Test of the Einstein relation in relativistic collisions

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Abstract

As an important ingredient of a nonequilibrium-statistical description of relativistic collisions, the validity of a weak-coupling dissipation-fluctuation theorem (Einstein relation) in rapidity space is tested from SIS via AGS to SPS energies. The theorem relates the rapidity relaxation time to the diffusion coefficient via the temperature of the equilibrium distribution. Whereas it is fulfilled at low SIS-energies, progressively larger deviations rising to an order of magnitude are found at AGS- and SPS-energies. The deviations are related to the strong-coupling character of the system which may also induce non-equilibrium quark-gluon plasma formation.

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1 Introduction

Relativistic heavy-ion collisions [1] offer a unique possibility to study equilibration processes in microscopic many-body systems. In particular, rapidity distributions for participant protons are valuable indicators for the time-evolution of the reactions. They can be used to investigate the approach to thermal equilibrium in the interaction region.

Isotropic thermal models fail to reproduce the broad, often flat or even dipped rapidity spectra that have been observed at AGS and SPS energies. Hence, these models have been supplemented [2] by collective expansion and anisotropic flow schemes to fit the data [4]. The broad rapidity distributions for participant protons may, however, also indicate nonequilibrium properties of the system. A nonequilibrium-statistical model based on a linear Fokker-Planck equation (FPE) [5] has therefore been introduced [6] to describe the gradual approach to local thermal equilibrium in the interaction zone.

This relaxation process occurs as a consequence of random collisions and creation of secondaries. The nonequilibrium rapidity spectrum for protons emerges analytically from the model. A similar relativistic diffusion model (RDM) has previously been used to evaluate and predict transverse energy distributions (integrated over all particle species) analytically [7]. As an important ingredient of such a nonequilibrium-statistical description, the validity of a weak-coupling dissipation-fluctuation theorem is tested here from SIS- via AGS- to SPS-energies.

2 Relativistic diffusion model

In the relativistic diffusion model (RDM) for rapidity spectra of participants, the relaxation of the initial projectile rapidity y_1 and of the target rapidity due to collisions, particle creations and decays is described as a diffusion process. The Fokker-Planck equation successfully accounts for similar nonequilibrium processes in many areas of physics [5] starting with the theory of Brownian motion. In relativistic systems the FPE for the probability distribution of the normalized center-of-mass rapidity Y becomes

$$\frac{\partial R(Y,t)}{\partial t} = \frac{1}{\tau_Y}\frac{\partial}{\partial Y}[(Y - Y_{eq})R(Y,t)] + D_Y \frac{\partial^2}{\partial Y^2}R(Y,t). \tag{1}$$

Two transport coefficients determine the time evolution of the rapidity distribution, the relaxation time τ_Y and the rapidity diffusion coefficient D_Y. Rapidity spectra calculated from eq.(1) will ultimately depend on two dimensionless parameters: τ_{int}/τ_Y with the interaction time τ_{int} (the final time in the solution of eq.(1)), and $\sqrt{D_Y \tau_Y}$. The first parameter describes how close to equilibrium the system is, and the second parameter determines the width of the rapidity distribution at equilibrium.

For a linear FPE, the fluctuations can be obtained without detailed consideration of the microscopic structure of the system, because the stationary solution R_{eq} of the FPE should agree with the isotropic thermal equilibrium distribution R_{th}. The latter depends on a temperature T_p that is to be identified with the freeze-out temperature, and on the particle mass m_p [2] .

3 Dissipation-fluctuation theorem

Equating a Gaussian approximation to the thermal distribution with the stationary solution of the FPE, I obtain the rapidity diffusion coefficient [6,17]

$$D_Y^{th} = \frac{1}{2\pi\tau_Y}[c_p^T m_p^2 T_p (1 + 2\frac{T_p}{m_p} + 2(\frac{T_p}{m_p})^2)]^{-2} \exp\left(\frac{2m_p}{T_p}\right). \qquad (2)$$

The constant c_p^T ensures that the distribution function in Y-space is normalized to 1 for each temperature T_p

$$c_p^T = \frac{y_1}{2m_p^2 T_p}[\int\limits_{-\infty}^{+\infty}(1 + 2\chi_T(y) + 2\chi_T(y)^2)\exp\left(\frac{-1}{\chi_T(y)}\right)dy]^{-1} \qquad (3)$$

with

$$\chi_T(y) = \frac{T_p}{m_p\cosh(y)}. \qquad (4)$$

For a given equilibrium distribution with temperature T_p, the rapidity diffusion coefficient is then solely determined by the dissipative constant τ_Y. This dissipation-fluctuation theorem is reminiscent of the well-known Einstein relation between diffusion and friction coefficient in the analysis of Brownian motion [12], $D \sim T/\tau$.

In relativistic collisions, however, the theorem need not be fulfilled because a large amount of energy resides in the interaction, and is used for the generation of secondary particles. Hence, in a comparison with data, values of the rapidity diffusion coefficient exceeding the ones obtained from the weak-coupling result may be expected, and are indeed found at SPS-energies.

4 Comparison with experiment

Comparing solutions of the FPE (fig.1) to the data from SIS via AGS to SPS energies, I have tested the validity of eq.(2). At SIS-energies, the light Ni + Ni - system provides data in the range 1.06 - 1.93 A GeV [15]. With the weak-coupling value of the rapidity diffusion coefficient the data are well reproduced at the lowest energy, as shown in the upper part of fig.2 (solid curve). Here, the dashed curve is the thermal equilibrium distribution with a temperature of 79 MeV (normalized to 38 protons). The thermal distribution is clearly too narrow to fit the data. In the relativistic diffusion model, the data find their natural explanation in terms of nonequilibrium contributions (incomplete stopping) as described via the solution of the FPE.The transport coefficients can be found in table 1. A small deviation from the dissipation-fluctuation theorem is found at the higher energies.

This deviation from the weak-coupling behaviour becomes more pronounced at AGS-energies. Here, the discrepancies between isotropic thermal distributions (dashed curves) [14] and data are very dramatic, fig.3. The solutions of the FPE represent the experimental results quite accurately. In case of the light Si + Al system, the nonequilibrium character is particularly evident through the dip in the rapidity spectrum, which reflects the initial situation at t=0 and appears to persist even for central collisions. The rapidity diffusion coefficients that represent the data deviate from the values obtained via the dissipation-fluctuation theorem by factors of 3.3 and 3.7 for Si + Al and Au + Au, respectively (table 1).

This violation of the dissipation-fluctuation theorem becomes even larger at SPS-energies where Pb + Pb - data [11] are used for the comparison [6], fig.4. Here, the weak-coupling theorem is violated by a factor of 9 (corresponding to a factor of 3 in FWHM).

This conclusion is not affected by the present uncertainties in the interaction times that enter the model calculation, because rescaled values of the interaction times (and correspondingly, of the rapidity relaxation time, keeping the ratio constant) would modify the diffusion coefficients, but maintain the variances. Hence, the violation of the dissipation-fluctuation theorem in Y-space with increasing energy occurs independently of a detailed description of the dynamical behaviour, it is a consequence of the nonequilibrium-statistical properties of the system.

It would be interesting to investigate in more detail the origin of this effect in relativistic collisions. Whereas it could be due to the strong-coupling behaviour that leads to an increase of the diffusion coefficient above the values that I have obtained from eq.(2), a rapid collective expansion of the system (before thermal equilibrium is attained) also leads to broader distributions and hence, could cause an artificial enhancement in the diffusion coefficients that are used to represent the data. The enhancement might eventually also be due to non-equilibrium quark-gluon plasma formation prior to hadronization. Developing unambiguous tests to discriminate these possibilities is very challenging.

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Table 1

Relaxation times τ_Y and rapidity diffusion coefficients D_Y at SIS-, AGS- and SPS-energies. The incident momentum per particle is p_L, the temperature of the thermal equilibrium distribution T_p. The theoretical value of the diffusion coefficient corresponding to the Einstein relation is D_Y^{th}, whereas D_Y is the fit value.

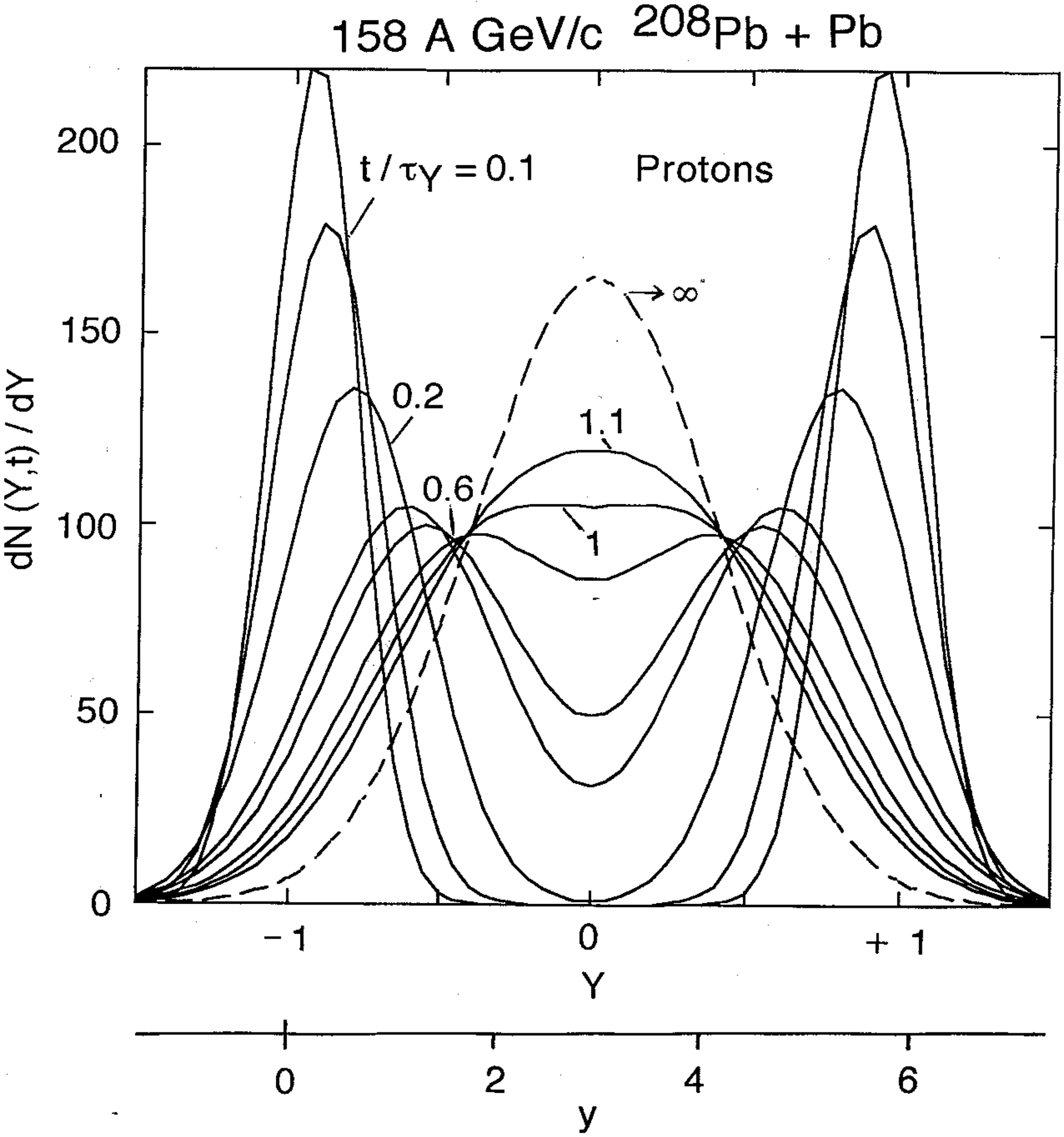
System	p_L (GeV)	y_1	τ_Y (fm)	τ_{int}/τ_Y	T_p (MeV)	D_Y^{th} (10^{-4}fm^{-1})	D_Y (10^{-4}fm^{-1})
Ni + Ni	1.77	1.388	360	1.03	79	4.7	4.7
	2.20	1.586	310	1.02	84	4.4	6.2
	2.71	1.782	290	0.95	92	4.1	6.5
Si + Al	14.6	3.44	60	0.80	140	7.8	26.0
Au + Au	1.77	1.39	1100	1.14	80	1.6	1.6
Au + Au	11.4	3.19	370	1.08	130	1.4	5.1
Pb + Pb	157.7	5.83	134	0.81	160	1.4	12.4
S + S	200	6.06	25	0.60	140	7.4	88.7
S + Au	200	6.06	72	0.84	175	2.1	25.5

Fig.1. Rapidity relaxation of participant protons in 158 A GeV/c Pb + Pb. Selected analytical solutions of the FPE for various values of time t/τ_Y are shown. The stationary distribution (dashed) is centered at midrapidity ($y_{eq} = 2.91, Y_{eq} = 0$).

Fig.2. Rapidity relaxation of participant protons in central relativistic Ni + Ni- collisions at SIS- energies. FOPI data are reflected about midrapidity. The dashed curves are thermal equilibrium distributions, solid curves are nonequilibrium results of the Relativistic Diffusion Model. At the lowest energy, the rapidity diffusion coefficient is obtained from the dissipation-fluctuation theorem, at the higher energies enhanced values are used to fit the data.

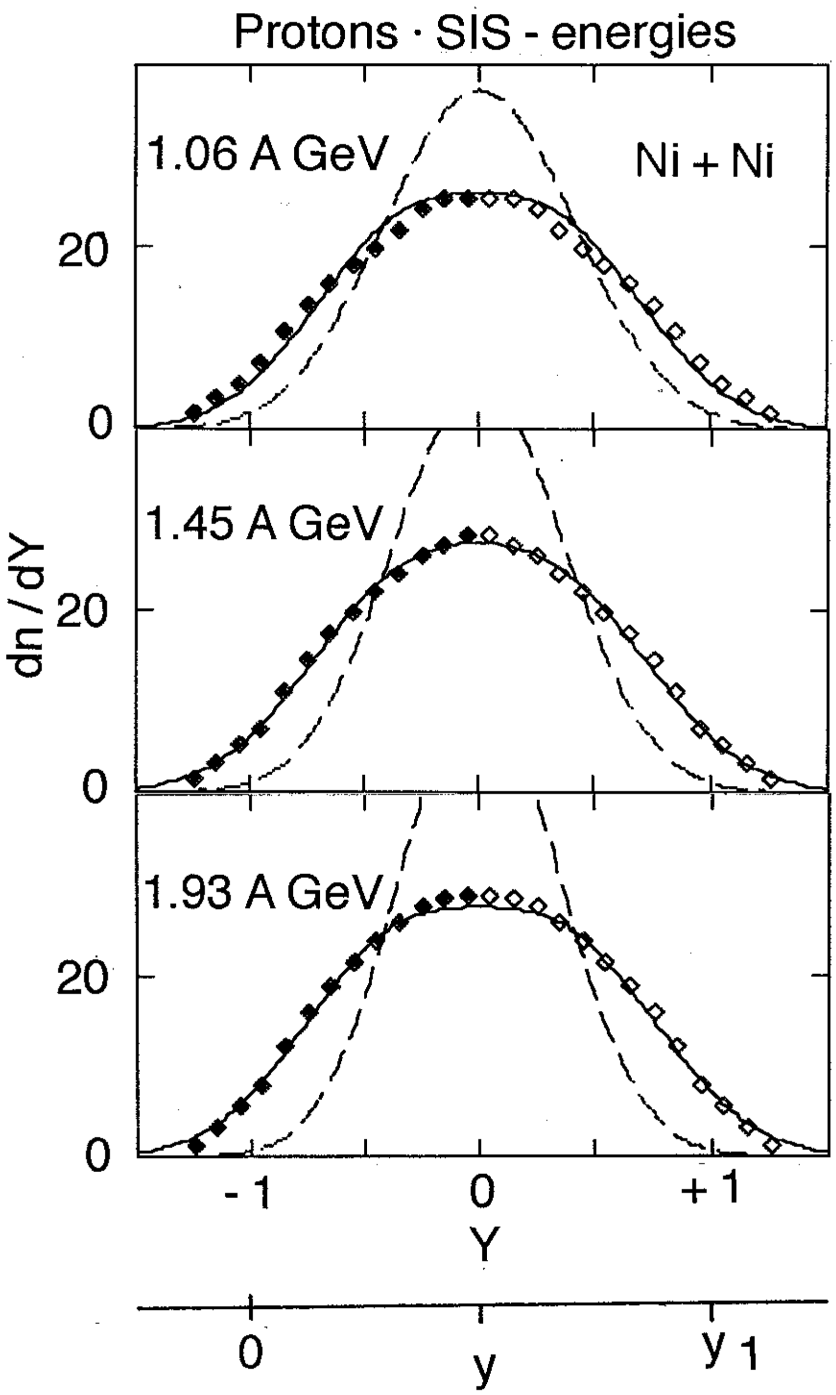
Fig.3. Rapidity relaxation of participant protons at AGS energies (cf. table 1). The Au + Au data and part of the Si + Al data have been reflected about midrapidity. Isotropic thermal equilibrium distributions (dashed curves) are too narrow to fit the data. Agreement is achieved through the nonequilibrium solutions in the RDM, solid curves. The fit values of the diffusion coefficients exceed the ones obtained from the dissipation-fluctuation theorem.

Fig.4. Calculated rapidity density spectra of participant protons in central S+Au, S+S and Pb+Pb- collisions at SPS- energies in comparison with NA 35 and NA 49 data (black squares; data for symmetric systems reflected at $y_{eq} = y_1/2$). The projectile rapidities are $y_1 = 6.06$ for S, and $y_1 = 5.83$ for Pb. For Pb+Pb the thermal equilibrium distribution (T=160 MeV) is also shown (dashed).



158 A GeV/c ^{208}Pb + Pb
$t / \tau_Y = 0.1$
Protons
$\to \infty$
0.2
1.1
0.6
1
dN (Y,t) / dY
200
150
100
50
0
−1
0
+ 1
Y
0
2
4
6
y

Fig.2.



Protons · SIS - energies
1.06 A GeV
Ni + Ni
1.45 A GeV
1.93 A GeV
dn / dY
20
0
- 1
0
+ 1
Y
y
y 1

FIG. 1.

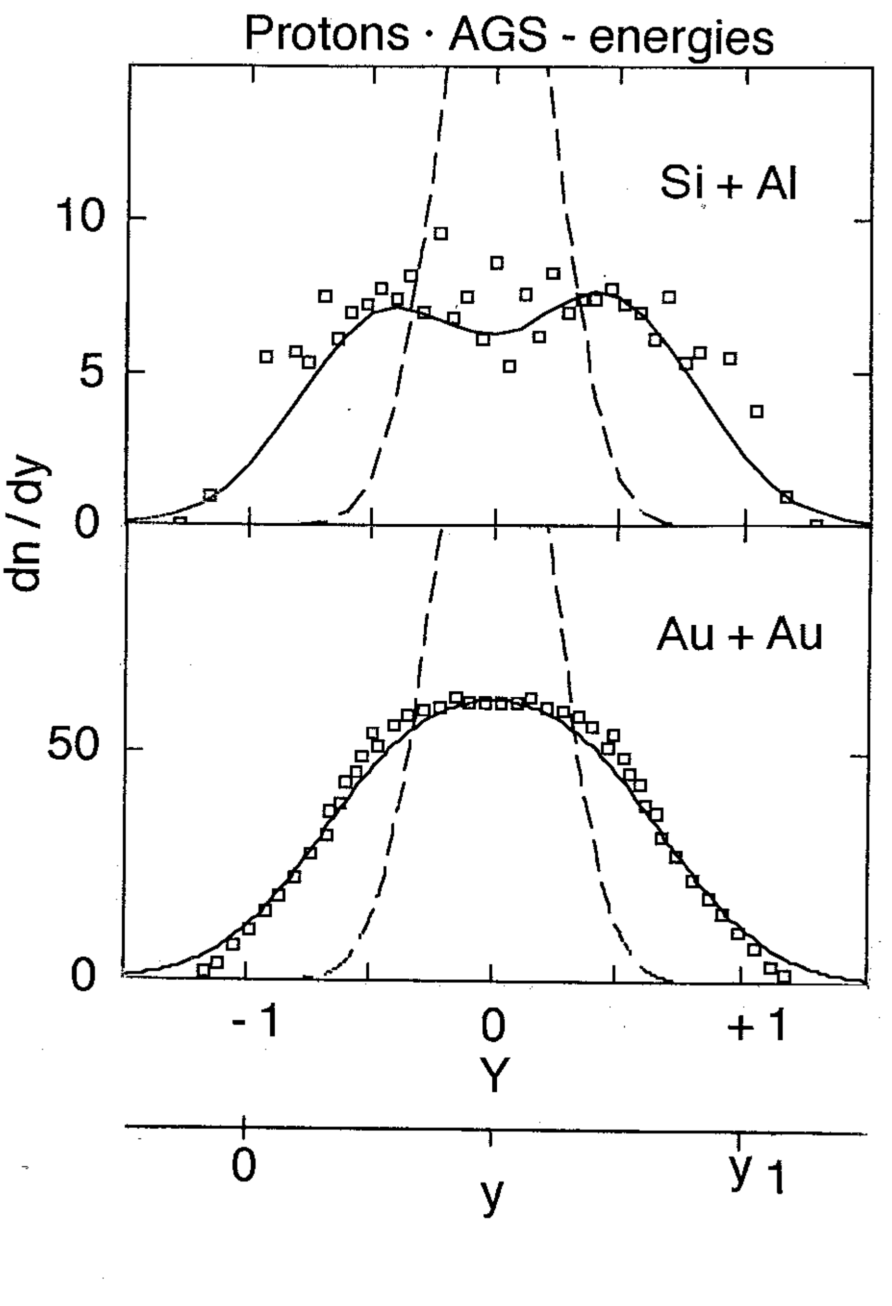


FIG. 3.