Particle Physics
and Cosmology

Dark Matter
What is our universe made of?

quintessence! fire, air, water, soil!
Dark Energy dominates the Universe

Energy - density in the Universe

= 

Matter + Dark Energy

25 % + 75 %
Composition of the universe

\[ \Omega_b = 0.045 \]

\[ \Omega_{dm} = 0.225 \]

\[ \Omega_h = 0.73 \]
critical density

- $\rho_c = 3 H^2 M^2$
  
  critical energy density of the universe
  
  (M: reduced Planck mass, H: Hubble parameter)

- $\Omega_b = \frac{\rho_b}{\rho_c}$
  
  fraction in baryons
  
  energy density in baryons over critical energy density

$H = \frac{\dot{a}}{a}$
Baryons/Atoms

- Dust
- $\Omega_b = 0.045$
- Only 5 percent of our Universe consist of known matter!

SDSS

~60,000 of >300,000 Galaxies
Abell 2255 Cluster
~300 Mpc
\( \Omega_b = 0.045 \)

from nucleosynthesis, cosmic background radiation
Matter : Everything that clumps
Dark Matter

- $\Omega_m = 0.25$ total “matter”
- Most matter is dark!
- So far tested only through gravity
- Every local mass concentration $\rightarrow$ gravitational potential
- Orbits and velocities of stars and galaxies $\rightarrow$ measurement of gravitational potential and therefore of local matter distribution
$\Omega_m = 0.25$
Matter: Everything that clumps

$$\Omega_m = 0.25$$

Abell 2255 Cluster
~300 Mpc
spatially flat universe

\[ \Omega_{\text{tot}} = 1 \]

- theory (inflationary universe)
  \[ \Omega_{\text{tot}} = 1.0000 \ldots x \]

- observation (WMAP)
  \[ \Omega_{\text{tot}} = 1.02 (0.02) \]
$\Omega_{\text{tot}} = 1$

Mean values:

$\Omega_{\text{tot}} = 1.02$

$\Omega_m = 0.27$

$\Omega_b = 0.045$

$\Omega_{\text{dm}} = 0.225$
Dark Energy

\[ \Omega_m + X = 1 \]
\[ \Omega_m : 25\% \]
\[ \Omega_h : 75\% \quad \text{Dark Energy} \]

h: homogenous, often \( \Omega_\Lambda \) instead of \( \Omega_h \)
dark matter candidates

- WIMPS
  weakly interacting massive particles

- Axions

- many others …
WIMPS

- stable particles
- typical mass: 100 GeV
- typical cross section: weak interactions
- charge: neutral
- no electromagnetic interactions, no strong interactions
- seen by gravitational potential
How many dark matter particles are around?
cosmic abundance of relic particles

- early cosmology: present in high temperature equilibrium
- annihilation as temperature drops below mass
- annihilation not completed since cosmic dilution prevents interactions
- decoupling, similar to neutrinos
- relic particles remain and contribute to cosmic energy density
cosmic abundance of relic particles

rough estimate for present number density $n$:
- $n a^3$ constant after decoupling
- $n/s$ approximately constant after decoupling
- compute $na^3$ at time of decoupling
- roughly given by Boltzmann factor for temperature at time of decoupling

and $z_{\text{dec}}$
cosmic abundance of relic particles

- number density for WIMPS much less than for neutrinos
  (Boltzmann factor at time of decoupling)
- \( \Omega = m n \)
- large mass: \( \Omega_m > \Omega_\nu \), possible
computation of time evolution of particle number

central aspects:
- decoupling of one species from thermal bath
- other particles in equilibrium
- Boltzmann equation
- occupation numbers

book: Kolb and Turner
occupation numbers

\[ f(\vec{p}) = \left[ \exp \left( \frac{(E - \mu)}{T} \right) \pm 1 \right]^{-1} \]

in equilibrium

\[ n = \frac{g}{(2\pi)^3} \int f(\vec{p}) d^3 p \]

\[ Q = \frac{g}{(2\pi)^3} \int E(\vec{p}) f(\vec{p}) d^3 p \]

\[ p = \frac{g}{(2\pi)^3} \int \frac{|\vec{p}|^2}{3E} f(\vec{p}) d^3 p \]
Boltzmann equation

\[ \dot{n}_\psi + 3Hn_\psi = -\int d\Pi_\psi d\Pi_a d\Pi_b \ldots d\Pi_i d\Pi_j \ldots \]
\[ \times (2\pi)^4 |M|^2 \delta^4 (p_i + p_j \ldots - p_\psi - p_a - p_b \ldots) \]
\[ \times [f_af_b \ldots f_\psi - f_if_j \ldots] \]

squared scattering amplitude

\[ |M|_{i+j\ldots\rightarrow\psi+a+b+\ldots}^2 = |M|_{\psi+a+b+\ldots\rightarrow i+j+\ldots}^2 \equiv |M|^2 \]

phase space integrals

\[ d\Pi \equiv \frac{g \frac{d^3 p}{2E}}{(2\pi)^3} \]
Dimensionless inverse temperature

\[ x \equiv \frac{m}{T} \]

\( m \) : particle mass

Small \( x \) : particle is relativistic \( x < 3 \)

Large \( x \) : particle is non-relativistic \( x > 3 \)

Dimensionless time variable in units of particle mass \( m \)

\[ t = 0.301 g_*^{-1/2} \quad \frac{M_p}{T^2} = 0.301 g_*^{-1/2} \quad \frac{M_p}{m^2} x^2 \]
particle number per entropy

\[ Y \equiv \frac{n_\psi}{s} \]

\[ \dot{n}_\psi + 3H n_\psi = s\dot{Y} \]
Boltzmann equation for $Y$

$$\frac{dY}{dx} = -\frac{x}{H(m)s} \int d\Pi_\psi d\Pi_ad\Pi_b \ldots d\Pi_id\Pi_j \ldots |\mathcal{M}|^2$$
$$\times (2\pi^4)\delta^4(p_i + p_j \ldots - p_\psi - p_a - p_b \ldots)$$
$$\times [f_af_b \ldots f_\psi - f_if_j \ldots]$$

$$H(m) = 1.67g_*^{1/2} m^2 / M_p$$

$$t = 0.301g_*^{-1/2} \frac{M_p}{T^2} = 0.301g_*^{-1/2} \frac{M_p}{m^2 x^2}$$

$$\dot{n}_\psi + 3Hn_\psi = -\int d\Pi_\psi d\Pi_ad\Pi_b \ldots d\Pi_id\Pi_j \ldots$$
$$\times (2\pi^4)\delta^4(p_i + p_j \ldots - p_\psi - p_a - p_b \ldots)$$
$$\times [f_af_b \ldots f_\psi - f_if_j \ldots]$$
2 – 2 - scattering

as dominant annihilation and creation process

particle X in thermal equilibrium (classical approximation)

energy conservation

detailed balance (also for large T)
Annihilation cross section

\[ \langle \sigma_{\psi\bar{\psi}\to X\bar{X}} | v | \rangle \equiv \left( n_{\psi}^{EQ} \right)^{-2} \int d\Pi_{\psi} d\Pi_{\bar{\psi}} d\Pi_X d\Pi_{\bar{X}} \times (2\pi)^4 \delta^4(p_{\psi} + p_{\bar{\psi}} - p_X - p_{\bar{X}}) |\mathcal{M}|^2 \times \exp(-E_{\psi}/T) \exp(-E_{\bar{\psi}}/T) \]
Boltzmann equation in terms of cross section

\[
\frac{dn_\psi}{dt} + 3H n_\psi = - \langle \sigma_{\psi\bar{\psi} \to X\bar{X}} |v| \rangle \left[ n_\psi^2 - \left( n_{EQ} \right)^2 \right]
\]

\[
\frac{dY}{dx} = \frac{-x \langle \sigma_{\psi\bar{\psi} \to X\bar{X}} |v| \rangle s}{H(m)} \left( Y^2 - Y_{EQ}^2 \right)
\]

\[
x \equiv \frac{m}{T}
\]

\[
\frac{dY}{dx} = \frac{x}{H(m) s} \int d\Pi_\psi d\Pi_\alpha d\Pi_b \ldots d\Pi_i d\Pi_j \ldots |\mathcal{M}|^2 \times (2\pi^4) \delta^4(p_i + p_j \ldots - p_\psi - p_\alpha - p_b \ldots) \times [f_\alpha f_b \ldots f_\psi - f_i f_j \ldots]
\]
particle number per entropy in equilibrium

\[
\frac{dY}{dx} = -\frac{x\langle\sigma_A|v|\rangle s}{H(m)} \left( Y^2 - Y_{EQ}^2 \right)
\]

\[
Y_{EQ}(x) = \frac{45}{2\pi^4} \left( \frac{\pi}{8} \right)^{1/2} \frac{g}{g^*_s} x^{3/2} e^{-x}
\]

\[
= 0.145 \frac{g}{g^*_s} x^{3/2} e^{-x} \quad (x \gg 3)
\]

\[
Y_{EQ}(x) = \frac{45\zeta(3) g_{eff}}{2\pi^4} \frac{g_{eff}}{g^*_s} = 0.278 \frac{g_{eff}}{g^*_s} \quad (x \ll 3)
\]

non–relativistic

relativistic
annihilation rate

\[ \Gamma_A \equiv n_{EQ} \langle \sigma_A \nu \rangle \]

\[
\frac{x}{Y_{EQ}} \frac{dY}{dx} = -\frac{\Gamma_A}{H} \left[ \left( \frac{Y}{Y_{EQ}} \right)^2 - 1 \right]
\]

freeze out when annihilation rate drops below expansion rate
hot and cold relics

- **hot relics** freeze out when they are relativistic
  
  hot dark matter, neutrinos

- **cold relics** freeze out when they are non-relativistic

  cold dark matter
hot relics

$x_f$: freeze out inverse temperature

\[ Y_\infty = Y_{EQ}(x_f) = 0.278 \frac{g_{eff}}{g_*(x_f)} \quad (x_f \lesssim 3) \]

\[ n_\psi_0 = s_0 Y_\infty = 2970 Y_\infty \text{cm}^{-3} \]
\[ = 825 \left[ \frac{g_{eff}}{g_*(x_f)} \right] \text{cm}^{-3} \]

$Y_{EQ}$ is constant, approach to fixed point!
approach to fixed point

\[ \frac{dY}{dx} = -\frac{x\langle \sigma_A | v | \rangle s}{H(m)} \left( Y^2 - Y_{EQ}^2 \right) \]

\( Y > Y_{EQ} \) : \( Y \) decreases towards \( Y_{EQ} \).

\( Y < Y_{EQ} \) : \( Y \) increases towards \( Y_{EQ} \).
hot relics

for hot relics:

Y does not change, independently of details

robust predictions
neutrinos

neutrino background radiation

\[ \Omega_\nu = \frac{\Sigma m_\nu}{(91.5 \text{ eV} \ h^2)} \]

\( \Sigma m_\nu \) present sum of neutrino masses

\( m_\nu \approx \) a few eV or smaller

comparison : electron mass = 511 003 eV
proton mass = 938 279 600 eV
cold relics

\[ \langle \sigma_A|v| \rangle \equiv \sigma_0(T/m)^n = \sigma_0 x^{-n} \quad \text{ (for } x \gtrsim 3) \]

\( n=0 \) for s-wave scattering \quad \text{(} n=1 \) p-wave \)

\[ s \sim T^3 \sim x^{-3} \]

\[ \frac{dY}{dx} = -\lambda x^{-n-2}(Y^2 - Y_{EQ}^2) \]

\[ \lambda = \left[ \frac{x \langle \sigma_A|v| \rangle s}{H(m)} \right]_{x=1} = 0.264(g_s/g_s^{1/2})M_pm\sigma_0 \]

\[ Y_{EQ} = 0.145(g/g_s)x^{3/2}e^{-x} \]

\[ \frac{dY}{dx} = -\frac{x \langle \sigma_A|v| \rangle s}{H(m)} \left( Y^2 - Y_{EQ}^2 \right) \]
approximate solution
of Boltzmann equation

\[ \Delta = Y - Y_{EQ} \]
\[ \Delta' = -Y_{EQ}' - \lambda x^{-n-2} \Delta (2Y_{EQ} + \Delta) \]

early time \( x < 3 \)

\[ \Delta' = 0 \]

\[ \Delta \approx -\lambda^{-1} x^{n+2} Y_{EQ}'/(2Y_{EQ} + \Delta) \]
\[ \approx x^{n+2}/2\lambda \]

\[ Y_{EQ} = 0.145(g/g_{*s})x^{3/2}e^{-x} \]
late time solution

\[ Y_{EQ} \ll \Delta \]

YEQ strongly
Boltzmann suppressed

\[ \Delta' = -\lambda x^{-n-2} \Delta^2 \]
stopping
solution

\[ Y_{\infty} = \Delta_{\infty} = \frac{n + 1}{\lambda} x^n f \]

\[ \Delta' = -Y_{EQ}' - \lambda x^{-n-2} \Delta (2Y_{EQ} + \Delta) \]
freeze out temperature

\[ x_f \text{ set by matching of early and late time solutions} \]

\[ \Delta \approx \frac{x^{n+2}}{2\lambda} \approx Y_{EQ}(x_f) \]
cold relic abundance

\[ Y_\infty = \frac{3.79(n + 1)x_f^{n+1}}{(g^s_s/g^*_s)^{1/2}M_pm\sigma_0} \]

\[ n_\psi_0 = s_0Y_\infty = 2970Y_\infty \text{cm}^{-3} \]
\[ = 1.13 \times 10^4 \frac{(n + 1)x_f^{n+1}}{(g^s_s/g^*_s)^{1/2}M_pm\sigma_0} \text{cm}^{-3} \]
cold relic fraction

\[ Y_\infty = \frac{3.79(n + 1)x_f^{n+1}}{(g_*s/g_*^{1/2})M_p m \sigma_0} \]

\[ \Omega_\psi h^2 = 2.8 \times 10^8 \frac{m}{GeV} Y_\infty \]
cold relic fraction

\[ \Omega_{\psi} h^2 = 1.07 \times 10^9 \frac{(n + 1)x_f^{n+1}\text{GeV}^{-1}}{(g_{*s}/g_{*}^{1/2})M_p\sigma_0} \]

not necessarily order one!

\[ Y_\infty = \frac{3.79(n + 1)x_f^{n+1}}{(g_{*s}/g_{*}^{1/2})M_p\mu m\sigma_0} \]
cold relic abundance, mass and cross section

\[ Y_\infty = \frac{3.79(n + 1)(g_*^{1/2}/g_* s) x_f}{M_P \langle \sigma_A | v \rangle} \]

inversely proportional to cross section, plus logarithmic dependence of \( x_f \)
baryon relics in a baryon symmetric Universe

\[ \sigma_A |\nu| = \frac{c}{m^2_{\pi}} \]

\[ x_f \approx 42 + \ln c_1, \]
\[ T_f \approx 22 \text{MeV}, \]
\[ Y_\infty = 7 \times 10^{-20} c_1^{-1} \]

observed: \( Y \approx 10^{-10} \)
anti-baryons in baryon asymmetric Universe

\[ Y_\infty(\bar{N}) = 10^{18} \exp(-9 \times 10^5) \]
relic WIMPS

\[ \sigma_0 \approx c_2 G_F^2 m^2 / 2\pi \]

\[ x_f \approx 15 + 3 \ln(m/\text{GeV}) + 1 \ln(c_2/5) \]

\[ Y_\infty \approx 6 \times 10^{-9} \left( \frac{m}{\text{GeV}} \right)^{-3} \left( \frac{5}{c_2} \right) \left[ 1 + \frac{3 \ln(m/\text{GeV})}{15} + \frac{\ln(c_2/5)}{15} \right] \]

only narrow range in m vs. \( \sigma \) gives acceptable dark matter abundance!
relic WIMP fraction

\[ Y_\infty \approx 6 \times 10^{-9} \left( \frac{m}{\text{GeV}} \right)^{-3} \left( \frac{5}{c_2} \right) \left[ 1 + \frac{3 \ln(m/\text{GeV})}{15} + \frac{\ln(c_2/5)}{15} \right] \]

\[ \Omega_\psi h^2 = 2.8 \times 10^8 \frac{m}{\text{GeV}} Y_\infty \]

strong dependence on mass and cross section
Wimp bounds

- Current CDMS limit
- Expected sensitivity of data in hand from first CDMS run at Soudan (5x better than current limit.)
- Projected sensitivity for CDMS at Soudan, with 5 towers 4 kg Ge, 1.5 kg Si: 0.1 events/kg/keV/year (100x better than current limit)

SUSY $g_{\mu}^{-2}$ (Baltz & Gondolo, PRL 86 (2001) 5004)

No SUSY $g_{\mu}^{-2}$ constraint
relic stable heavy neutrinos

\[ \Omega h^2 = 3(m/\text{GeV})^{-2} \left[ 1 + \frac{3 \ln(m/\text{GeV})}{15} \right] \]

stable neutrinos with mass > 4-5 GeV allowed
next week no lecture