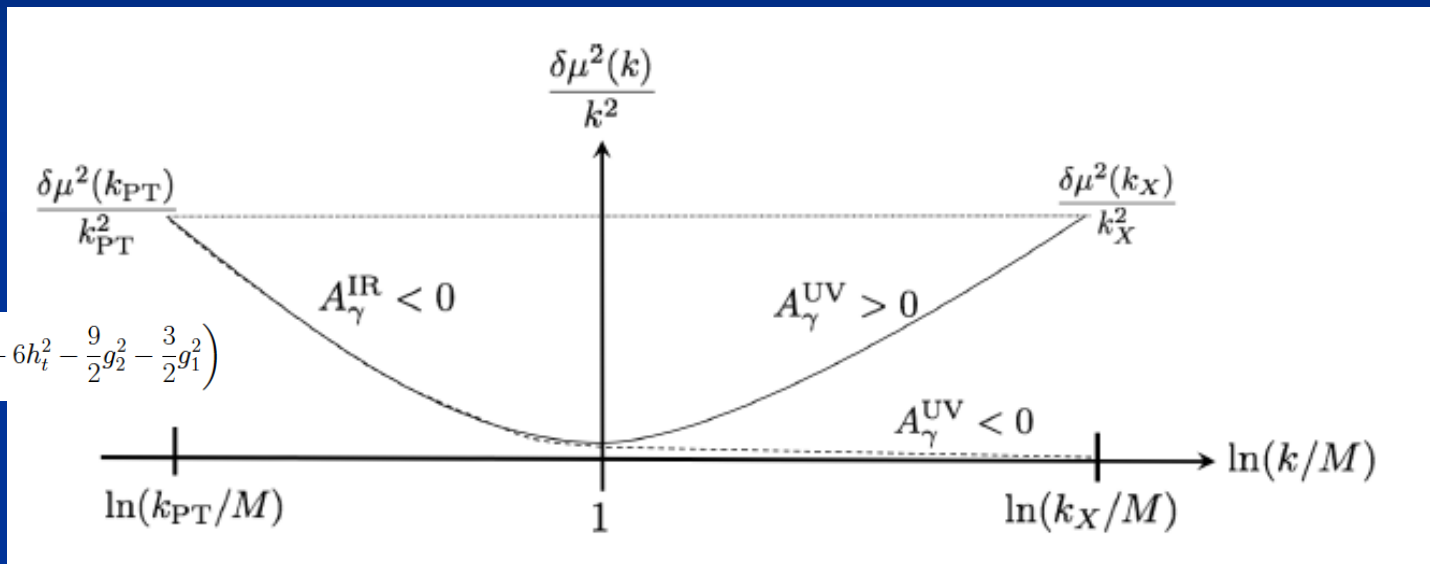
There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalous dimension $A_\lambda > 0$ the running of the quartic scalar self interaction λ at scales beyond the Planck mass is determined by a fixed point at zero. This results in $m_H = m_{\min} = 126$ GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well.

s in $m_H = m_{\min} = 126$ GeV, with o

Possible explanation of gauge hierarchy

$$A_\gamma^{\mathrm{IR}} = -2 + A,$$

$$A = \frac{1}{16\pi^2}\left(2\lambda_H + 6h_t^2 - \frac{9}{2}g_2^2 - \frac{3}{2}g_1^2\right)$$



Gauge hierarchy problem in asymptotically safe gravity –the resurgence mechanism

Christof Wetterich[1] and Masatoshi Yamada[1]

Phys.Lett. B770 (2017) 268-271

For A>2 : self organized criticality

Quantum scale symmetry in cosmology

Quantum gravity with scalar field – the role of scale symmetry for cosmology

Exact scale symmetry ?

Precisely on fixed point :

- Exact scale symmetry

Vicinity of fixed point :

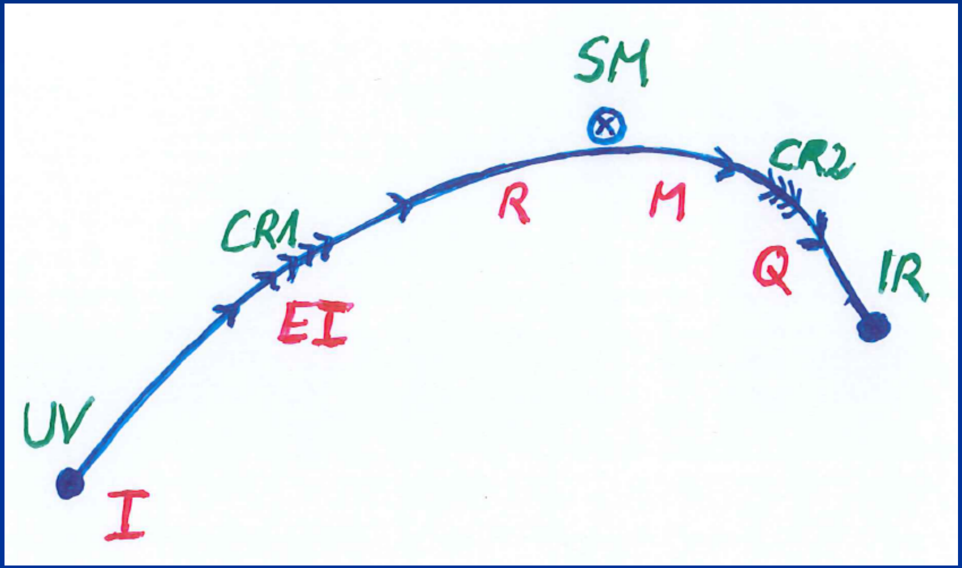
- Relevant parameters induce intrinsic scales by flow away from fixed point.
- Approximate scale symmetry in vicinity of fixed point.
- Fixed point with exact scale symmetry only reached in extreme UV or IR limit.

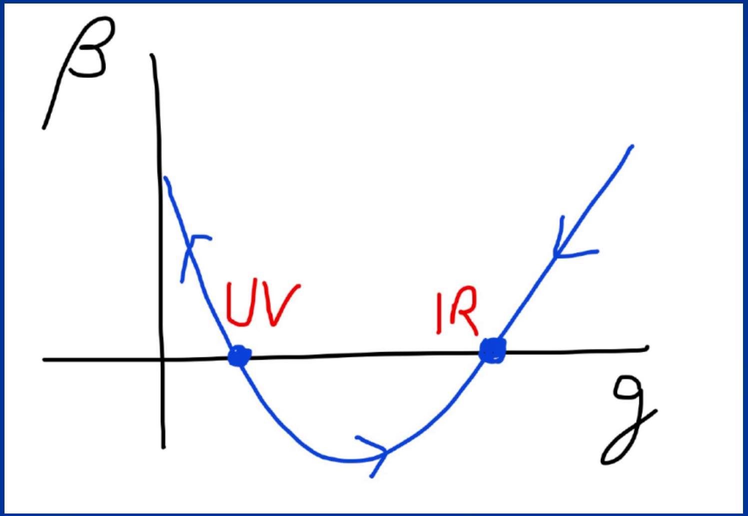
Approximate scale symmetry near fixed points

- UV : approximate scale invariance of primordial fluctuation spectrum from inflation

- IR : cosmon is pseudo Goldstone boson of
  spontaneously broken scale symmetry,
  tiny mass,
  responsible for dynamical Dark Energy

Possible consequences of crossover in quantum gravity





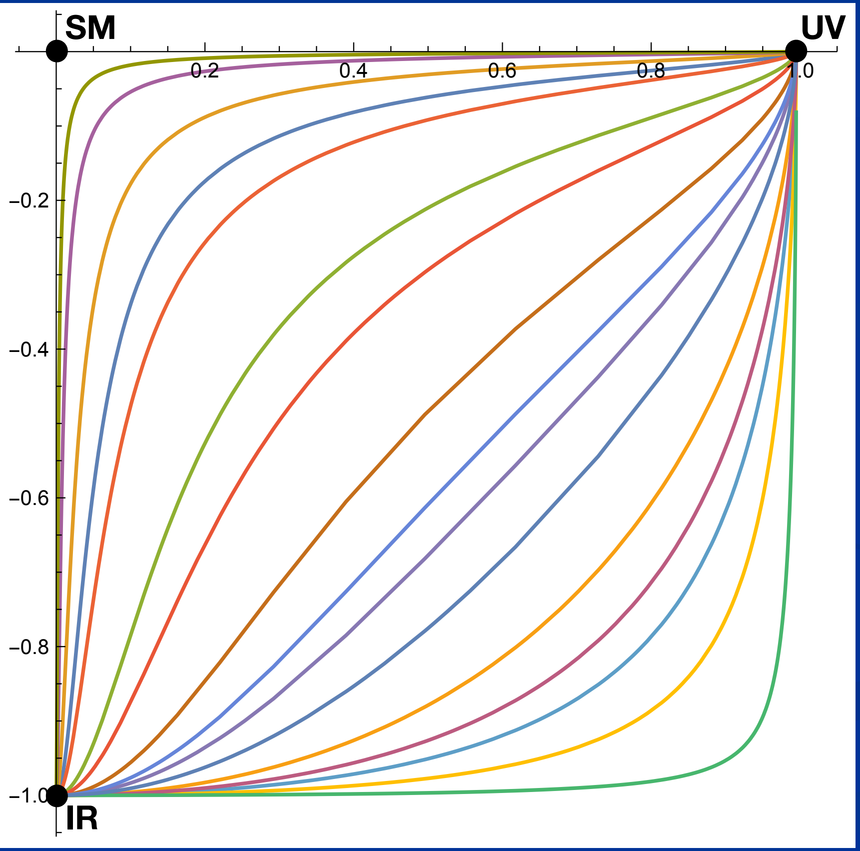
Realistic model for inflation and dark energy with single scalar field

Scale symmetry and fixed points

Inflation

Relative strength of gravity

Particle scale symmetry



Gravity scale symmetry

Distance from electroweak phase transition

Cosmic scale symmetry

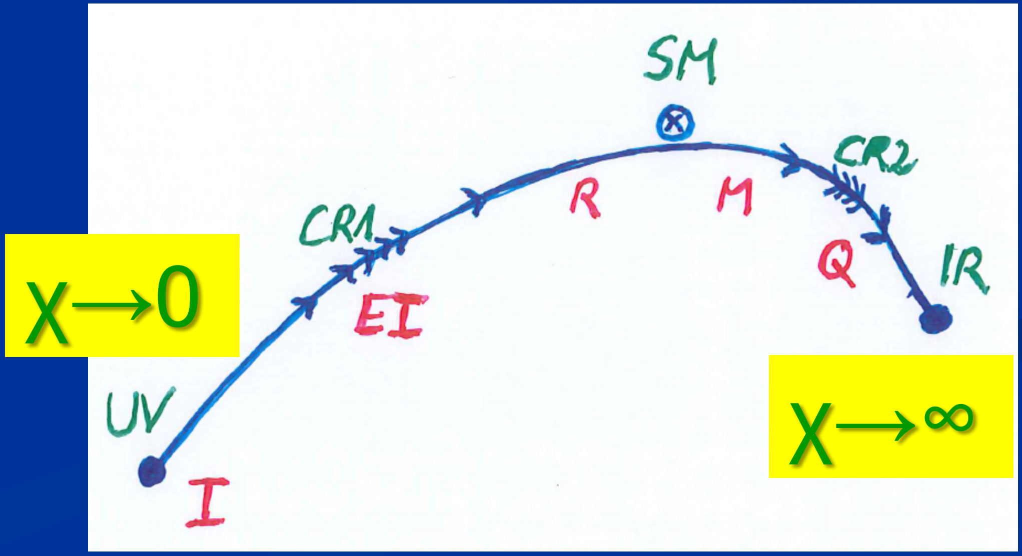
Dynamical dark energy

Cosmological solution : crossover from UV to IR fixed point

- Dimensionless functions as B depend only on ratio μ/χ .
- IR: $\mu \to 0$, $\chi \to \infty$
- UV: $\mu \to \infty$, $\chi \to 0$

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi \right\}$$

Cosmology makes crossover between fixed points by variation of χ .



Renormalization flow and cosmological evolution

- renormalization flow as function of μ

  is mapped by dimensionless functions to

- field dependence of effective action on scalar field χ

  translates by solution of field equation to

- dependence of cosmology an time t or η

variable gravity

“Newton’s constant is not constant – and particle masses are not constant”

Πάντα ῥεῖ

Variable Gravity

$$\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi\right\}$$

quantum effective action,
variation yields field equations

Einstein gravity : $\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2} M^2 R\right\}$

What is Dark Energy ?

Dark energy is energy density of scalar field χ

$$\rho = V + \text{kinetic term}$$

$$V = \mu^2 \chi^2$$

$$p = -V + \text{kinetic term}$$

Dark energy is dynamical if χ changes with time

asymptotically vanishing cosmological „constant“

- What matters : Ratio of potential divided by fourth power of Planck mass

$$\frac{V}{\chi^4} = \frac{\mu^2\chi^2}{\chi^4} = \frac{\mu^2}{\chi^2}$$

$V = \mu^2 \chi^2$

- vanishes for $\chi \to \infty$!

small dimensionless number ?

- needs two intrinsic mass scales
- standard approach :V and M (cosmological constant and Planck mass)
- variable gravity : Planck mass moving to infinity , with fixed or moderately increasing V

⇨ ratio vanishes asymptotically !

Quintessence

Dynamical dark energy ,

generated by scalar field (cosmon)

C.Wetterich, Nucl.Phys.B302(1988)668, 24.9.87

P.J.E.Peebles, B.Ratra, ApJ.Lett.325(1988)L17, 20.10.87

Prediction :

homogeneous dark energy influences recent cosmology

- of same order as dark matter -

Original models do not fit the present observations

…. modifications

(different growth of neutrino mass)

Variable Gravity

$$\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi\right\}$$

quantum effective action,
variation yields field equations

Einstein gravity : $\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2} M^2 R\right\}$

+ scale symmetric standard model

- Replace all mass scales by scalar field χ

(1) Higgs potential

$$U = \frac{\lambda_H}{2}(\varphi^\dagger\varphi - \epsilon\chi^2)^2 \Rightarrow \varphi_0^2 = \epsilon\chi^2$$

(2) Strong gauge coupling, normalized at $\mu = \chi$, is independent of χ

$$g(\chi) = \bar{g} \Rightarrow \Lambda_{QCD} = \chi \exp\left(-\frac{1}{b_0 \bar{g}^2}\right) \quad b_0 = \frac{1}{16\pi^2}\left(22 - \frac{4}{3}N_f\right)$$

+ scale invariant action for dark matter

Scale symmetry in variable gravity
(IR – fixed point)

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \cancel{\mu^2 \chi^2} + \frac{1}{2}\left(B(\cancel{\chi/\mu}) - 6\right)\partial^\mu \chi \partial_\mu \chi \right\}$$

quantum effective action,
variation yields field equations

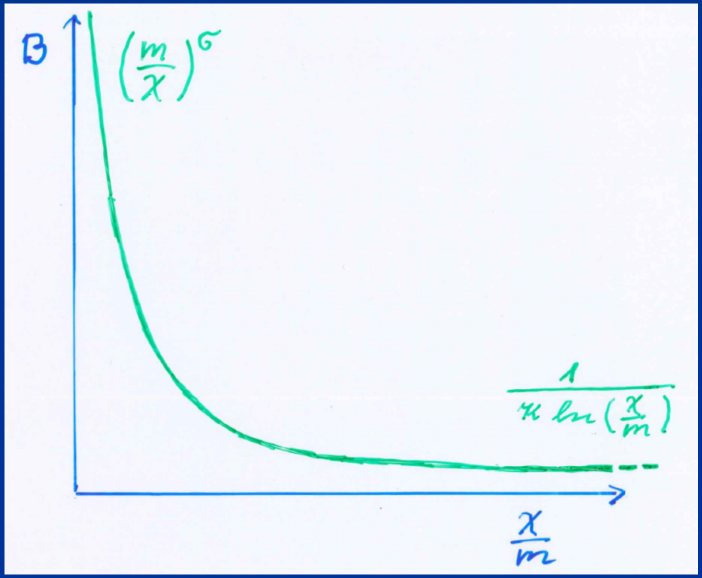
Einstein gravity : $\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} M^2 R \right\}$

Variable Gravity

- Scalar field coupled to gravity
- Effective Planck mass depends on scalar field
- Simple quadratic scalar potential involves intrinsic mass μ
- Nucleon and electron mass proportional to dynamical Planck mass

$$\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi\right\}$$

Kinetial B :
Crossover between two fixed points



assumption:
running coupling obeys flow equation

$$\mu\frac{\partial B}{\partial \mu} = \frac{\kappa\sigma B^2}{\sigma + \kappa B}$$

$$B^{-1} - \frac{\kappa}{\sigma}\ln B = \kappa\left[\ln\left(\frac{\chi}{\mu}\right) - c_t\right] = \kappa\ln\left(\frac{\chi}{m}\right)$$

m : scale of crossover
can be exponentially larger than intrinsic scale μ

Four-parameter model

- model has four dimensionless parameters
- three in kinetial :

  $\sigma \sim 2.5$

  $\varkappa \sim 0.5$

  $c_t \sim 14$ (or m/μ)
- one parameter for growth rate of neutrino mass over electron mass : $\gamma \sim 8$
- + standard model particles and dark matter : sufficient for realistic cosmology from inflation to dark energy
- no more free parameters than ΛCDM

No small parameter for dark energy

Cosmology

Add matter and radiation (standard model + dark matter)

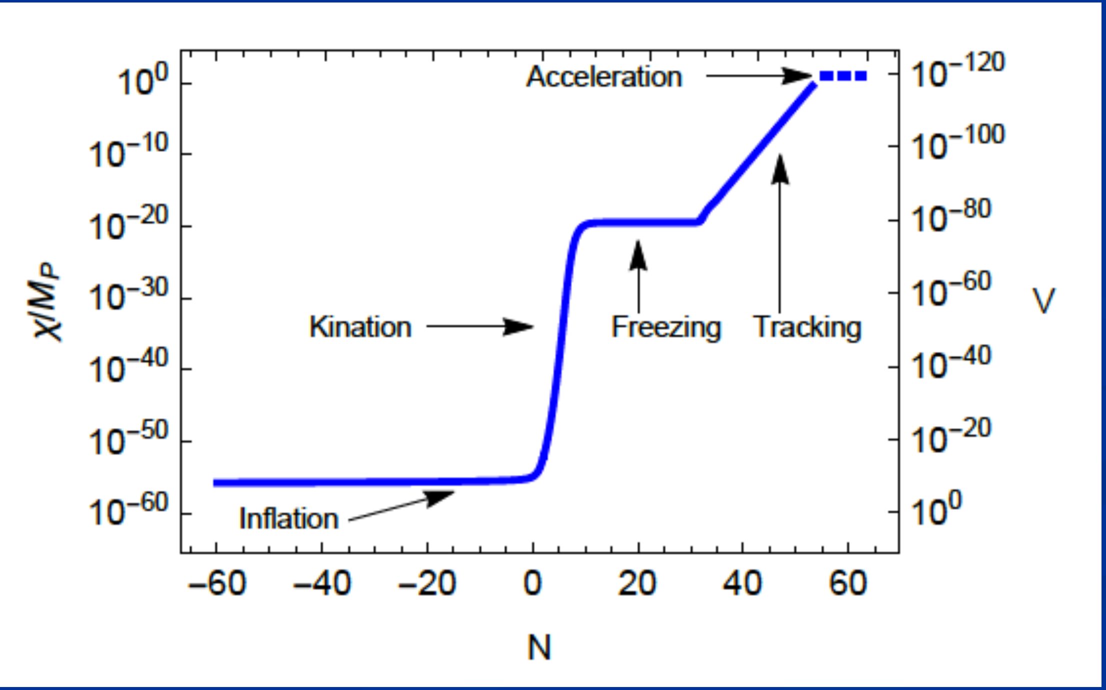
Solve field equations…

$$\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi\right\}$$

$$B^{-1} - \frac{\kappa}{\sigma}\ln B = \kappa\left[\ln\left(\frac{\chi}{\mu}\right) - c_t\right] = \kappa\ln\left(\frac{\chi}{m}\right)$$

Cosmological solution

- scalar field χ vanishes in the infinite past
- scalar field χ diverges in the infinite future



J.Rubio,…

Model is compatible with present observations

Together with variation of neutrino mass over electron mass in present cosmological epoch :

model is compatible with all present observations, including inflation and dark energy

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi \right\}$$

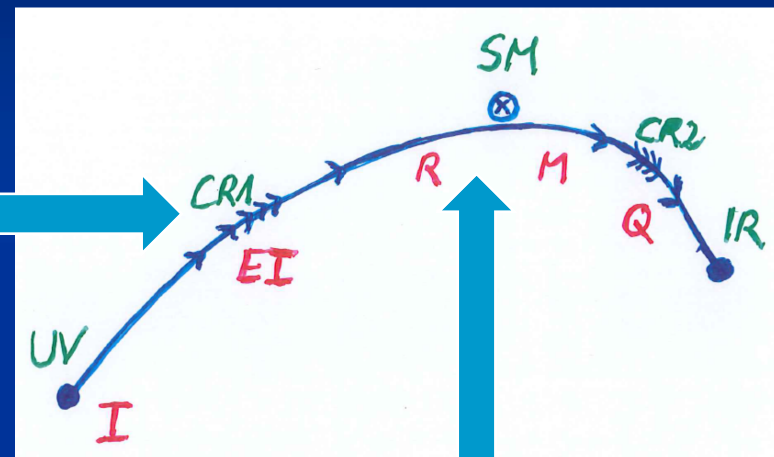
$$B^{-1} - \frac{\kappa}{\sigma}\ln B = \kappa\left[\ln\left(\frac{\chi}{\mu}\right) - c_t\right] = \kappa\ln\left(\frac{\chi}{m}\right)$$

No tiny dimensionless parameters
(except gauge hierarchy)

- one mass scale $\mu = 2 \cdot 10^{-33}$ eV
- one time scale $\mu^{-1} = 10^{10}$ yr
- Planck mass does not appear as parameter
- Planck mass grows large dynamically
- Dark energy is tiny because Universe is old

Scaling solution

after end of inflation



scaling solution with
few percent
of Early Dark Energy

Dark Energy decreases similar to radiation and matter

Conclusions

- Quantum scale symmetry is realized at fixed points of running couplings or flowing effective action

- Crossover between different fixed points:

  Quantum scale symmetry, particle scale symmetry,

  cosmic scale symmetry

- Quantum scale symmetry is predictive:

  Mass of the Higgs boson (and more …?)

  Properties of inflation

  Properties of dark energy

end